

Game analysis of decision behavior evolution of reverse supply chain enterprises under detection error

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Abstract

The implementation of reverse supply chain can realize the harmonious development of economy and ecological environment, and is an effective way for recycling enterprises to fulfill their social responsibilities. However, the detection error in the recycling process has caused a waste of reproducible resources to a certain extent. In this paper, evolutionary game theory is used to establish an evolutionary game model for remanufacturers and recyclers to reduce the detection error behavior under the cost sharing mechanism, and the model is solved, analyzed and numerically simulated to draw the corresponding research conclusions and management enlightenment. The results show that reducing the work cost of both sides of the game is conducive to the adjustment of the evolution path to a positive win-win situation; In different cases, there is an optimal cost sharing ratio to promote the cooperation between the two sides of the game.

Keywords

Reverse supply chain, Detection error, Cost sharing contract, Evolutionary game.

1. Introduction

With the development of the Internet recycling platform, many large and capable enterprises such as Gree and other manufacturers or remanufacturers began to build their own online recycling channels. However, in the process of implementing the reverse supply chain, many enterprises have negative behaviors: on the one hand, some reverse supply chain enterprises do not handle waste products according to environmental protection standards, and only extract things with high value, which leads to low disassembly costs and high profits, resulting in serious pollution problems in the recovery process; On the other hand, there are two kinds of errors in the process of recycling and testing of waste products. One is the error of treating non remanufactured products as "remanufactured products" (where "remanufactured products" include some products that are cannot be actually remanufactured). The other is the error of regarding remanufactured products as "non remanufactured products" (where "non remanufactured products" include some products that can be actually remanufactured). The existence of these two kinds of errors has greatly reduced the recovery efficiency of enterprises, but also caused a waste of recyclable resources to a certain extent^[1].

Without corresponding measures to supervise them, the relevant enterprises will hide the recovery information, reduce the recovery requirements, reduce the cost of disassembly and processing, and obtain high profits under the drive of interests. When remanufacturers carry out online recycling and recyclers carry out offline recycling, they need to pay a certain amount of cost to reduce the recovery detection error, whether by means of personnel training or equipment updating. If they pay more costs but receive less revenue, their participation enthusiasm will be affected to some extent, and even the phenomenon of "free riding" will occur. Therefore, in order to solve the above problems, the reverse supply chain needs to introduce a

coordination mechanism to promote member enterprises to avoid hitchhiking while fulfilling their corporate responsibilities.

In terms of supply chain coordination, Savaskan et al. [2] discussed the management coordination of reverse logistics and forward logistics from the perspective of household appliance manufacturers. Govindan and Popiuc [3] found that the revenue sharing mechanism can effectively solve the coordination problem of the two-stage and three-stage supply chains. Lei Wang et al. [5] established a tripartite revenue sharing mechanism to achieve Pareto optimality of the model. Saha et al. Fudong Wang and Meihua Zhou [6] adopted a penalty contract for supply chain coordination in the case of third-party logistics participation. In addition, Chen et al. [7] used a combination of two pricing mechanisms and profit sharing mechanisms to achieve supply chain coordination. Huaizhen Yang and Gaoda Lu [8] considered transaction credit on the basis of revenue sharing contract strategy.

According to the existing literature, the two-tariff pricing mechanism and revenue sharing mechanism are often used to solve the coordination problem of supply chain members. From the perspective of cost sharing, this paper proposes a new coordination and cooperation mode, considers the bounded rationality of supply chain members, and establishes an evolutionary game model to analyze whether the online remanufacturer recovery testing center and offline recycler recovery testing center can cooperate effectively under the cost sharing mechanism to reduce the detection error in the recovery link in the reverse supply chain operation process. At the same time, this paper uses Matlab software to carry out simulation research, which directly reflects the changes of evolution tracks under different circumstances. To explore its evolution mechanism and influencing factors, and then put forward corresponding countermeasures and suggestions.

