ISSN: 1813-4890

Research on mathematical model of Rubik's Cube and its application

Kepan Tu

King's College the British School of Madrid, Tres Cantos, 28760, Spain;

kepantu@163.com

Abstract

This paper reviews the origin, development history and development status of Rubik's cube at home and abroad. The basic definition of the Rubik's cube concept is also used. By using the mathematical knowledge of linear algebra bra and group theory, the group related knowledge points involved in Rubik's cube are given, such as action, every mutation, transitivity, conjugation, transposition, etc. This paper mainly studies the mathematical reduction model of Rubik's cube, and finally summarizes the application of Rubik's cube in mathematics teaching and computer.

Keywords

Rubik's cube, Change seats, Mathematical application.

1. Introduction

The Rubik's Cube, also known as the Rubik's Cube or the Rubik's Cube puzzle, is a three-dimensional puzzle game composed of small squares. Its earliest version was invented in 1974 by the Hungarian sculptor and architect Ern Rubik, who initially thought it was intended to help his students understand the concept of three-dimensional geometry. In 1975, he received an international patent for the Rubik's Cube. However, it was not until 1977 that he successfully developed the prototype into a reversible and challenging spelling map.

Then, in August 1978, Bella Szalai, the founder of the American Logic Game company, first produced and sold the Rubik's cube. In November 1978, Viennese businessman Tibauer Leszi began trying to restore the Rubik's cube to its original state, and he approached Rubik to learn from him how to restore the cube. Subsequently, Leszi introduced the Rubik's cube to the famous British toy expert Tom Kramer, and under the strong promotion of Kramer, ABC TV launched a program called "Magic Cube" in September 1983, which attracted countless viewers. This was followed by the Rubik's spherical cube "Rubik 360" at the 2009 Nuremberg International Toy Fair. In the 1880s, the Rubik's cube gradually went to the European continent, and then spread around the world, many Rubik's cube enthusiasts tried to use the fastest way to restore the cube, the idea is their ultimate goal.

As a kind of educational toy, the Rubik's cube takes a very long time to restore from the chaotic state to the initial state, which can only show that its memory is good and its reaction speed is fast. The Rubik's cube is also the research object of mathematicians, they found that the Rubik's cube contains a lot of relevant mathematical knowledge, as early as the end of the 19th century, held in Helsinki, the 18th Global mathematician seminar, the Rubik's cube has been the attention of mathematicians around the world. American mathematician David Singmaster liked the Rubik's Cube since he saw it, and then wrote Notes on Rubiks Magic Cube, Handbook of Cubik Math and other monographs, in which he gave the Rubik's cube symbol notation. Magic is studied from the aspects of mathematical group theory and combinatorics. Later, physicists discovered that the Rubik's cube rotation rules bear striking similarities to some important concepts in quantum physics. This was first discovered by Solomon Golom, a professor at the

University of Southern California, and published in the American Journal of Physics in 1981. Similarly, computer scientists study algorithms for Rubik's cubes and the minimum optimal number of rotations required to restore the cube.

Since many group theory knowledge in mathematics, such as action, displacement, transitivity, conjugation, commutation and other concepts, are reflected in the Rubik's cube, studying the mathematical transformation rule of the Rubik's cube, studying the mathematical reduction model and representation method of the Rubik's cube, can greatly test one's mathematical logical reasoning ability and ability to conduct mathematical induction and research on complex processes. Therefore, this paper will use relevant mathematical knowledge to deeply study the mathematical transformation rules and models of Rubik's cube.

2. Mathematical knowledge of Rubik's cube

2.1. The basic definition of Rubik's Cube

Most of the Rubik's cubes we see in daily life are third-order, in fact, there are many kinds of Rubik's cubes, common ones are: ordinary Rubik's cube; Second order Rubik's Cube; Third level Rubik's Cube; Fourth level Rubik's cube; Five-step Rubik's Cube; Sixth level Rubik's Cube; Mutant Rubik's Cube, etc.

(1) Six faces (F) : top (up,U), bottom (down,D), front (F), back (B), left (L), right (R).

(2) 12 edges.

(3) 8 corners.

(4) Face Cubie, referred to as face block, is each small face

The little piece in the middle, there are six of them.

(5) Edge Cubie, referred to as edge block, is the small block in the center of each side of the Rubik's cube, a total of 12.

