

Price Decisions for Extended Warranty in Closed-Loop Supply Chain Based on Uncertainty of Collection Quality

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Abstract

Based on the two-level closed-loop supply chain formed by a manufacturer and a retailer, a multi-stage Stackelberg model is established, considering the extended warranty and its impact on the uncertainty of the quality of the collected used products. The conclusion indicates that the improvement of collecting quality and product quality retention by consumers who purchase extended warranties increase the quantity of collecting and reduce the manufacturing cost in the second stage, thus increasing the profits of the manufacturer and the retailer.

Keywords

Extended warranty, Uncertainty of collect quality, Closed-loop supply chain, Multi-stage model.

1. Introduction

An extended warranty is a form of after-sale and warranty service vendors offer for products after their standard warranty period. With the continued development of modern technology and market-oriented operations, the extended warranty has gradually become a part of enterprise development strategies. In addition to providing better after-sale service to customers and improving customer satisfaction and loyalty, it can also enable enterprises to gain more significant market share and create new revenue streams. Moreover, an extended warranty can ensure product quality, extend service life, and align with the national call for energy conservation and environmental protection.

With the market growth and rise in consumer demands, extended warranties have become a common feature for many enterprises, constantly evolving and expanding. Therefore, the research on extended warranties in the academic community is also increasing. First, research is conducted on the factors that influence extended warranty from a consumer's perspective. Some scholars design extended warranty contracts from the perspective of the risk preferences of consumers and manufacturers [1], [2]. Song et al. considered consumers' willingness to pay for it while studying the product's extended warranty strategy. [3]. Peng et al. investigate network coordination strategies based on extended warranty, considering sale cost and sale effort level in the process [4]. Among the sale issues in an extended warranty, some scholars studied the impact of innovation investment on the extended warranty model of strong retailers [5], while others have considered government subsidies when studying manufacturers' remanufacturing extended warranty decisions [6]. In the problem of competition and coordination regarding the extended warranty, Sun et al. established a multi-product supply chain network model and analyzed the impact of factors such as extended warranty time and quality effort cost on equilibrium decision-making and profits. [7]. Ma et al. revealed the impact of retailers providing extended warranties and market competition on sale rebate contract selection strategies by developing a competitive model of two manufacturers and two retailers offering extended warranties [8]. For extended warranty model selection,

some scholars studied the interaction strategy between an extended warranty and a basic warranty in a two-level supply chain [9]. Zhang conducted research and comparative analysis on the extended warranty of three models in the dual-channel supply chain of products [10]. Yi et al. considered network externalization and channel power structure issues [11]. Li et al. modeled both wholesale and consignment models to consider the optimal timing pricing strategy for extended warranties provided by e-commerce platforms [12].

In addition, the quality of the product and the extended warranty are both critical factors to consider when evaluating an extended warranty. Some scholars have found that the difference in consumers' time, process, and habits in using the product can lead to uncertainty in the quality of collected used products [13]. Therefore, there is a need to classify the collection and remanufacturing of the used products. Regarding the quality issues related to extended warranties, some scholars have studied the service quality of extended warranties and discussed it as an influencing factor for consumers to purchase extended warranties, while others have focused on the quality of the product itself. Kou et al. provide joint pricing and inventory strategies for the multiple effects of product quality on the demand for products and extended warranty in the supply chain [14]. Zhang et al. investigate a supply chain extended warranty model based on product quality, considering the importance consumers attach to product quality, manufacturers' efforts to improve product quality, and product failure rate. [15]. Kou et al. studied a supply chain model of extended warranty provided by retailers under quality constraints [16]. Quality constraints only apply to the product itself but do not consider the impact of extended warranty on the collecting quality of the used products.

However, the quality of collected products is lower for those who have purchased an extended warranty and prolonged their service lives than those who have not. So, it raises the cost of remanufacturing the product for the remanufacturer. Therefore, when examining the added income generated by offering extended warranties, it is crucial to explore the effects of extended warranties on the quality of collected used products and the expenses of remanufacturing them post-collection.

