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Application of invariant theory in computer vision

Zehao Fu [a], Chaoqi Hu [b]
Tianjin University of Technology and Education, Guangzhou 510000, China;
[a]2290112047@qq.com, [b]2606126649@qq.com

Abstract

In the field of computer vision, the motion of objects and the change of viewpoints often affect the judgment of target recognition. In planar images, whether curves or curves fitted by discrete points have invariance determines the anti-noise performance of recognition methods. Invariant theory is consistent with this, if the transformation group of the object is known, finding its characteristic invariants can effectively improve the accuracy of target recognition. Based on the invariant theory, this paper analyzes the application of the projective invariance fitting problem of the conic curve of the plane curve, transforms the fitting problem into an optimization problem that is easy to solve, constructs the Lagrange function, and calculates the numerical solution of the function through the mathematical theory method of Groebner bases, and the fitting problem is finally solved.

Keywords

Projective transformation ;computer vision;invariant;Lagrange function;Groebner bases.

1. Introduction

The concept of invariant was proposed by Cayley, Salmon, Clebsh et al.[2]. After perfecting its theoretical knowledge by Hilbert and completing the proof of finite principle (Finteness Theorem) in 1888, people felt that this theory had been complete and was gradually shelved by [3]. It was not until the late 1980s that the theory of invariant in mathematics and physics was introduced into the research of machine vision. In 1991, European and American scholars formally established a seminar in Iceland and put forward the theory of visual invariant. At the mathematical level, the visual invariant is a functional form in which the target geometry condition remains constant under a certain transformation group, thus distinguishing the important parameter [4] for different targets.

2. Preparatory knowledge

In three-dimensional space, it is set that the line harness passing through point A respectively passes through plane Ⅰ and plane Ⅱ, and the line harness corresponds to the intersection of two planes one by one, which is called perspective correspondence. If the line harness passing through another point B passes through plane Ⅰ and plane Ⅲ, the line harness also corresponds to the intersection of two planes one by one, then plane Ⅱ and plane Ⅲ are connected through a series of perspective correspondence, which is called projective correspondence. Then the image in plane Ⅱ projects to the corresponding image in plane Ⅲ, this process is called projective transformation. The projective transformation from one two-dimensional plane to another can also be expressed as:

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$$\lambda \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{1}$$

Where, λ is a non-zero constant, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix}$ are the coordinates of a point on the plane and the corresponding point on the other plane of its projective transformation, $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a non-singular transformation matrix. If only considering plane finite far point, projective transformation and the coordinates of each point can use nonhomogeneous coordinates (x, y) and (x‘,y‘), (1) can be expanded for nonhomogeneous coordinates:

$$\begin{cases} x' = \frac{a_{11}x+a_{12}y+a_{13}}{a_{31}x+a_{32}y+a_{33}} \\ y' = \frac{a_{21}x+a_{22}y+a_{23}}{a_{31}x+a_{32}y+a_{33}} \end{cases} \tag{2}$$

Projective invariant is a characteristic of projective transformation, which refers to the quantity that the figure does not change through any projective transformation. The main characteristic of this transformation is the retention of associativity. For example, the associativity between point and line and point and plane, etc., such as the most basic projective invariant - intersection ratio. This paper discusses how to construct a conic curve with projective invariants in a plane.

3. Groebner based method

Groebner's fundamental theory belongs to computational algebraic geometry. This theory was first introduced by Hirona-ka H in 1964 and called as the standard basis. In 1965, Buchberger first proposed the Groebner based algorithm of polynomial ideal, and in the following decades, he constantly improved and optimized the algorithm. In the research process of his theory, problems were generally converted into nonlinear polynomial problems, and then Groebner based theory was used to solve the transformed mathematical problems.

In the process of constructing plane conic curve, the key is to establish the multiple parameter values of the solution curve of Lagrange equation, that is, to solve the higher order polytropic equations. This kind of problem is often difficult to calculate in the case of too many parameters and constraints. In this paper, the Groenber basis method is introduced. The Groebner basis theory is used to solve the transformed mathematical problems in the process of research, so that these problems can be transformed into simple single variable equations, which greatly reduces the computational.

