ISSN: 1813-4890

A New Method to Probe the Admissibility Conditions in Symbolic Dynamics of Trimodal Maps

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Abstract

Admissibility conditions played an important role in the study of symbolic dynamics for one-dimension maps, for example, a n-period sequence is called admissible, the simplest proof is that the parameters of iterative maps must be calculated by the word-lifting technique. In this paper, we construct a decision tree to judge if a triply-superstable keading sequence (TSSKS) is admissible, without using the famous admissibility conditions. The key problem is to find a way to transform un-equal length TSSKS to a numerical feature of a data table, by the TSSKS and sort rule of sequence, four logical features are defined as the columns of the data table, the class label is got by the admissibility conditions. The decision tree is constructed by the data table, a higher model score is attained.

Keywords

Symbolic dynamics; Decision tree; Admissibility conditions.

1. Introduction

Symbolic dynamics is an important tool and discipline for the study of dynamic system and chaos. Admissibility conditions are introduced by B.-L. Hao in symbolic dynamics of unimodal maps [1]. These rules can produce so-called 'skelton', 'muscle' and 'joint' which are composition of the symbolic space [2], for example, in one dimensional trimodal maps, the joint is a TSSKS, the corresponding parameters can be calculated by the word-lifting tecnique [3], a route to chao may be described by the star product [4], Feigenbaum's metric universities and scale factor [5] may be reproduced and researched. So, admissibilty conditions are inherent important rules in the research of symbolic dynamics.

In the paper, we found a few of logical and numerical features are strong related with the admissibility conditions, these logical features come from order rule of the geometric property of TSSKS without using admissibility conditions, while the largest eigenvalue λ from its stefan matrix is calculated by a given TSSKS, we named $\log(\lambda)$ topological entropy [6] as the numerical feature, during the computing process, the comparison rule of sequence is used. For an arbitray TSSKS, it's class labels is 1 for admissible and 0 for not admissible by the admissibility conditions. A data table is setup up to build a decision tree. We apply the C4.5 algorithm by the famous python machine learning sklearn on the data table. For a given TSSKS, it' admissibility is obtained by querying the decision tree. Finally, the evaluation of the decision tree model is given and has a higher points.

The paper is organized as follows. In sec.2, the concise decription of symbolic dynamics of trimodal maps, including sequence comparison rule and the admissibility condition; in sec.3, the construction of data set and decision tree; in sec.4, conclusion.

2. Fundmental of Symbolic Dynamics of Trimodal Maps

2.1. Symbolic Dynamics of Trimodal Maps

Consider the genernal trimodal maps $f(x)=k\int(x-c)(x-d)(x-e)dx+b$, on the interval [-1,1] of endomorphism, by the two boundary conditions *f* (-1)=1 and *f* (1)=1, parameters *k* and *b* are eliminated. The iterated map of timodal are written as

$$x_{n+1}=f(x_n,c,d,e) \tag{1}$$

Here, *c*,*d* and *e* are horizontal coordinates of three critical points *C*,*D*,and *E* (see Fig.1.),

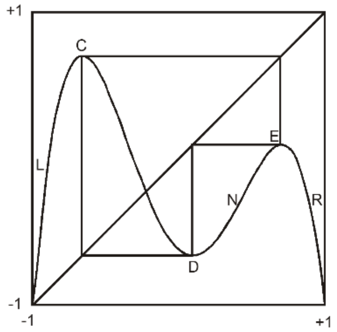

Fig. 1 Schematic iterative map for TSSKS EDC

L,M,N and R are four monotonous limbs, by the MSS order [1], $L\prec C\prec M\prec D\prec N\prec E\prec R$ holds. For an initial point x_0, by iterative map (1), a numerical sequence is obtained as x_0, x_1,..., x_n, it can be converted into a symbilic sequence $Q_0Q_1\cdots Q_n\cdots$, by the following rule (2), the coarse granulation process is quintessence of the symbolic dynamics.

$$Q_k=\begin{cases} L & if\ \ x_k<c, \\ C & if\ \ x_k=c, \\ M & if\ \ c<x_k<d, \\ D & if\ \ x_k=d, \qquad k\in Z^+ \\ N & if\ \ d<x_k<e, \\ E & if\ \ x_k=e, \\ R & if\ \ x_k>e. \end{cases} \tag{2}$$

If the sequence is periodic, for example $(ZEXDYC)^\infty$ or $(XDZEYC)^\infty$, it is simply normalized as $ZEXDYC$ or $XDZEYC$, where Z,X,Y are sequences composed of $\{L,M,N,R\}$, these sequences are called TSSKS which are periodic and passed through the three critical points *C*, *D* and *E* respectively. A TSSKS is important to calculate its corresponding parameters *c*, *d* and *e*.

