# Temperature Stress Analysis of Building Roofs

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**Abstract.** Stress distribution of roof in three working conditions is analyzed in this paper, the results show that stress generated by temperature load is about 3 times larger than stress generated by the vertical load, the maximum stress is discovered in the 45 °area of corner, and radiation steel should be added to corner in roof design.

**Keywords:** roof; temperature load; stress distribution

## 1. Introduction

In most areas of China, temperature range is large in summer, the effect of temperature load on the roof is often ignored, and cracks can often be discovered in the use process. There are many studies on the roof temperature stress. The insulation performance of roof was analyzed in the literature [1-4], and the method to reduce the temperature load was put forward. The factor which can influence roof temperature stress was analyzed in the literature [5-6], and the method to control the roof cracks was put forward. In this paper, by using ANSYS software, roof stress distribution of three working conditions is analyzed, and the corresponding measures of crack control are proposed.

## 2. Theoretical analysis of temperature stress

### 2.1 Temperature stress equation

The temperature T can be composed of the following partial differential equation:

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{W_0}{c\rho}$$

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

In the equation, ρ indicates density, kg/m$^3$; c indicates specific heat, kJ/(kg· ℃), λ indicates thermal conductivity, kJ/(m·h· ℃); α indicates coefficient of temperature conductivity, the equation is:

$$\alpha = \frac{\lambda}{c\rho}$$

Link the heat dq to the temperature gradient $\frac{\partial \mathrm{T}}{\partial \mathrm{n}}$, the equation is:

$$dq = -\lambda \cdot \frac{\partial T}{\partial n} \cdot dF \cdot dt$$

If the temperature along the Z direction is constant, the temperature field is a plane field, so the heat conduction equation can be simplified as:

$$\frac{\partial T}{\partial t} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{W_0}{c\rho}$$

If the temperature along the Y and Z direction is all constant, then the heat conduction equation can be simplified as:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + \frac{W_0}{c\rho}$$

If the temperature is independent of time, then the distribution of temperature is a three-dimensional space function. In order to simplify the calculation, the most unfavorable temperature state at a certain stage of the instable state is often calculated as stable state, the partial differential equation is:

$$\frac{W_0}{\lambda} + \nabla^2 T = 0$$

When the heat source can be neglected, $W_0$=0, the equation is:

$$\nabla^2 T = 0$$

**2.2 Initial and boundary conditions**

The initial condition is that the initial temperature is assumed as 0℃.

The boundary condition is:

(1) $t=t_1$,the temperature distribution of structure surface is known:

$$T(x,y,z,t_1) = f(x,y,z,t_1)$$

In this paper, the outdoor temperature is assumed as 40℃ , and the indoor temperature is assumed as 26℃.

(2) $t=t_1$, the heat transfer from the environment to the structure surface is known:

$$(\frac{\partial T}{\partial n})_0 = \frac{k}{\lambda}(T_b - T_a)$$

In the equation, $T_b$ indicates structure surface temperature, $T_a$ indicates environment temperature.

In this paper, the outdoor heat transfer coefficient is 19W/ ($m^2$· ℃), the indoor heat transfer coefficient is 8.7 W/ ($m^2$· ℃)

## 3. Numerical analysis of temperature stress

### 3.1 Roof model

(1) Element model

The element model is Solid 45. It is a dimensional solid element, and each node has three degrees of freedom along the xyz direction.

(2) Model properties

Floor: concrete strength grade is C25, and reinforcement ratio is 0.6% (two directions respectively 0.3%); beam: concrete strength grade is C30, and reinforcement ratio is 0.6%; column: concrete strength grade is C30, and reinforcement ratio is 2%. The constitutive relation of concrete is MISO (Multi-linear isotropic hardening mode), and the constitutive relation of reinforcement is BISO (bi-linear isotropic hardening model). Material parameters are shown in Table 1.

Table 1 Material parameters

| Material | $E_c$ | $f_c$ (N/mm$^2$) | $f_t$ (N/mm$^2$) |
|---|---|---|---|
| C25 | $2.8\times10^{10}$ | $11.9\times10^6$ | $1.27\times10^6$ |
| C30 | $3.0\times10^{10}$ | $14.3\times10^6$ | $1.43\times10^6$ |

(3)Model sizes

Floor: 6m×6m×0.12m, beam:0.3m×0.6m×6m, column:0.5m×0.5m×1.8m.