2. Model construction

In the process of recycling waste products, the recycling enterprise has the responsibility to invest special costs for special rectification to reduce the waste of resources. As the member units of the reverse supply chain, remanufacturers and recyclers have their optimal strategies under decentralized decision-making. Therefore, remanufacturers and recyclers may not necessarily consider their social responsibilities when making separate decisions, and may be reluctant to reduce the detection error, thus failing to achieve Pareto optimization of the reverse supply chain as a whole. In this case, this paper establishes the following evolutionary game model.

2.1. Evolutionary Game Model Assumptions

Assumption 1: Both the remanufacturer and the recycler have two strategies (reduce, not reduce) to choose. Assume that the probability of the remanufacturer to reduce the recovery detection error is x , and the probability of not reducing the recovery detection error is $1-x$; The probability of the recycler to reduce the recovery detection error is y , and the probability of not reducing the recovery detection error is $1-y$.

Assumption 2: If both the remanufacturer and the recycler choose not to reduce the recovery detection error, it means that when both sides of the game make a negative choice, the profits of both sides remain unchanged, it means that the original profit level when making independent decisions is maintained. Assume that the remanufacturer's profit level is π_m and the recycler's profit level is π_r .

Assumption 3: If both parties take a positive choice, it means that both the remanufacturer and the recycler choose to rectify to reduce the recovery detection error. At this time, for the remanufacturers and recyclers who make positive decisions, the unnecessary costs caused by

the existence of detection errors will be effectively reduced, so the enterprise profits will rise to a certain extent. Suppose that the profit growth rate of the remanufacturers is μ , and the profit growth rate of the recyclers is λ .

2.2. Establishment of evolutionary model of remanufacturer and recycler

Based on the above assumptions, the game payment matrix of remanufacturer and recycler can be obtained, as shown in the table 1.

Table1 Remanufacturer and recycler game payment matrix

Remanufacturer	Recycler	
	Reduce	Not reduce
Reduce	$(1+\mu)\pi_m - \eta c_c$	$(1+\mu)\pi_m - c_c$
	$(1+\lambda)\pi_r - (1-\eta)c_c$	π_r
Not reduce	π_m	π_m
	$(1+\lambda)\pi_r - c_c$	π_r

Table 1 shows that the expected utility of the remanufacturer when adopting the strategy of reducing detection error is:

$$W_1 = y[(1+\mu)\pi_m - \eta c_c] + (1-y)[(1+\mu)\pi_m - c_c]$$

The expected utility when the remanufacturer does not adopt the strategy of reducing detection error is:

$$W_2 = y\pi_m + (1-y)\pi_m$$

The average utility of the remanufacturer is:

$$\bar{W} = x \cdot W_1 + (1-x) \cdot W_2$$

According to evolutionary game theory, the dynamic equation of game replication for remanufacturer strategy selection is:

$$F(x) = \frac{dx}{dt} = x(1-x)(W_1 - W_2) = x(1-x)\{\mu\pi_m + [(1-\eta)y - 1]c_c\}$$

The expected utility of the recycler when adopting the strategy of reducing detection error is:

$$G_1 = x[(1+\lambda)\pi_r - (1-\eta)c_c] + (1-x)[(1+\lambda)\pi_r - c_c]$$

The expected utility of the recycler when adopting the strategy of not reducing the detection error is:

$$G_2 = x\pi_r + (1-x)\pi_r$$

The average utility of the recycler is:

$$\bar{G} = y \cdot G_1 + (1-y) \cdot G_2$$

According to evolutionary game theory, the dynamic equation of game replication for the recycler's strategy selection is:

$$H(y) = \frac{dy}{dt} = y(1-y)(G_1 - G_2) = y(1-y)[\lambda\pi_r - (1-x\eta)c_c]$$

Unilever $F(x)$ and $H(y)$ can obtain the two-dimensional replication powertrain of remanufacturers and recyclers. The Jacobian matrix of the system is:

$$J = \begin{pmatrix} \frac{\partial F(x, y)}{\partial x} & \frac{\partial F(x, y)}{\partial y} \\ \frac{\partial H(x, y)}{\partial x} & \frac{\partial H(x, y)}{\partial y} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

The specific expression of $a_{11}, a_{12}, a_{21}, a_{22}$ in Jacobian matrix is shown in Table 2:

Table 2 Expressions of factors in Jacobian matrix

Symbol	expression
a_{11}	$(1-2x)\{\mu\pi_m + [(1-\eta)y-1]c_c\}$
a_{12}	$x(1-x)(1-\eta)c_c$
a_{21}	$y(1-y)\eta c_c$
a_{22}	$(1-2y)[\lambda\pi_r - (1-x\eta)c_c]$

Let $\begin{cases} F(x) = 0 \\ H(x) = 0 \end{cases}$, get the five evolution equilibrium points of the replication dynamic system,

named $A(0,0), B(0,1), C(1,0), D(1,1), E(x^*, y^*)$, and $x^* = \frac{c_c - \lambda\pi_r}{\eta c_c}, y^* = \frac{c_c - \mu\pi_m}{(1-\eta)c_c}$.

3. Stability Analysis of Evolutionary Game

The local stability of the evolutionary system can be obtained by analyzing the Jacobian matrix. The determinant of the matrix is:

$$\begin{aligned} Det(J) &= a_{11} \cdot a_{22} - a_{12} \cdot a_{21} \\ &= (1-2x)\{\mu\pi_m + [(1-\eta)y-1]c_c\} \cdot (1-2y)[\lambda\pi_r - (1-x\eta)c_c] \\ &\quad + x(1-x)(1-\eta)c_c \cdot y(1-y)\eta c_c \end{aligned}$$

The trace of the matrix is:

$$\begin{aligned} Tr(J) &= a_{11} + a_{22} \\ &= (1-2x)\{\mu\pi_m + [(1-\eta)y-1]c_c\} + (1-2y)[\lambda\pi_r - (1-x\eta)c_c] \end{aligned}$$

The trace and determinant of each equilibrium point are obtained by bringing in the value of the equilibrium point, as shown in Table 3:

Table 3 Determinants and traces of each equilibrium point

equilibrium point	$Det(J)$	$Tr(J)$
$A(0,0)$	$(\mu\pi_m - c_c)(\lambda\pi_r - c_c)$	$(\mu\pi_m - c_c) + (\lambda\pi_r - c_c)$
$B(0,1)$	$-(\mu\pi_m - \eta c_c)(\lambda\pi_r - c_c)$	$(\mu\pi_m - \eta c_c) - (\lambda\pi_r - c_c)$
$C(1,0)$	$-(\mu\pi_m - c_c)[\lambda\pi_r - (1-\eta)c_c]$	$\lambda\pi_r - \mu\pi_m + \eta c_c$
$D(1,1)$	$(\mu\pi_m - \eta c_c) \cdot [\lambda\pi_r - (1-\eta)c_c]$	$-\mu\pi_m - \lambda\pi_r - c_c$
$E(x^*, y^*)$	$-\eta(1-\eta)c_c^2 \cdot x^*(1-x^*)y^*(1-y^*)$	0

When all characteristic roots of the Jacobian matrix corresponding to the equilibrium point are negative, that is, $Det(J) > 0$ and $Tr(J) < 0$, the equilibrium point is the evolutionary stability

strategy (ESS) of the evolutionary game system. According to the judgment conditions of the equilibrium point, the stability of the equilibrium point can be obtained under the following four conditions. See Table 4 to Table 7 for details:

Table 4 Local stability of equilibrium point under different conditions

equilibrium point	A(0,0)	B(0,1)	C(1,0)	D(1,1)	$E(x^*, y^*)$
① $\mu\pi_m > c_c, \lambda\pi_r > c_c$					
$Det(J)$	+	-	-	+	uncertain
$Tr(J)$	+	uncertain	uncertain	-	0
stability	instable	saddle point	saddle point	ESS	saddle point
② $\mu\pi_m > c_c, (1-\eta)c_c < \lambda\pi_r < c_c$					
$Det(J)$	-	+	-	+	uncertain
$Tr(J)$	uncertain	+	uncertain	-	0
stability	saddle point	saddle point	saddle point	ESS	saddle point
③ $\eta c_c < \mu\pi_m < c_c, \lambda\pi_r > c_c$					
$Det(J)$	-	-	+	+	uncertain
$Tr(J)$	uncertain	uncertain	+	-	0
stability	saddle point	saddle point	saddle point	ESS	saddle point
④ $\eta c_c < \mu\pi_m < c_c, (1-\eta)c_c < \lambda\pi_r < c_c$					
$Det(J)$	+	+	+	+	+
$Tr(J)$	-	uncertain	+	-	0
stability	ESS	saddle point	instable	ESS	center point
⑤ $\mu\pi_m > c_c, \lambda\pi_r < (1-\eta)c_c$					
$Det(J)$	-	+	+	-	uncertain
$Tr(J)$	uncertain	+	-	uncertain	0
stability	saddle point	instable	ESS	saddle point	saddle point
⑥ $\mu\pi_m < \eta c_c, \lambda\pi_r > c_c$					
$Det(J)$	-	+	+	-	uncertain
$Tr(J)$	uncertain	-	+	uncertain	0
stability	saddle point	ESS	instable	saddle point	saddle point
⑦ $\mu\pi_m < \eta c_c, \lambda\pi_r < (1-\eta)c_c$					
$Det(J)$	+	-	-	+	+
$Tr(J)$	-	uncertain	uncertain	+	0
stability	ESS	saddle point	saddle point	instable	center point

It can be found from Table 4 that ESS (1,1) can only be realized when the parameters meet the conditions ①~④. The remanufacturer and the recycler reach an agreement to actively cooperate to reduce the detection error to achieve a win-win situation for both parties. It can be seen that $\mu\pi_m > \eta c_c$ and $\lambda\pi_r > (1-\eta)c_c$ should be satisfied if we want to achieve Pareto optimality of win-win cooperation.

Specifically, after the implementation of the decision to reduce the detection error, the increased profit of the enterprise must be greater than its shared cost, otherwise the enterprise is unwilling to take measures to reduce the detection error, and the two sides of the game will not reach an agreement to reduce the detection error. Therefore, remanufacturers and recyclers should properly evaluate and budget when investing in special costs to reduce detection errors, try to avoid waste of funds, and strive to maximize input and output.

At this time, the profit of the remanufacturer and the recycler from actively reducing the detection error is greater than the profit of their negative measures. Both parties share the cost of personnel training and equipment upgrading to reduce the detection error according to a certain proportion, and at the same time, obtain a higher profit after reducing the detection error. Moreover, this kind of environmental protection behavior reflects the social responsibility of enterprises, which is conducive to forming a good reputation for reverse supply chain enterprises, and also provides more possibilities for game players to achieve higher profit levels after making positive decisions.

In case ④, there are two evolutionary stability strategies (0,0) and (1,1), that is, through repeated games, both players may choose negative decisions to maintain the status quo, or they may also choose positive decisions to achieve win-win results. The specific probability is determined by the area of area S enclosed by point ABC and area S enclosed by point BCD. When $A > B$, both sides of the game are more likely to make decisions. When $B > A$, the probability of both players making positive decisions is high. When $A = B$, the possibility of the system developing to two kinds of decisions is equal.

See Figure 1 for details, where $S_1 = \frac{1}{2} \left(\frac{c_c - \lambda\pi_r}{\eta c_c} + \frac{c_c - \mu\pi_m}{(1-\eta)c_c} \right)$, $S_2 = \frac{1}{2} \left(\frac{\lambda\pi_r - (1-\eta)c_c}{\eta c_c} + \frac{\mu\pi_m - \eta c_c}{(1-\eta)c_c} \right)$.

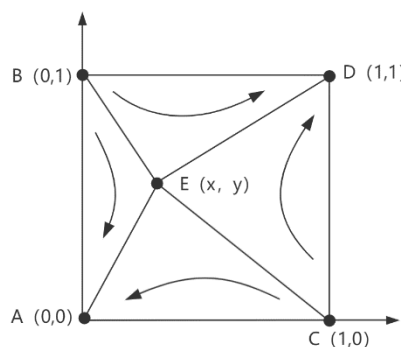


Fig. 1 Evolution trend chart

The following propositions can be obtained through further derivation:

Proposition 1: The smaller the cost of the remanufacturer and the recycler to reduce the detection error, the greater the probability that both sides of the game will make a positive decision.