(6) Cornor Cubie, referred to as corner block, is the Rubik's cube

The corner blocks. There are eight of them.

(7) Rotation: refers to the clockwise rotation of $\pi/2$ of all blocks on a certain face of the Rubik's cube, and the reversal is the counterclockwise rotation of $\pi/2$.

(8) Level: The number of small pieces on the side of the Rubik's cube is several levels of the Rubik's cube.

(9) Recovery: The process of restoring the Rubik's cube from the mixed wheel state to the initial state is recovery.

The middle blocks of the cube are: top (U), bottom (D), front (F), back (B), left (L), right (R); Each side block is denoated as: uf/fu, ur/ru, ub/bu, ul/lu, fl/lf, fr/rf, bl/lb, br/rb, df/fd, dl/ld, dr/rd, db/bd; Each corner block is denoted as urf/rfu/fur, ufl/flu/luf, ulb/lbu/bul, ubr/bru/rub, dfr/frd/rdf, dlf/lfd/fdl, dbl/bld/ldb, drb/rbd/bdr.

(1) Rubik's Cube Group

Let the resultant operation of the Rubik's cube rotation be from left to right. For X1X2 ∈{U, D, L, R, F, B}, X1X2 means that X1 is rotated first, and then X2 is rotated. For example, FB means that F is rotated first, and then B. The state of the Rubik's cube is represented by m, and X(m) represents the new state formed after X's rotation in the state of m. For the rotation of the Rubik's cube, (M1M2) (c) = (M2 (c)). The set formed by all the rotations of the Rubik's cube, the operation is based on the principle of synthesis, and the group formed is called the Rubik's cube group.

(2) Rubik's Cube is a subgroup of S48. There are 54 small faces on the surface of the Rubik's cube, and the position change of the middle block can be ignored, because the middle block cannot be rotated independently, and only the change of the middle block cannot cause the

ISSN: 1813-4890

change of the current state of the cube, so the Rubik's cube group can be regarded as the transformation limited to the other 48 small faces. Through the above, we can have a clear understanding of the Rubik's cube group, which is the Rubik's cube group of S48. Number the cube, for example:

F= (6254316) (1183041) (7284213) (24221719) (18212320)

B= (1144827) (2124729) (394632) (33354038) (34373936)

L= (1174140) (9111614) (4204437) (10131512) (6224635)

R= (25273230) (3384319) (5364521) (8334824) (26293128)

U= (9332517) (2574) (1386) (10342618) (11352719)

D= (14223038) (15233239) (41434846) (16243240) (42454744)

2.2. Restoration of Rubik's Cube

2.2.1. Five changes of the Rubik's Cube

(1) Change the direction of the two corner blocks and keep their position and the position of all other small blocks unchanged.

(2) Change the direction of the two edge blocks, keeping their position and the position of all other small blocks unchanged.

(3) Exchange the positions of any two diagonal blocks, keeping their direction and the positions of all other small blocks unchanged.

(4) Exchange the positions of any two pairs of side blocks, keeping their positions and those of all other small blocks unchanged.

(5) Exchange the positions of any diagonal block and any pair of side blocks, keeping their direction and all small block positions unchanged.

2.2.2. State and principle of Rubik's Cube

The eight corner blocks of the Rubik's cube are marked as follows. ufl on U is marked as 1; The ufr on U is labeled 2; The ubr on U is labeled 3; ubl on U is marked 4; The dbl on D is labeled 5; The dfl on D is labeled 6; The dfr on D is labeled 7; dbr on D is labeled 8. The 12 sides of the cube are marked as follows. ub on U is marked as 9; ur on U is marked with 10; uf on U is labeled 11; The ul on the U is labeled 12; bl on B is labeled 13; The br on B is labeled 14; The fr on F is labeled 15; fl on F is labeled 16; db on D is labeled 17; The dr On D is marked as 18; df on D is labeled 19; The dl on D is labeled 20.

The Rubik's cube has a total of 26 small blocks, including 20 face blocks and 20 corner blocks. In this section, only the transformations that change the positions of these 20 small blocks are considered. We define the displacement of the basic rotation of small blocks as follows.

Fp= (1672) (11151916); Bp= (3854) (9141713); Rp= (2783) (10131815); Lp= (1456) (12162014); Up= (1234) (9101112); Dp= (5876) (17201918). The above six permutation groups are denoted as,Tp,Tp ∈S8, and S12 are subgroups of S20.