(4)Loads and boundary conditions

After calculation, vertical load is 2.6kN/$m^2$. According to “Design standard for energy efficiency of residential buildings in hot summer and cold winter area”, the outdoor heat transfer coefficient is 19W/ ($m^2$•℃), and the indoor heat transfer coefficient is 8.7W/ ($m^2$•℃); the outdoor temperature is assumed as 40℃ , and the indoor temperature is assumed as 26℃.

The roof is only applied vertical load in condition 1, and it is only applied temperature load in condition 2, and two loads are applied in condition 3. Three working conditions are shown in Table 2.

Table 2 Three working conditions

| Condition | Vertical load (kN/m$^2$) | Temperature load (℃) |
|---|---|---|
| Condition 1 | 2.6 | 0 |
| Condition 2 | 0 | 14 |
| Condition 3 | 2.6 | 14 |

### 3.2 Analysis of results

As is shown in Fig. 1 and Fig. 2, in condition 1, the tensile stress of roof corner is smaller, the maximum tensile stress is discovered in the edge of beam, and the top surface of center is in the condition of compression, but the bottom surface is all in the condition of tension, and it is the same to the condition of reinforcement calculation in the engineering. As is shown in Fig. 3 and Fig. 4, in condition 2, the maximum tensile stress is discovered in the 45 ° area of corner, and the bottom surface is also in the condition of tension, so the cracks appeared in engineering is through cracks. As is shown in Fig. 5 and Fig. 6, in condition 3, the stress in the edge of beam and in the 45 °area of corner is larger, especially the corner.