Proposition 2: When the detection error is reduced, the greater the profit that the enterprise may increase, the greater the probability that both sides of the game will make positive decisions.

It can be known through calculation: $\frac{\partial S_2}{\partial \mu} = \frac{\pi_m}{2(1-\eta)c_c} > 0$, $\frac{\partial S_2}{\partial \lambda} = \frac{\pi_r}{2\eta c_c} > 0$.

Proposition 3: Under different circumstances, both sides of the game will take positive measures to improve their own profits. It can be found by calculating and comparing the profits of different strategies under different circumstances:

When the remanufacturer does not reduce the detection error by taking negative measures, compared with the choice of negative decision, the revenue when the recycler chooses to actively reduce the detection error is: $\pi_1 = \lambda\pi_r - c_c$. When the remanufacturer takes positive measures to carry out environmental protection rectification, compared with negative decisions, the revenue of the recycler when making positive decisions is: $\pi_1 + \Delta_1 = \lambda\pi_r - (1-\eta)c_c$, where $\Delta_1 = \eta c_c$.

When the recycler chooses not to reduce the detection error, compared with the negative decision, the benefit of the remanufacturer's choice of rectification is: $\pi_2 = \mu\pi_m - c_c$. When the recycler chooses to reduce the detection error, compared with no reduction, the benefit of the remanufacturer's choice of positive decision is $\pi_2 + \Delta_2 = \mu\pi_m - \eta c_c$, where $\Delta_2 = (1-\eta)c_c$.

To sum up, $\Delta_1 > 0$ and $\Delta_2 > 0$. Through the above derivation, we can find that under different circumstances, when both sides of the game adopt positive strategies, their benefits are higher. At the same time, this is also the reason why the first four situations reflected in Table 4 above are always stable points (that is, both sides of the game tend to make positive decisions).

Proposition 4: Under different circumstances, the cost sharing proportion η has the corresponding optimal value.

It can be obtained through calculation $\frac{\partial S_2}{\partial \eta} = \frac{c_c - \lambda\pi_r}{2\eta^2 c_c} - \frac{c_c - \mu\pi_m}{2(1-\eta^2)c_c}$, let $\frac{\partial S_2}{\partial \eta} = 0$, $\eta = \left(\sqrt{\frac{c_c - \mu\pi_m}{c_c - \lambda\pi_r}} + 1\right)^{-1}$. It can be seen that the cost sharing ratio η has the corresponding optimal value under different income and cost inputs, so as to promote the consensus between the remanufacturer and the recycler to cooperate to reduce the detection error. Under different circumstances, the implementation of cost sharing mechanism in accordance with the optimal proportion will make the players more inclined to cooperate.

4. Evolutionary simulation analysis

In order to show the decision-making situation of both sides of the game more intuitively, this paper uses MATLAB to simulate different situations, as follows:

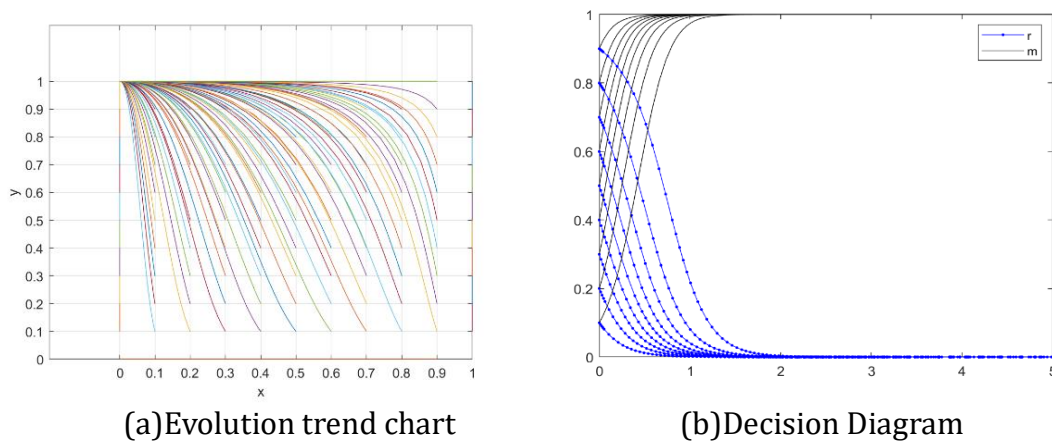


Fig. 2 Simulation image when $c_c > \mu\pi_m > \eta c_c$, $c_c > \lambda\pi_r > (1-\eta)c_c$