In the following article, the excellent elimination ability of Groebner based method is utilized to simplify a relatively complex nonlinear equation set that needs to be solved in the experimental process, and finally its numerical solution is obtained.

4. Construction of projective invariance curve

Given curve fitting, the plane point set $(x_i,y_i)(1 \le i \le N)$, fitting out of the closed curve of a point set.

If a good plane curve is fitted, two properties need to be satisfied: (1) Good curve approximation, small changes in data will only lead to small changes in the fitting curve. (2) projective invariance, that is, the fitting curve of the projective point set is exactly the projection of the fitting curve of the original point set.

4.1. Quadratic invariant curve fitting theory

Suppose the desired conic curve is:

$$Q_2(x, y) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \tag{3}$$

The curve can also be expressed in the form of matrix and vector multiplication, i.e$\boldsymbol{Q_2}(\boldsymbol{x},\boldsymbol{y}) = \mathbf{v}'\boldsymbol{P}\mathbf{v}$，Where **v** represents the argument vector, **v′** is **v** transpose and **v**=[x,y,1]; P is the matrix of coefficients, It is

$$\mathrm{P} = \begin{pmatrix} A & \frac{B}{2} & \frac{D}{2} \\ \frac{B}{2} & C & \frac{E}{2} \\ \frac{D}{2} & \frac{E}{2} & F \end{pmatrix}$$

The distance from any point (x_i,y_i) to the curve $Q_2(x,y)$ is defined as $Q_2^2(x_i,y_i)$, and the distance from the given fixed point set to the curve is$\sum_{i=1}^{N} Q_2^2(x_i,y_i)$In order to fit a conic curve with good properties, it is necessary to satisfy projective invariance and its good approximation to the original curve. First of all, in order to satisfy the second property we need to make the distance between the set of points and the given curve as small as possible; Secondly, although $Q_2(x,y)$ and $kQ_2(x,y)$ represent the same curve, if the point (x_i,y_i) is not on the curve, then the algebraic distance calculated with different k values is different. In order to keep the point set in office a little to the uniqueness of algebraic distance curve, request to fit quadratic curve coefficient matrix determinant | P| = 1. In this way, invariance fitting of conic curve is finally transformed into an optimization problem with constraints, namely:

$$\begin{cases} min \sum_{i=1}^{N} Q_2^2(x_i,y_i) \\ s.t. |P| = 1 \end{cases}$$

By using Lagrange multiplier method, the optimization problem with constraints is further transformed into an optimization problem without constraints:

$$\mathrm{L} = \sum_{i=1}^{N} Q_2^2(x_i,y_i) + \lambda(|p| - 1)$$

Where λ is the Lagrange multiplier, a seven-element cubic nonlinear system can be obtained.

However, the above equations are nonlinear, and according to the general theorem of algebraic equations, it can be known that there are at most 192 solutions in the complex field. Therefore, it will be very complicated to discuss the relationship between the form and parameters of projective curves in general cases. The following is a concrete experiment to discuss the projective invariance fitting process of a conic curve.

4.2. The inuniqueness of point set curve fitting

We know that the target crosses point q_1:(2,2), q_2:(-2,2), q_3:(2,-2), q_4:(-2,-2), q_5:(1,0), q_6:(-1,0), It is tried to fit the target curve with a conic curve to satisfy projective invariance.

According to the fitting theory of quadratic invariant curve above, the following nonlinear equations can be obtained:

$$\begin{cases} 132\mathrm{A} + 128\mathrm{C} + + 36\mathrm{F} + \lambda\left(\mathrm{CF} - \frac{\mathrm{E}^2}{4}\right) = 0 \\ 128\mathrm{B} + \lambda\left(\frac{\mathrm{DE}}{4} - \frac{\mathrm{BF}}{2}\right) = 0 \\ 128\mathrm{A} + 128\mathrm{C} + 32\mathrm{F} + \lambda\left(\mathrm{AF} - \frac{\mathrm{D}^2}{4}\right) = 0 \\ 36\mathrm{D} + \lambda\left(\frac{\mathrm{BE}}{4} - \frac{\mathrm{CD}}{2}\right) = 0 \\ 32\,\mathrm{E} + \lambda\left(\frac{\mathrm{BD}}{4} - \frac{\mathrm{AE}}{2}\right) = 0 \\ 36\,\mathrm{A} + 32\,\mathrm{C} + 12\,\mathrm{F} + \lambda\left(\mathrm{AC} - \frac{\mathrm{B}^2}{2}\right) = 0 \\ \mathrm{ACF} + \frac{\mathrm{BDE}}{4} - \frac{\mathrm{CD}^2}{4} - \frac{\mathrm{AE}^2}{4} - \frac{\mathrm{B}^2\mathrm{F}}{4} - 1 = 0 \end{cases} \quad (4)$$

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By determining the lexicographical order of the above eight variables as $\{\lambda, A, B, C, D, E, F\}$, 13 equations containing the above variables can be obtained, and 6 satisfactory polynomials can be screened out. One of the polynomials contains only F single arguments, so we can first find the root of this polynomial and plug it into the other five polynomials. Find the solution equivalent to the problem, Consider in the field of real numbers, the rest of the solution see Table 1:

Table 1 Real number solution

Numble	A	B	C	D	E	F
1	-1	0	0.25	0	2.24	1
2	-1	0	0.25	0	-2.24	1
3	0.25	1.12	0.83	0	0	-4.3
4	0.25	-1.12	0.83	0	0	-4.3
5	0.19	0	-0.99	2.18	0	0.95
6	0.19	0	-0.99	-2.18	0	0.95
7	-0.24	-0.73	-0.3	1.75	-1.78	-0.24
8	-0.24	0.73	-0.3	-1.75	-1.78	-0.24
9	-0.24	0.73	-0.3	1.75	1.78	-0.24
10	-0.24	-0.73	-0.3	-1.75	1.78	-0.24
11	0.65	0	0.68	0	0	2.28
12	-0.68	0	-0.43	0	0	3.47
13	1.1	0	-0.83	0	0	-1.1

By substituting the above parameters into Equation (3), seven equations are obtained, corresponding to the following seven fitting curves respectively, a see Fig 1:

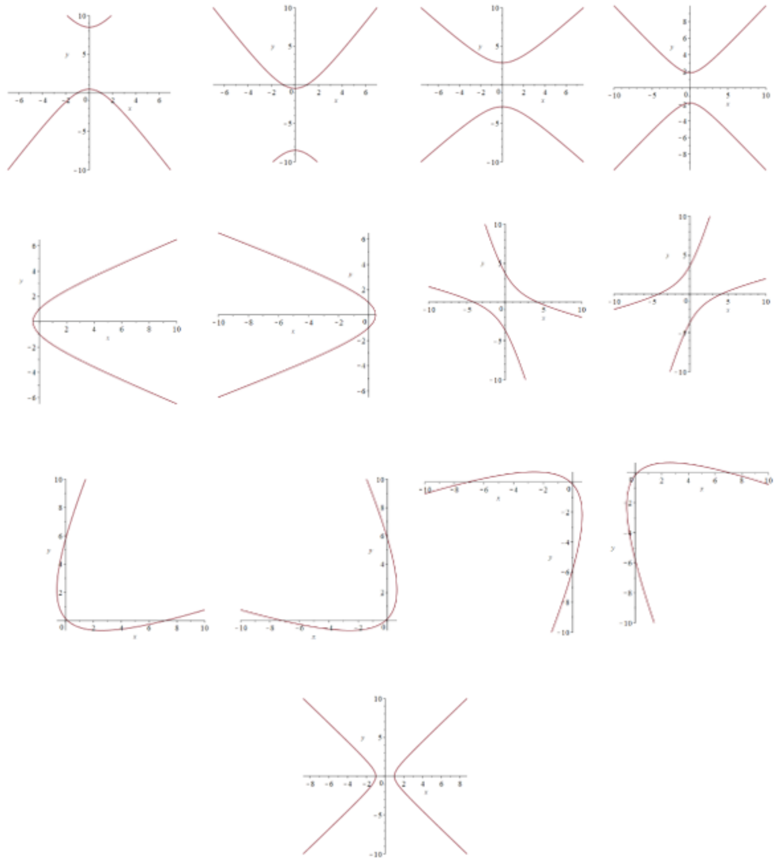

Fig 1 The image of the fitted curve

Among the 13 fitting curves, there are 6 parabolas and 7 groups of hyperbolas. It is proved that the invariant curves fitted by this method are not unique. However, the degree of fitting of not every curve is excellent, which is due to the limitation of conic curve, which can only be fitted through ellipse, hyperbola and parabola. Once the point set exceeds this category, overfitting phenomenon will occur in the calculation process, leading to inaccurate results. In the above example, it can be seen that the best fitting curve should be curve 13: $1.1x^2 - 0.83y^2 - 1.1 = 0$.

4.3. Projective invariance verification of curves

According to the definition in Section 2, the projective invariance of curve 13 can be verified by a projective transformation as follows:

Suppose point set $q_1, q_2, q_3, q_4, q_5, q_6$ changes to point set $q_1', q_2', q_3', q_4', q_5', q_6'$ after transformation matrix τ, and the projective transformation matrix τ is:

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The coordinates of the six points are see in Table 2:

Table 2 Coordinates under projective transformation

Numble	Before transformation	After transformation
1	(2,2)	(8,8)
2	(-2,2)	(4,-1)
3	(2,-2)	(-1,3)
4	(-2,-2)	(-4,-5)
5	(1,0)	(3,3)
6	(-1,0)	(1,-1)

After the transformation of hyperbolic equation $1.1x^2 - 0.83y^2 - 1.1 = 0$ by the projective matrix τ, we get:

$$-0.25x^2 - 0.12xy + 0.4y^2 + 1.11x - 0.55y - 1.93 = 0$$

Fitting curve after projective transformation，see Fig 2.

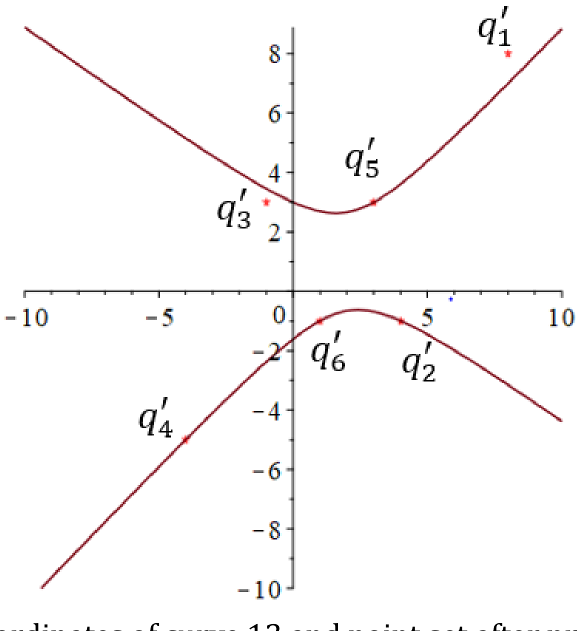

Fig 2 Image and coordinates of curve 13 and point set after projective transformation

It can be seen that the points after the projection transformation all fall on the fitted curve after the transformation, and its topological structure is still hyperbolic, and its equation is still quadratic equation. Therefore, the curve $1.1x^2 - 0.83y^2 - 1.1 = 0$ fitting points not only meet the good approximation of the curve, but also have the property of projective invariant curve. It is proved that the proposed method can be used for curve fitting efficiently and accurately.Conclusion

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5. Conclusion

In this paper, a hyperbolic projective invariance fitting method for plane curves is constructed based on Groebner theory and quadratic invariant curve fitting theory. The non-uniqueness of the fitting curve and the topological structure and degree invariance of the curve during affine transformation are discussed through a concrete example, and the feasibility of the method is proved. However, a large number of complex solutions in the curve fitting process have not been deeply discussed, and the practical significance and use of complex solutions will be considered in the future research.

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