There are three principles of Rubik's cube reduction: (1) inversion principle; (2) partial inversion principle; (3) Conjugate principle.

2.2.3. Mathematical reduction model of Rubik's Cube

Stipulation: clockwise rotation is indicated by FBLRUD, counterclockwise rotation is indicated by $F^{-1}B^{-1}L^{-1}R^{-1}U^{-1}D^{-1}$.

(1) Cambridge mathematician Conway's reduction method: ① restore the bottom four side blocks; ② Restore the bottom four corner blocks; ③ Restore the four side blocks in the middle layer; (4)

The top four side blocks return; ⑤ Pairs of exchange top corner blocks; ⑥ Turn the overturned piece right over.

ISSN: 1813-4890

The first step only needs to consider the position of the completed edge blocks, and does not need to consider whether other small blocks are scrambled.

Step 2 Perform the following operations to restore: B1:$F^{-1}U^{-1}F$； B2： RUR^{-1}； B3： $F^{-1}UFRU^{2}R^{-1}$.

Step 3 Perform the following operations to restore the data: C1: $URU^{-1}R^{-1}U^{-1}F^{-1}UF$;

C2: $U^{-1}F^{-1}UFURU^{-1}R^{-1}$.

Step 4 Perform the following operations to restore the data: D: $UFRUR^{-1}U^{-1}F^{-1}$.

Step 5 Perform the following operations to restore the data: E: $FDF^{2}D^{2}F^{2}D^{-1}F^{-1}$.

Step 6 Restore: F1: $(F^{-1}RFR^{-1})^{2}$=Ma; F2: $(RF^{-1}R^{-1}F^{-1})^{2}$=Mc; F3: $(MDR)^{4}$=Me.

(2) Group theory expert Joyner's "first corner behind method" : First let 8 corner blocks in place, and then let all 12 side blocks in place.

(3) The reduction methods commonly used at present are described by mathematical models as follows.

The first step: Return the original bottom layer of the cross, on the basis of $FU^{2}B^{2}\,FBR^{-1}F^{-1}B^{-1}U^{-1}F^{-1}B^{-1}L^{-1}FBD^{-1}$.

Step 2: Restore the bottom four corners of the cube and perform $LU^{-1}L^{-1}UR^{-1}\,UR^{-1}U^{-1}R^{-1}B^{-1}UBL^{-1}ULR^{-1}U^{2}R$. At this time, the first layer of the cube has been returned

Original, get state.

Step 3: Return the second layer $LUL^{-1}UFU^{-1}F^{-1}$; $ULU^{-1}L^{-1}U^{-1}BUB^{-1}$; $URU^{-1}R^{-1}U^{-1}F^{-1}UF$; $U^{-1}R^{-1}URUB^{-1}U^{-1}B$.

Step 4: The top cross, then the top corner block position return; Perform $R^{-1}ULU^{-1}RU^{-1}U^{2}$ multiple times.

Step 5: Top corner block direction return; Carry out $RU^{2}R^{-1}U^{-1}RU^{-1}R^{-1}$.

Step 6: Top side block return; $RU^{-}1RURURU^{-1}R^{-1}U^{-1}\,R^{2}$ is performed to obtain the final state, and the matrix is described as:

$$\begin{pmatrix} 17 & 2 & 33 & 4 & 13 & 19 & 7 & 35 \\ 11 & 10 & 25 & 21 & 37 & 14 & 15 & 16 \\ 8 & 18 & 9 & 12 & 29 & 22 & 23 & 24 \\ 1 & 20 & 3 & 36 & 26 & 30 & 31 & 32 \\ 27 & 34 & 6 & 5 & 28 & 38 & 39 & 40 \\ 41 & 42 & 43 & 44 & 45 & 46 & 47 & 48 \end{pmatrix} \to \begin{pmatrix} 27 & 4 & 17 & 7 & 2 & 8 & 5 & 1 \\ 3 & 18 & 19 & 12 & 13 & 14 & 15 & 16 \\ 25 & 26 & 35 & 20 & 21 & 22 & 23 & 24 \\ 9 & 34 & 6 & 28 & 29 & 30 & 31 & 32 \\ 11 & 10 & 33 & 36 & 37 & 38 & 39 & 40 \\ 41 & 42 & 43 & 44 & 45 & 46 & 47 & 48 \end{pmatrix} \to \begin{pmatrix} 9 & 2 & 33 & 4 & 5 & 11 & 7 & 19 \\ 35 & 10 & 17 & 12 & 13 & 14 & 15 & 16 \\ 6 & 18 & 25 & 20 & 21 & 22 & 23 & 29 \\ 8 & 26 & 3 & 28 & 29 & 30 & 31 & 32 \\ 27 & 34 & 1 & 36 & 37 & 38 & 39 & 40 \\ 41 & 42 & 43 & 44 & 45 & 46 & 47 & 48 \end{pmatrix} \to$$

