Parameter Estimation of Near-field Sources Based On Fourth-order Cumulants

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Abstract. An algorithm based on fourth-order cumulants is presented for jointing two-dimensional arrival angles and range estimation of incoming non-Gaussian narrow-band near-field sources. It can give the signal parameter estimation without any spectral searching and only needs five guiding sensors configured in a certain position without any constrains on other elements of the array. The algorithm can be applied to any Gaussian noise environment in the case of fourth-order cumulants. Simulation results show that the proposed algorithm is effective.

Keywords: near-field source, fourth-order cumulants, two-dimensional arrival angles, range.

1. Introduction

Array signal parameter estimation is one of the important researches in array signal processing. The usual array signal parameter estimation methods assume the signal source is located in the far field. Thus the received signals are before the plane wave. In this way the position of the signal sources is determined by the two-dimensional angle of arrival. When the signal sources fall in Fresnel area of array aperture, which means in the near field, assuming a plane wave front will no longer be set up. At this point spherical wave required to come to be described for near-field source. Location of the signal source needs to be determined by jointing two-dimensional arrival angles and range estimation. Near-field source localization has important application value in many places, such as sonar, voice source localization and separation, electronic surveillance, seismic exploration and so on.

Challa and Shamsunder first introduced fourth-order cumulants into arrival angle and range estimation [1], it could be used in any Gaussian noise environment without searching. However, it takes advantage of the central symmetric structure of the array, with large loss of array aperture. On this basis Chen Jianfeng etc. introduced frequency estimation to form a joint estimation algorithm of near-field source's range, frequency and degree of arrival [2]. Wu Yuntao proposed another joint estimation algorithm [3], compared with literature [2] it avoided array aperture loss since without using central symmetrical structure. This paper proposed an algorithm based on fourth-order cumulants for jointing two-dimensional arrival angles and range estimation of incoming non-Gaussian narrow-band near-field sources.

2. Data model

Consider P near-field sources are incident on M array elements antenna with five omnidirectional antenna elements and (M-5) arbitrary array element. The location of five omnidirectional antenna elements is shown in Figure 1. The spacing of array element $u_1(t)$ and the other array elements is d.

The output signal of antenna array can be expressed:

$$\boldsymbol{u}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) = \sum_{k=1}^{P} \boldsymbol{a}_k s_k(t) + \boldsymbol{n}(t) \tag{1}$$

Where, $\boldsymbol{u}(t) = [u_1(t), \cdots, u_M(t)]^T$ is m output array vectors. $\boldsymbol{A} = [\boldsymbol{a}_1, \cdots, \boldsymbol{a}_P]$ is $M \times P$ dimensional steering vector matrix, and $\boldsymbol{a}_i = [a_{1i}, \cdots, a_{Mi}]^T, i = 1, \cdots P$ is the steering vector. $\boldsymbol{s}(t) = [s_1(t), \cdots, s_P(t)]^T$ is the source vector, and $s_k(t) = E_k e^{j(\omega_k t + \psi_k)}$ stands for kth input signal, E_k is amplitude, $\omega_k = 2\pi f_k$ is angular frequency of signal, ψ_k is the initial phase. Noise vector is $\boldsymbol{n}(t) = [n_1(t), \cdots, n_M(t)]^T$.

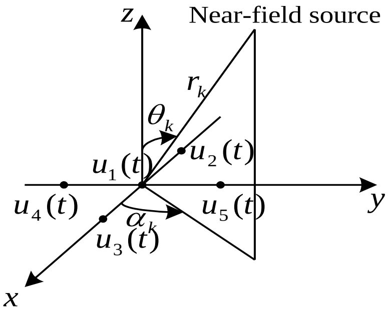

Fig 1. Structure of Array

Assume that unit length of axis is d, the coordinates of (x,y) is (m,n) ($m,n \in Z$), where is an array element. If reference array element is coordinate origin, the output of coordinates (m,n) is:

$$u_{m,n}(t) = \sum_{k=1}^{P} s_k(t) e^{j\tau_{m,n}(k)} + n_{m,n}(t) \tag{2}$$

Where $s_k(t)$ is narrow-band signal, $n_{m,n}(t)$ is noise, $\tau_{m,n}(k)$ is the phase difference of array element in (m,n) and coordinate origin. If the signal sources fall in Fresnel area of array aperture, $\tau_{m,n}(k)$ can be denoted as approximately[4]:

$$\tau_{m,n}(k) \approx \gamma_{xk} m + \phi_{xk} m^2 + \gamma_{yk} n + \phi_{yk} n^2 + \beta_k mn \tag{3}$$