2. Problem Description

2.1. Structure Description

This paper considers a two-level closed-loop supply chain game model consisting of a manufacturer and a retailer. Established a Multi-stage model where the manufacturer is the leader, the retailer provides extended warranty and collects used products. The manufacturer will reclaim the used products from the retailer for remanufacturing and sales.

The manufacturer produces and sells products to the retailer at wholesale price and the retailer sells products to the consumer at the retail price, while providing paid extended warranty, consumers can choose whether to purchase the extended warranty. Then the retailer collects the used products from consumers, including products that have not purchased extended warranty and those that have purchased extended warranty. Finally, the manufacturer collects and remanufactures used products from the retailer, sells the new products and remanufactured products to the retailer and the retailer sells products and extended warranty to the consumer. As shown in Fig. 1:

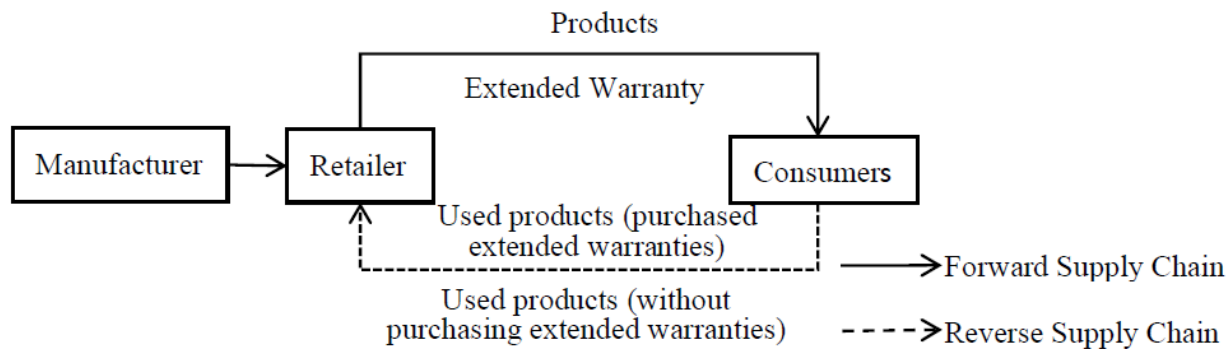


Fig. 1 Model structure diagram

2.2. Research Hypothesis

(1) New products have the same quality and price as remanufactured products. The market demand is $Q = \alpha - \beta p$. α is the maximum market demand for the products, β is the price sensitivity coefficient of the consumer, p is the unit retail price of the products. In the first stage, the manufacturer and the retailer only produce new products for sale to consumers. The manufacturer sells the products to the retailer at the wholesale price w , and the retailer sells the products to consumers at the unit retail price p . The collected used products are used for remanufacturing, and both remanufactured and new products will be sold to consumers simultaneously in the new stage.

(2) The total demand for extended warranty is $Q' = Q - \varepsilon p_e$, p_e is the unit retail price for the extended warranty, ε is the sensitivity coefficient of consumers to the price of an extended warranty. The retailer sells the extended warranty to consumers at the price p_e . Consumers are more sensitive to the unit retail price of an extended warranty than the price of products, $0 < \beta < \varepsilon$. The cost of the extended warranty is φ .

(3) The quality of collected used products is uncertain. $q_i (i=1,2)$ represents the quality of recycled waste products obtained from different levels of product wear and tear, $0 < q_i < 1$. The larger q_i is, the lower the degree of product loss and the higher the level of collecting quality. q_1 represents the quality of the products that did not purchase the extended warranty, q_2 represents the quality of the products that have purchased the extended warranty, $q_2 = \gamma q_1$, $\gamma (0 < \gamma < 1)$ is the degree to which consumers who have purchased extended warranty services maintain the quality of their products. The higher the degree of maintenance, the lower the degree of product wear and tear, and the higher the quality of collected products.