2.2. Comparison Rule of Sequence and Admissibilty Conditions

Based on the MSS order $L\prec C\prec M\prec D\prec N\prec E\prec R$, the oder rule of sequences are defined as follows: For two sequences $U=u_1u_2\cdots u_n\cdots=\Delta a\cdots$ and $V=v_1v_2\cdots v_n\cdots=\Delta b\cdots$, let Δ be the common leading part, $a,b\in\{L,C,M,D,N,E,R\}, a\neq b$. If the number of decreasing limb M and R sums even (including Δ is empty set), then $if\ a\prec b$ then $U\prec V$, else $if\ a\succ b$ then $U\succ V$; If the number of decreasing limb M and R sums odd, then $if\ a\prec b$ then $U\succ V$, else $if\ a\succ b$ then $U\prec V$.

The first operator notation is subsequence, if W is a sequence which is composed of elements in $\{L,C,M,D,N,E,R\}$, then $\bar{L}(W)$ is all subsequences of *L* in *W*, $\bar{C}(W)$ is subsequence of *C* in

W, $\bar{M}(W)$ is all subsequences of M in W, $\bar{D}(W)$ is subsequence of D in W, $\bar{N}(W)$ is all subsequences of N in W, $\bar{E}(W)$ is subsequence of E in W, $\bar{R}(W)$ is all subsequences of R in W. For instance, if TSSKS W = RLLEDLC, $\bar{L}(W)$ are three sequences LEDLC, EDLC and C, $\bar{C}(W)$ = RLLEDLC, $\bar{M}(W) = \bar{N}(W) = \varnothing$, $\bar{D}(W)$ = LC, $\bar{E}(W)$ =DLC, $\bar{R}(W)$ = LLEDLC.
Admissibility conditions are defined as follows [1]:

$$\begin{cases} \bar{L}(W) \prec \bar{C}(W) \succ \bar{M}(W) \\ \bar{M}(W) \prec \bar{D}(W) \prec \bar{N}(W) \\ \bar{N}(W) \prec \bar{E}(W) \succ \bar{R}(W) \end{cases} \tag{3}$$

We introduced the subsequence operator in place of Hao's shift operator [1] here because the later unsuited sequences in trimodal maps.

2.3. N-period TSSKSs from Permutation and its Class Labels

For a given n, all n-3 letters in {L,M,N,R} are permutated while n-2 positions are for E and n-1 positions for D, C is fixed on the tail. So, $4^{n-3}(n-2)(n-1)$ TSSKSs are produced by permutation and combination algorithm, each of them is checked according to algorithm of (3) by Matlab programming. Table 1 presents the numberof period 3-8 TSSKSs and admissible ones, the original data table (DT) has 52186 rows, it is so long and omitted here.

Table 1 Period 3-8 TSSKSs from permutation and admissibility conditions

Period n	Number of TSSKSs	Admissible Number	Not Admissible
3	2	1	1
4	24	4	20
5	192	18	174
6	1280	70	1210
7	7680	261	7419
8	43008	960	42048

For each TSSKS, if it is admissible, in another word, meet the conditions (3), we label it 1, else label it 0 in the columns of the data table as class label.

3. Construction of Decision Tree

3.1. Calculation of Topological Entropy of a TSSKS in Data Table

First, the toplogical entropy of a TSSKS is an important numerical feature for the construction of the decision tree. A TSSKS may get it's toplogical entropy whether it is admissible or not, it only relates with the order of the subsequences. A period-n TSSKS in DT can get n subsequences which form n-1 intervals, numbered intervals as 1, 2, ..., n-1. from left to right of the interval endpoints transfers to another interval by shift operator. Finally, we get a matrix $A_{n-1,n-1}$. Here, we present an example for understanding easily. Take $W = EDLC$ in DT, we get a subsequences $EDLC, DLC, LC, C$, then order them by the MSS order, $LC \prec C \prec DLC \prec EDLC$, it forms three intervals $[LC, C]$,$[C, DLC]$,$[DLC, EDLC]$,noted as 1,2,3 repectively. By shift operator, first interval $[LC, C]$ maps to $[C, EDLC]$, the second interval maps to $[LC, EDLC]$,the third interval $[DLC, EDLC]$ maps to $[LC, DLC]$, so the stefan matrix is obtained as $A_{3,3} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, the largest eigenvalue $\lambda \approx 2.4142136$, the toplogical entropy

$te = \log(\lambda) \approx 0.88137359$. The calculation of the process can be easily programed by Matlab, so the 52186 TSSKS can produce corresponding toplogical entropy values as a numerical feature in the DT. So far, the DT has three feature named TSSKS, TE, Admissible.