Where, $$\gamma_{xk} = -2\pi \frac{d}{\lambda_k} \sin\theta_k \cos\alpha_k \tag{4}$$

$$\phi_{xk} = \pi \frac{d^2}{\lambda_k r_k}(1 - \sin^2\theta_k \cos^2\alpha_k) \tag{5}$$

$$\gamma_{yk} = -2\pi \frac{d}{\lambda_k} \sin\theta_k \sin\alpha_k \tag{6}$$

$$\phi_{yk} = \pi \frac{d^2}{\lambda_k r_k}(1 - \sin^2\theta_k \sin^2\alpha_k) \tag{7}$$

$$\beta_k = -\frac{\pi d^2}{\lambda_k r_k} \sin^2\theta_k \sin 2\alpha_k \tag{8}$$

And α_k, θ_k, r_k is azimuth, elevation and range of the kth input signal respectively, as shown in Figure 1. λ_k is the wavelength corresponding to signal with frequency f_k.

3. About algorithm

In order to estimate the parameter of two-dimensional arrival angles and range of near-field sources, we construct six fourth-order cumulants matrix as follows:

$$\boldsymbol{C}_0 \overset{\Delta}{=} cum(u_1(t), u_1^*(t), \boldsymbol{u}(t), \boldsymbol{u}^H(t)) = \boldsymbol{A}\Lambda\boldsymbol{A}^H \tag{9}$$

$$\boldsymbol{C}_1 \overset{\Delta}{=} cum(u_1(t), u_2^*(t), \boldsymbol{u}(t), \boldsymbol{u}^H(t)) = \boldsymbol{A}\Lambda\Phi_1\boldsymbol{A}^H \tag{10}$$

$$\boldsymbol{C}_2 \overset{\Delta}{=} cum(u_3(t), u_2^*(t), \boldsymbol{u}(t), \boldsymbol{u}^H(t)) = \boldsymbol{A}\Lambda\Phi_2^H\boldsymbol{A}^H \tag{11}$$

$$\boldsymbol{C}_3 \overset{\Delta}{=} cum(u_1(t), u_4^*(t), \boldsymbol{u}(t), \boldsymbol{u}^H(t)) = \boldsymbol{A}\Lambda\Phi_3^H\boldsymbol{A}^H \tag{12}$$

$$\boldsymbol{C}_4 \overset{\Delta}{=} cum(u_5(t), u_4^*(t), \boldsymbol{u}(t), \boldsymbol{u}^H(t)) = \boldsymbol{A}\Lambda\Phi_4^H\boldsymbol{A}^H \tag{13}$$

$$\boldsymbol{C}_5 \overset{\Delta}{=} cum(u_1(t), u_1^*(t+1), \boldsymbol{u}(t), \boldsymbol{u}^H(t)) = \boldsymbol{A}\Lambda\Phi_5^H\boldsymbol{A}^H \tag{14}$$

It can be seen from formula (9) to (14) that the generalized eigenvalues of matrix pair $(\boldsymbol{C}_0,\boldsymbol{C}_1)$, $(\boldsymbol{C}_0,\boldsymbol{C}_2)$, $(\boldsymbol{C}_0,\boldsymbol{C}_3)$, $(\boldsymbol{C}_0,\boldsymbol{C}_4)$, $(\boldsymbol{C}_0,\boldsymbol{C}_5)$ is the diagonal element of Φ_1, Φ_2, Φ_3, Φ_4, Φ_5. Thus the generalized eigenvalues can be solved through egienvalue decomposition to matrix pair, which can obtain the values of $(\gamma_{xk},\varphi_{xk},\gamma_{yk},\varphi_{yk})$.

Because Φ_1, Φ_2, Φ_3, Φ_4, Φ_5 are characterized by five independent generalized decompositions, five generalized eigenvalues may be arranged differently. So we need to pair five generalized eigenvalues using the correlation of each generalized vector.

Thus the values of (α_k,θ_k,r_k) can be solved with equation (4) to (7). In this way the parameter of two-dimensional arrival angles and range of the near-field source can be solved via the generalized eigenvalues of the pairing matrix. The process is expressed as follows:

$$\alpha_k = \arctan(\gamma_{yk}/\gamma_{xk}),\quad \theta_k = \arcsin\left(\frac{\lambda_k}{2\pi d}\sqrt{\gamma_{xk}^2+\gamma_{yk}^2}\right),\quad r_k = (r_{xk}+r_{yk})/2 \tag{15}$$

Where, $\gamma_{xk} = -\arg(\hat{\Phi}_2(k,k))/2$, $\phi_{xk} = (2\arg(\hat{\Phi}_1(k,k)) - \arg(\hat{\Phi}_2(k,k)))/2$, $\gamma_{xy} = -\arg(\hat{\Phi}_4(k,k))/2$, $\phi_{xy} = (2\arg(\hat{\Phi}_3(k,k)) - \arg(\hat{\Phi}_4(k,k)))/2$, $r_{xk} = \dfrac{(2\pi d)^2-(\lambda_k\gamma_{xk})^2}{4\pi\lambda_k\phi_{xk}}$, $r_{yk} = \dfrac{(2\pi d)^2-(\lambda_k\gamma_{yk})^2}{4\pi\lambda_k\phi_{yk}}$.