(4) The unit cost of producing a new product is c_n and the unit production cost of the remanufactured product is a linear decreasing function of the quality of the used product, $c_r = c_n - \lambda q_i (i=1,2)$, λ is the product quality cost coefficient, which indicates the effect of collecting quality on the remanufacturing cost.

(5) The Manufacturer's collecting price for a used product F is a linear increasing function of collecting quality, $F_i = \theta q_i (i=1,2)$, the manufacturer collects used products from the retailer. The retailer collects used products from consumers at a price $f_i = (1-r)F_i$, θ is the products quality value coefficient, it represents the extent to which the quality of collecting affects the price of collecting. $r (0 < r < 1)$ is the unit profit rate collected by the retailer. In this process, the manufacturer can reduce its cost by collecting used products for remanufacturing to make it profitable, $\theta < \lambda$.

(6) The collecting quantity G is a linear increasing function of the collecting price, which includes two parts: collecting used products from consumers who did not purchase the extended warranty and collecting the used products from consumers who purchased the extended warranty. $G_i = g_i + kF_i (i=1,2)$, g_i are the used products donated by consumers for free,

which refers to the level of environmental awareness among consumers. k is the consumer's sensitivity coefficient to the collecting price.

(7) The supply chain is a multi-stage model. The recycled used products will be used for the next production stage of remanufactured products. The quantity of the collected used products $G_1 + G_2$ is not enough to meet the demand for products in the next stage Q , $G_1 + G_2 < Q$. Therefore, the manufacturer will produce new products and sell them together with remanufactured products to meet the next stage of product market demand.

2.3. Notations

Based on the above research hypotheses, the relevant variables and meanings involved in the established multi-stage model are listed below in Table 1.

Table 1 Symbol description

Variables	Meaning
p	The unit retail price of a product.
p_e	The unit retail price for an extended warranty.
c_n	The unit manufacturing cost of a new product.
c_{ri}	The unit manufacturing cost of a remanufactured product.
φ	The unit cost of extended warranty.
λ	The quality cost coefficient of a product.
γ	The degree to which consumers who purchase extended warranties maintain the quality of their products.
q_i	The quality of the collected used products.
F_i	The unit price for the manufacturer to collect a used product.
f_i	The unit price for the retailer to collect a used product.
θ	The quality margin coefficient of the product.
g_i	The amount of the used products donated by consumers without compensation.
k	Sensitivity coefficient of consumers to collecting price.
Q	The demand for the products.
Q'	The demand for extended warranty.
G_i	The collection quantity of used products.
Π_R	The total profit of the retailer.
Π_M	The total profit of the manufacturer.

3. Model Construction and Solution

The profits of the manufacturer and the retailer are established separately according to the multi-stage model description and research hypotheses.

$$\Pi_M = Q(w - c_n) - G_1 F_1 - G_2 F_2 + G_1(w - c_{r1}) + G_2(w - c_{r2}) + [Q - (G_1 + G_2)](w - c_n) \quad (1)$$

$$\Pi_R = Q(p - w) + Q'(p_e - \varphi) + G_1(F_1 - f_1) + G_2(F_2 - f_2) \quad (2)$$

By substituting the parameter calculation formula from the model assumption into the above equation, we get:

$$\Pi_M = 2(\alpha - \beta p)(w - c_n) + q_1(\lambda - \theta)[g_1 + \gamma g_2 + k\theta q_1(1 + \gamma^2)] \quad (3)$$

$$\Pi_R = (\alpha - \beta p)(p + p_e - w - \varphi) - \varepsilon p_e(p_e - \varphi) + r\theta q_1[g_1 + \gamma g_2 + k\theta q_1(1 + \gamma^2)] \quad (4)$$

To solve the optimal solution of the model, the optimal pricing strategies of the manufacturer and the retailer are determined and solved by the inverse induction method. Taking the first

derivative of the product price and extended warranty price in the total profit function of the retailer.