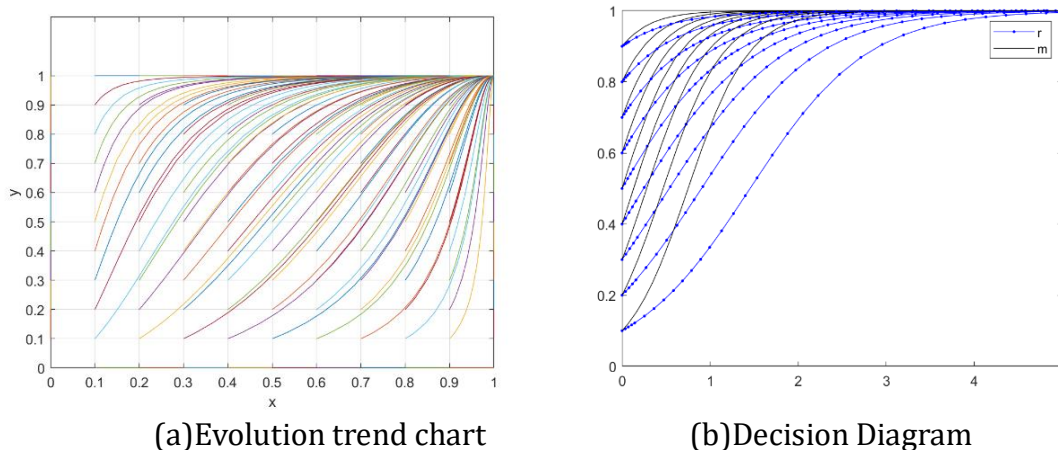


Fig. 3 Simulation image when $c_c < \mu\pi_m, c_c < \lambda\pi_r$

By comparing Figure 2 and Figure 3, it can be found that when the cost used by enterprises to reduce detection errors is high (greater than the new income that enterprises may obtain), recyclers are often unwilling to invest additional costs to reduce detection errors due to small volume and thin profits, so they choose to take negative measures to maintain the status quo. At this time, it is difficult to achieve Pareto optimization of waste product recycling in the entire reverse supply chain. When the cost used by the enterprise to reduce the detection error is controlled within a certain range, the decision-making parties tend to make positive decisions - jointly reduce the recovery detection error in the system, and obtain higher benefits while reducing the waste of resources and sinking costs. Therefore, the lower the work cost of remanufacturers and recyclers when they choose to actively reduce the detection error, the greater the possibility of cooperation between the two parties, whether it is to conduct new learning and training for detection personnel or to upgrade the recovery detection system. Therefore, both decision-making parties need to make joint efforts, plan in advance, constantly optimize the cost budget, minimize unnecessary costs, and achieve the maximum benefit return with the minimum investment.

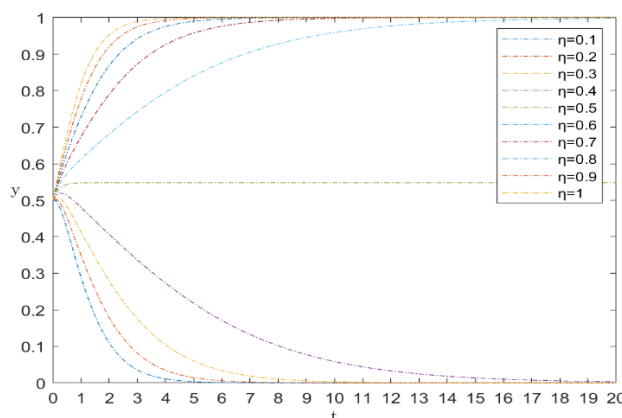


Fig. 4 sensitivity analysis about η

Compared with remanufacturers, recycling stores are scattered and have weak assets. They need to reduce unnecessary cost waste due to the existence of detection errors, but they cannot afford high rectification costs. Therefore, recyclers cannot reduce their recovery detection errors independently. They can only choose to rely on larger and stronger remanufacturers to help them jointly complete the rectification work to reduce the recovery detection errors of the reverse supply chain. Through the simulation in Figure 4, it is found that with the increase of the cost sharing proportion of the remanufacturer, the willingness of the recycler to make positive decisions continues to increase, and only when the cost sharing proportion of the