$$\begin{pmatrix} 1 & 4 & 3 & 2 & 7 & 6 & 5 & 8 \\ 9 & 34 & 11 & 12 & 13 & 14 & 15 & 16 \\ 17 & 26 & 19 & 20 & 21 & 22 & 23 & 24 \\ 25 & 18 & 27 & 28 & 29 & 30 & 31 & 32 \\ 33 & 10 & 35 & 36 & 37 & 38 & 39 & 40 \\ 41 & 42 & 43 & 44 & 45 & 46 & 47 & 48 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\ 17 & 18 & 19 & 20 & 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 \\ 33 & 34 & 35 & 36 & 37 & 38 & 39 & 40 \\ 41 & 42 & 43 & 44 & 45 & 46 & 47 & 48 \end{pmatrix}$$

2.3. Mathematical conclusion in Rubik's Cube

The Rubik's cube is a tool that embodies many definitions and related properties in mathematical group theory, such as action, displacement, transitivity, originality, orbit and other concepts are represented in the Rubik's cube, and the main function of conjugation and commutation in the process of rotating the Rubik's cube is to simplify complexity. Use E_A to represent all sides of the cube, V_A to represent all corners of the cube, E_B to represent all sides of the cube, and V_B to represent all corners of the cube. Obviously the union of E_A and V_A is A, and the intersection is the empty set; The intersection of E_B and V_B is B and is the empty set.

Theorem 1 Inintersectable loops in a Rubik's cube group S are commutative.

Theorem 2 All exchanges in a Rubik's cube group S are products of disjoint cycles.

Theorem 3 Disjoint permutations in a Rubik's cube group are commutative.

Theorem 4 The rotations of disjoint surfaces of a Rubik's cube group are commutative.

Theorem 5 The action of the Rubik's cube group S on E_A, E_B, V_A and V_B is transitive.

Theorem 6 If two elements of a Rubik's cube group have the same cyclic structure, they are conjugate (omitted from the above proof).

Commutator: In the process of rotating the Rubik's cube, there are 5× 4=20 facets redistributed every time the cube is rotated, and the commutator only changes a small part of the facets or blocks during the rotation process, which simplifies the function of the cube restoration process. Assuming $[h, i]=hih^{-1}i^{-1}$, it means that the Rubik's cube is rotated by hi first, and then by $H^{-1}i^{-1}$. In this process, it can be verified that: $[h, i]^2$ changes 3 edge blocks without changing the corner blocks; $[h, i]^3$ changes 2 diagonal blocks without changing the side blocks.

3. Conclusion

In this paper, the origin, development history and current situation of Rubik's cube at home and abroad are reviewed, and a series of concepts of Rubik's cube are basically defined. In addition, the mathematical knowledge of linear algebra and group theory is used to give the relevant knowledge points of group theory involved in Rubik's cube, such as action, displacement, transitivity, conjugation, commutation, etc. The mathematical reduction model and related application of Rubik's cube are studied through mathematical knowledge.

References

[1] Diaconu, A. V., & Loukhaoukha, K. (2013). An improved secure image encryption algorithm based on rubik's cube principle and digital chaotic cipher. [J] Mathematical Problems in Engineering, 2013,(3): 1-10.

[2] Hong Shick Jang. Math Modelling, Storytelling, the Metaphor and Metonymy in modelling Permutation of Rubik`s Cube.[J].2013(9).

[3] Hobbs, J. Rubik's cube as a teaching tool for mathematics and science. School Science & Mathematics, 1985, 85(5): 367–373.

[4] Diaconu, A. V, & Loukhaoukha, K. (2013). An improved secure image encryption algorithm based on rubik's cube principle and digital chaotic cipher[J]. Mathematical Problems in Engineering,2013:1-10.