$$\frac{\partial \Pi_R}{\partial p} = 2\alpha - 2\beta(2p + p_e - c_n - \varphi) \quad (5)$$

$$\frac{\partial \Pi_R}{\partial p_e} = 2(\alpha - \beta p) - 2\varepsilon(2p_e - \varphi) \quad (6)$$

Let $\frac{\partial \Pi}{\partial p} = 0$, $\frac{\partial \Pi}{\partial p_e} = 0$, we can get the expression for the unit retail price of the product and the unit retail price for the extended warranty:

$$p = \frac{\alpha - \varepsilon\varphi}{\beta} - \frac{2\varepsilon(\alpha - \beta w - 2\varepsilon\varphi)}{\beta(4\varepsilon - \beta)} \quad (7)$$

$$p_e = \varphi + \frac{\alpha - \beta w - 2\varepsilon\varphi}{4\varepsilon - \beta} \quad (8)$$

Substitute the above two function expressions (7) (8) into the manufacturer's profit function Π_M , and let $\frac{\partial \Pi_M}{\partial w} = 0$, we can get the optimal value of the unit wholesale price of the product.

$$w^* = \frac{2\alpha + 2\beta c_n - \varepsilon\varphi}{4\beta} \quad (9)$$

Finally, the above equation is substituted back into the functional expressions (7) and (8), and we can get the optimal unit retail price of the new product and the optimal unit retail prices of the remanufactured product.

$$p^* = \frac{\alpha}{\beta} - \frac{\alpha\varepsilon}{\beta(4\varepsilon - \beta)} + \frac{\varepsilon(2c_n + \varphi)}{2(4\varepsilon - \beta)} \quad (10)$$

$$p_e^* = \frac{\varphi}{2} + \frac{2\alpha - \beta(2c_n + \varphi)}{4(4\varepsilon - \beta)} \quad (11)$$

4. Numerical Examples

To verify the rationality and effectiveness of the above results, numerical simulations were conducted using MATLAB software. The numerical settings of the relevant parameters involved in the model are shown in Table 2, referring to some literature and the relevant requirements research hypothesis in this paper.

Table 2 Setting for parameters

Parameters	Values	Parameters	Values
α	2000	c_r	15
β	8	θ	40
ε	15	g_1	40
c_n	100	g_2	20
φ	5	k	10
λ	95	r	0.2

Substituting the parameter values set above into the optimal result expression in the model. The quantity of recycled used products and the profits of the manufacturer and the retailer are shown in Fig. 2 to Fig. 4.

After numerical simulation, it was found that the quality of collected used products and the degree of preservation of products by consumers who purchase extended warranties in this model do not affect the unit wholesale price of the product, the unit retail price of the product, and the unit retail price of extended warranty.

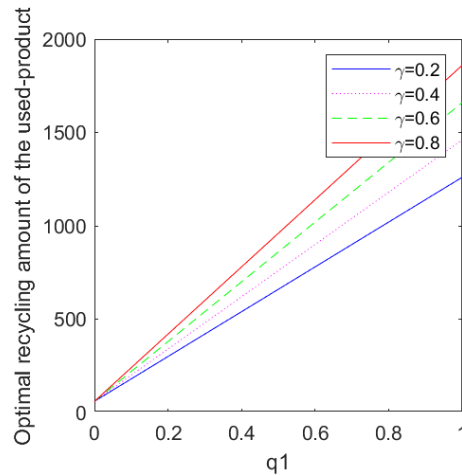


Fig. 2 Changes of the optimal quantity of recycled used products under different quality levels q_1 and γ .

In Fig. 2, the curves from top to bottom show the effect of the quality of collected used products q_1 on the collected volume G_2 when consumers who purchase extended warranty maintain the quality of their products at $\gamma=0.2$, $\gamma=0.4$, $\gamma=0.6$ and $\gamma=0.8$ respectively. The higher the quality of the collected used products, the higher the collection price. Therefore, more consumers are willing to collect their used products. As the amount of collection increases, more products will be used for collection and manufacturing. Thus, the collection amount is directly proportional to the quality of used products and the degree to which consumers who purchase extended warranties maintain product quality.