remanufacturer, the recycler will choose to participate in the rectification work to reduce the detection error.

5. Conclusion and enlightenment

In order to urge the members of the reverse supply chain, the remanufacturer and the recycler, to rectify the detection errors in the recovery process, this paper proposes a coordination mode based on the positive win-win of both parties in the game, establishes a game model of evolution between the remanufacturer and the recycler, obtains the system stability strategy (ESS) through stability analysis, and analyzes the factors that affect the evolution path of both parties through MATLAB simulation, and obtains the following conclusions:

(1) When the profit growth rate obtained by the enterprise after the rectification to reduce the detection error is small and the cost of the enterprise after making positive decisions is large, the system will eventually have two ESS, that is, both sides of the game make positive choices or both make negative choices. Therefore, enterprises can effectively improve the probability of both sides making positive choices at the same time by reducing their rectification costs and improving their work efficiency.

(2) When the cost of the remanufacturer and the recycler to reduce the detection error is smaller, and the profit that the enterprise may increase after the detection error is reduced is larger, the more quickly the game players can evolve into a bilateral positive state. In addition to reducing the silent cost of the recovery system itself, the rectification of the enterprise to reduce the detection error also bears the social responsibility of saving the reproducible resources. Therefore, the government or the media should also publicize scientific recycling knowledge, improve citizens' awareness of environmental protection, promote the growth of recycling of waste products, and increase the probability that both sides of the game can obtain more benefits when making positive decisions, so as to better realize the win-win cooperation of the member enterprises of the reverse supply chain.

(3) When the enterprise controls the input cost within the range that can achieve Pareto optimality for both sides of the game, through sensitivity analysis of the cost sharing ratio, it is found that only when the remanufacturer bears at least 60% of the input cost can the recycler participate in the work of reducing the recovery detection error. As the proportion of remanufacturers increases, recyclers can make positive decisions more quickly.

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References

- [1] Gu Q, Gao T. Management strategy of inspection error rate in reverse supply chain of remanufacturing[J]. Computer Integrated Manufacturing Systems, 2016,22(10).
- [2] R. Canan Savaskan, Shantanu Bhattacharya, Luk N. van Wassenhove. Closed-Loop Supply Chain Models with Product Remanufacturing[J]. Management Science,2004,50(2).
- [3] Govindan K, Popiuc M N. Reverse Supply Chain Coordination by Revenue Sharing Contract: A Case for the Personal Computers Industry [J]. European Journal of Operational Research, 2014, 233(2).
- [4] Wang L, Jing Q U, Liu X. Pricing Strategy and Coordination of Closed-loop Supply Chain Considering Horizontal Fairness under Dual Channel Sales[J]. Industrial Engineering Journal, 2014, 21(3).
- [5] Saha S, Sarmah S P, Moon L. Dual-channel Closed-loop Supply Chain Coordination With a Reward-driven Remanufacturing Policy [J]. International Journal of Production Research, 2016, 54(5).

- [6] Wang F, Zhou M, Management S O. Research on Three-level Supply Chain Coordination Mechanism Based on Participation of Third-party Logistics[J]. Statistics & Decision, 2018, (14).
- [7] Jing C, Hui Z, Ying S. Implementing coordination contracts in a manufacturer Stackelberg dual-channel supply chain[J]. Omega, 2012, 40(5).
- [8] Yang H, Lu G, Business S O. Benefit Coordination of Fresh Agricultural Products Three-level Supply Chain Based on Trade Credit[J]. Statistics & Decision, 2018, (22).