3.2. Four Logical Features and Height Order Relation

For an arbitray TSSKS $W = w_0 w_1 \cdots w_{|W|-1}$ in the DT, the shift operator σ^k is indroduced as follows:

$$\sigma^k(W) = w_k w_{k+1} \cdots, k = 0,1,2,\cdots,|W|-1 \tag{4}$$

$|W|$ stands for the period of the TSSKS W, Let $S = \left\{\sigma^k(W) \middle| k = 0,1,2,\cdots,|W|-1\right\}$, four logical varables are named C_is_Highest, E_is_Highest, D_is_Lowest and D_is_Up, they are defined as follows:

$$\forall s \in S,\ \text{if } \bar{C}(W) \succ s \text{ holds, then C\_is\_Highest} = 1, \text{else } 0. \tag{5}$$

$$\forall s \in S,\ \text{if } \bar{E}(W) \succ s \text{ holds, then E\_is\_Highest} = 1, \text{else } 0. \tag{6}$$

$$\forall s \in S,\ \text{if } D(W) \prec= s \text{ holds, then D\_is\_Lowest} = 1, \text{else } 0. \tag{7}$$

$$\text{If } \bar{D}(W) \prec \sigma^1(\bar{D}(W)) \text{ holds, then D\_is\_Up} = 1, \text{else } 0. \tag{8}$$

All these rules (5)-(8) applied the MSS order and the oder rule of sequences defined in Sec. 2.2 without using the admissibility conditions (3). Essentially, the MSS orders in (5)-(8) are height order relation by the geometry properties of TSSKS *W*. Four logical varables are embedded into the columns of the DT as the features. These rules are all programmable in Matlab, Matrix $L_{52186,4}$ is computed and filled the DT.

3.3. Decision Tree

From sec. 2.3, sec. 3.1 and sec. 3.2, The DT which meets the requirement of decision tree is finnished. (see Tab. 2)

Table 2 Schematic of DT from Period 3-8 TSSKSs

S. N.	TSSKS	Period	Toplogical Entropy	C_is_Highest	E_is_Highest	D_is_Lowest	D_is_Up	Admissible
1	DEC	3	0.48119082	0	1	0	1	0
2	EDLC	4	0.88137359	1	0	1	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
52185	NRNRERDC	8	0.85916064	0	0	1	0	0
52186	RMRMMEDC	8	0.66079092	1	0	1	0	1

The algorithm selection of the decision tree in this paper uses the C4.5 algorithm [7], which takes information gain ratio as attribute choice criterion and is mature and stable. The algorithm principle of C4.5 is open and omitted here. We select the famous python machine learning library sklearn to construct the decision tree, while numpy, matplotlib and pandas are appplied .

First, the train data and test data are divided from gray part of the DT (see Tab. 2), the python codes are follows:

```
from sklearn.model_selection import train_test_split
X_train,X_test,Y_train,Y_test = train_test_split(DT_feature, Admissible,test_size = 0.3,random_state = 2023)
```

Second, construct the cart decision tree from the train set:

```
from sklearn import tree
clf = tree.DecisionTreeClassifier(random_state = 2023)
clf = clf.fit(X_train,Y_train)
```

Third, predict and rate the model:

```
y_pre = clf.predict(X_test)
from sklearn.metrics import accuracy_score
print("Score:",accuracy_score(y_pre,Y_test))
```

Finally, about 86 points for the decision tree model is obtained. It' worth noting that random_state is fixed to avoid the other factors which may effect the score of the model.

4. Conclusion

In fact, when we increase or decrease the features in Tab. 2, there are little effects on the score of the decision tree model. 86 points is high enough for a DT with 52186 rows. The possible method to increase score may to add other logical features from the geometry properties of TSSKS by the MSS order. On the other hand, (5)-(8) may played an important role on the initial value issue of word-lifting technique, it will present a way to define the distance from two TSSKS in the symbolic space efficiently.

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