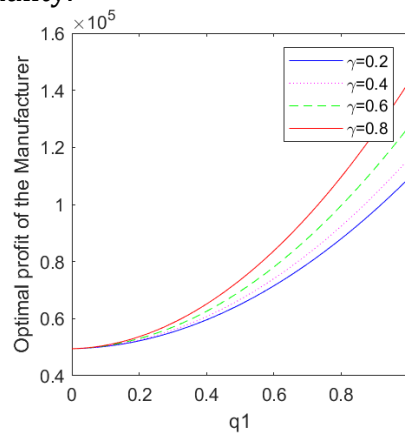


Fig. 3 Changes of the optimal manufacturer's profit under the different quality of recycled used products.

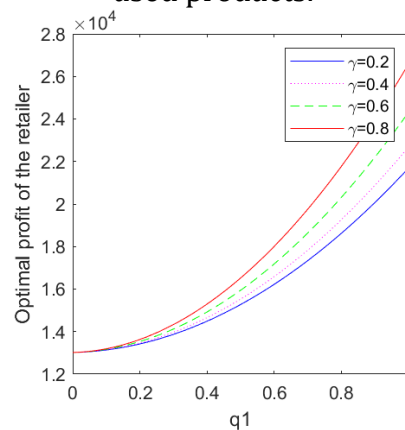


Fig. 4 Changes of the optimal retailer's profit under different quality of recycled used products.

Fig. 3 and Fig. 4 show the trend of the manufacturer's and retailer's profits as the quality of the collected used products q_1 increases. As shown in the figures, both profits increase as the quality of the collected used products q_1 improves. The curve from top to bottom in the graphs shows the trend of the manufacturer's profit and the retailer's profit being affected by the quality of collected used products when the degree of product quality maintenance by consumers who purchase extended warranties is $\gamma=0.2$, $\gamma=0.4$, $\gamma=0.6$ and $\gamma=0.8$. It can be seen that as consumers who purchase extended warranties maintain a higher quality of their products, the optimal profits of the manufacturer and the retailer increase. Moreover, the manufacturer's profit is higher than the retailer's profit.

When the retailer provides extended warranties and collects the used products, the profits of both the manufacturer and the retailer increase as the quality of the collected used products q_1 increases. A thorough analysis of the figures above reveals that the retailer's profit increases with a rise in both the collecting price and the collection volume. This is attributed to an improvement in the quality of the collected used products q_1 and the increased retention of product quality by consumers who purchase the extended warranty. Based on the comprehensive analysis of the above figures, the collection of used products has become more profitable due to the improved quality of the collected used products and the increased maintenance levels of products by consumers who purchase extended warranties. The collection price has increased as a result, which has led to an increase in the quantity of products being collected, resulting in increased profits for the retailer. Although the rise in the collection price leads to an uptick in the manufacturer's collection cost, the boost in quantity and quality of the collected used products decreases the manufacturing cost of the manufacturers in the second stage. Thus, an increase in both variables increases the manufacturer's profits.

5. Conclusion

In this study, we investigated the effect of the quality of used products and the extent to which consumers who purchase extended warranties maintain product quality on various pricing and profitability measures. Specifically, we analyzed the unit wholesale and retail prices, the retail price of extended warranty, the collection price and quantity of used products, and the impact on manufacturer and retailer profits. Based on our findings, it is evident that:

The price and quantity of collected used products will rise with an increase in used product quality and the degree of maintenance of product quality by consumers who purchase extended warranties. The improvement of the quality of the used products and the increase in the maintenance of product quality by consumers who purchase extended warranties have led to an increase in the collecting price of the used products, resulting in an increase in the quantity of the used products being collected and an increase in the collecting cost for the manufacturer. Due to the improved quantity and quality of the collected used products, the manufacturing cost in the second stage has decreased, thereby increasing profits for both the manufacturer and retailer.

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