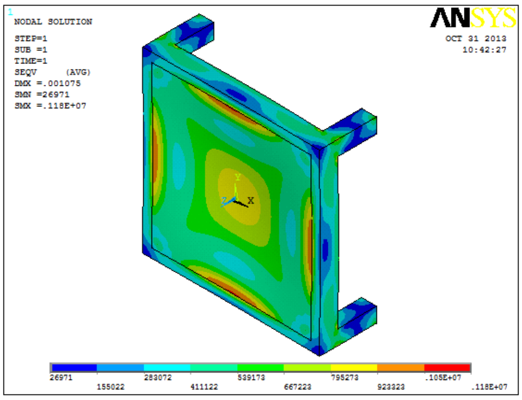

Fig.1 Stress distribution in condition 1

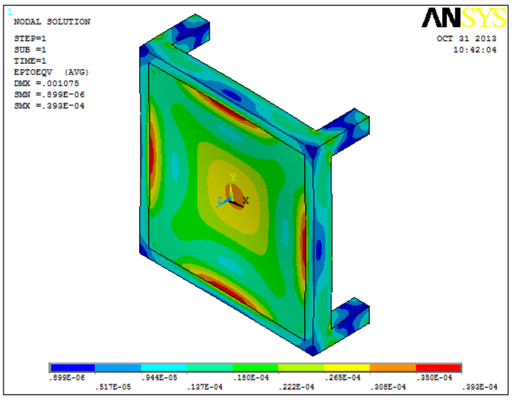

Fig.2 Strain distribution in condition 1

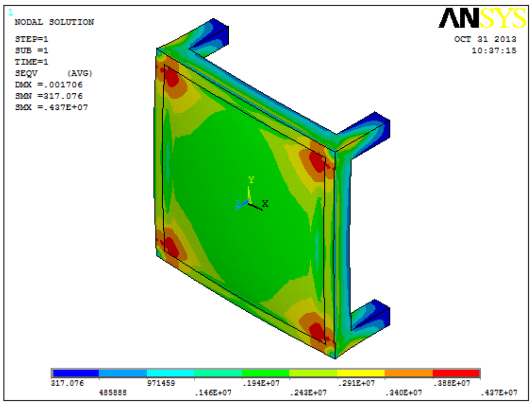

Fig.3 Stress distribution in condition 2

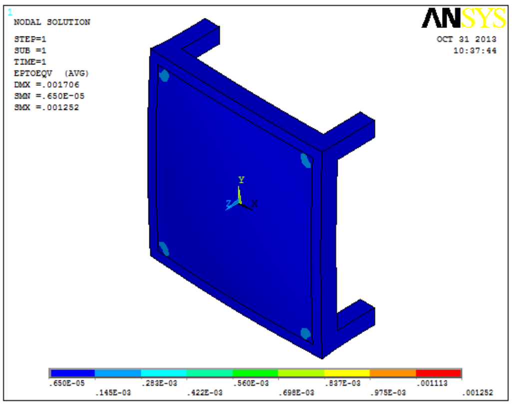

Fig.4 Strain distribution in condition 2

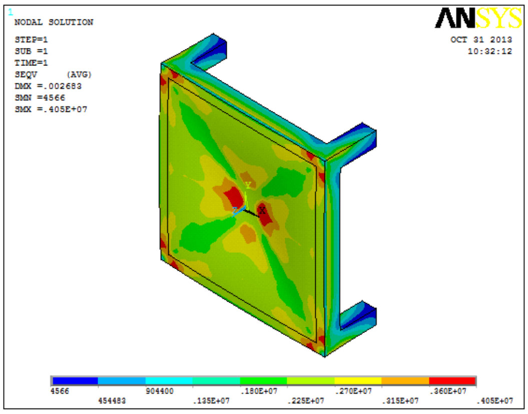

Fig.5 Stress distribution in condition 3

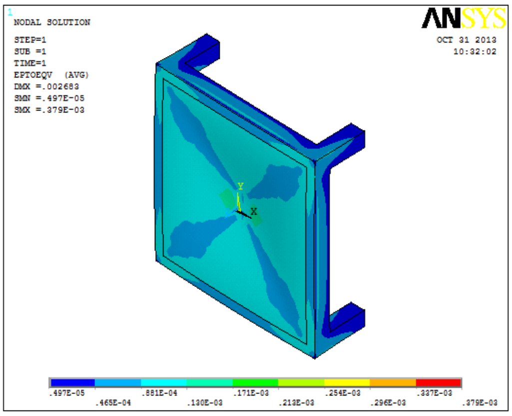

Fig.6 Strain distribution in condition 3

Table 3 Maximum value of stress and strain

| Condition | Condition 1 | Condition 2 | Condition 3 |
|---|---|---|---|
| Stress(Pa) | $0.118\times10^{7}$ | $0.437\times10^{7}$ | $0.405\times10^{7}$ |
| Strain(m) | $0.768\times10^{-4}$ | $0.256\times10^{-3}$ | $0.379\times10^{-3}$ |

After calculation, in condition 1, the maximum stress value in the edge of beam is 0.118 ×107Pa; in condition 2, the maximum stress value in the 45 °area of corner is 0.437 ×107Pa, the maximum stress is larger in condition 2, so stress generated by temperature load is about 3 times larger than stress generated by the vertical load. The maximum value of stress and strain is shown in Table 3.

**3.3 Optimization of structure**

Crack is easier developed in the 45° area of corner in condition 3. In structural design, the insulation performance of roof should be strengthened to reduce the temperature load, and radiation steel should be added to the 45° area of corner to reduce the crack. The arrangement of radiation steel is shown in Fig. 7.

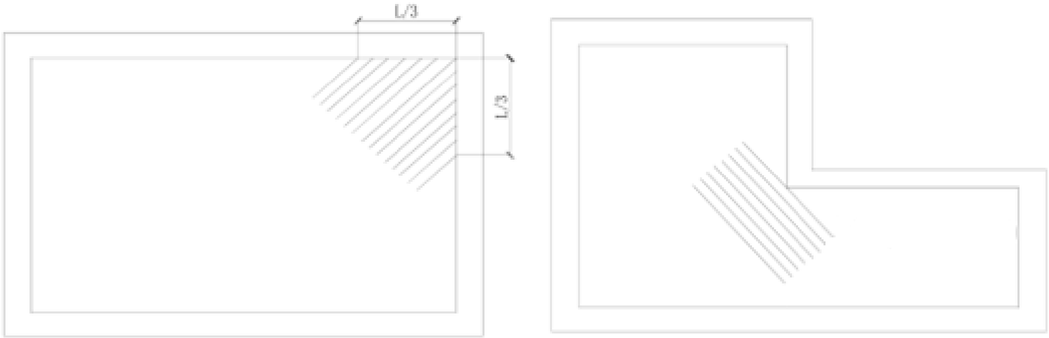


Fig.7 Arrangement of radiation steel

## 4. Conclusion

Stress and strain distribution of roof under the vertical load and temperature load is analyzed in this paper, and the results show that stress generated by temperature load is about 3 times larger than stress generated by the vertical load, and radiation steel should be added to the 45 ° area of corner to reduce the crack.

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