$\hat{\Phi}_1(k,k)$, $\hat{\Phi}_2(k,k)$, $\hat{\Phi}_3(k,k)$, $\hat{\Phi}_4(k,k)$, $\hat{\Phi}_5(k,k)$, $k=1,\cdots P$ is the generalized eigenvalues of the pairing matrix $(\boldsymbol{C}_0,\boldsymbol{C}_1)$, $(\boldsymbol{C}_0,\boldsymbol{C}_2)$, $(\boldsymbol{C}_0,\boldsymbol{C}_3)$, $(\boldsymbol{C}_0,\boldsymbol{C}_4)$, $(\boldsymbol{C}_0,\boldsymbol{C}_5)$ respectively.

4. Simulation

Consider a cross array build up by ULA, which of the element number is 9. The spacing of array element is $\lambda/4$. One spatial signal with the normalized frequency of 0.5 arrives at the array in the position of $\alpha=30^\circ$, $\theta=60^\circ$, $r=0.6\lambda$. 2000 snapshots.

In order to verify the performance of the proposed algorithm, we use the root mean square error (RMSE) as performance criteria:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{\alpha}-\alpha)^2}$$

Where, N is number of experiments, $\hat{\alpha}$ is the estimated value, α is the actual value.

SNR changes from 0dB to 30dB, we calculate it every 5dB. The results of simulation are shown in Figure 2.

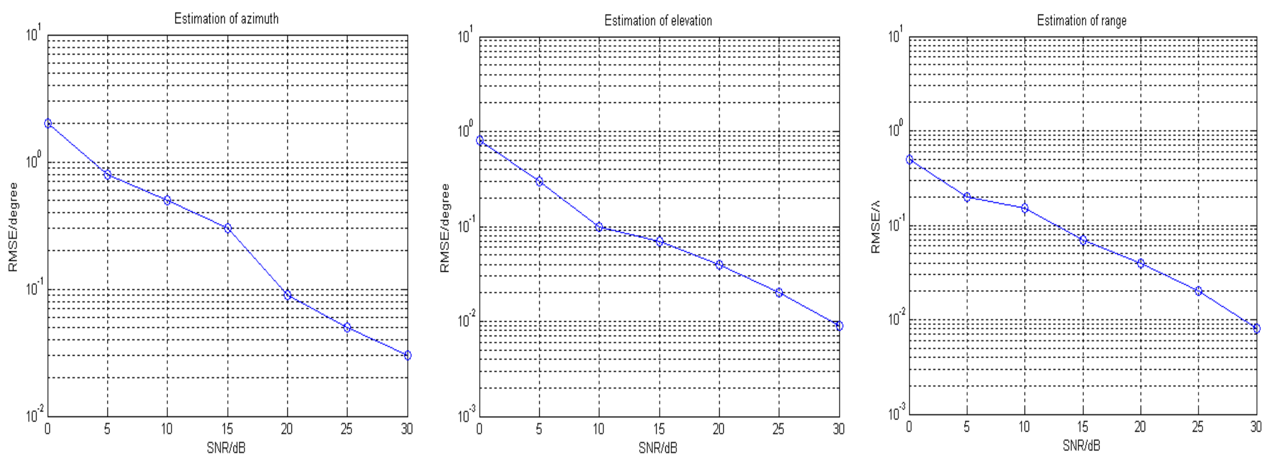

Fig. 2 Relation of SNR and RMSE

From the figure, we can see that the root mean square error of azimuth, elevation and range of near-field source decrease rapidly as the SNR increases. And in the case of lower SNR, it still has a smaller mean square error, which confirms the validity of the proposed algorithm.

5. Conclusion

This paper proposed an algorithm based on fourth-order cumulants for jointing two-dimensional arrival angles and range estimation of incoming non-Gaussian narrow-band near-field sources. The algorithm firstly constructs six fourth-order cumulant matrices, then computes the generalized eigenvalues of the five matrix pairs formed by the six constructed matrices. We can get several medium parameters after pairing the generalized eigenvalues. The parameters of near-field sources will be solved from the medium parameters. Simulation results show that the proposed algorithm is effective.

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