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A brief review of semi-Markov jump systems

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Abstract

Semi-Markov Jump Systems (SMJSs), as a special stochastic control system, has become a research hotspot in the field of automatic control and probability analysis because of its wide application in model uncertainty and decision process. In this paper, the basic concept, main theoretical basis, main achievements and research prospects of semi-Markov jump systems are briefly reviewed. We first introduce the basic knowledge of semi-Markov process and the dynamic characteristics of the jump system, then discuss the stability analysis of the semi-Markov jump system, the research progress of the semi-Markov jump system, and the application in complex networks and multi-agent systems. Finally, we explore current challenges and possible future research directions in this field.

Keywords

Semi-markov jump system, complex network synchronization, multi-agent consensus.

1. Introduction

Semi-Markov Jump Systems (SMJSs) are a class of dynamic systems with random jump characteristics, which are very common in practical applications, such as economic systems, industrial process control, biomedical engineering, communication networks, etc[1]. The characteristic of this kind of system is that the transformation of the system state not only depends on the current state, but also is affected by the experienced time, which makes the semi-Markov jump system more accurate than the traditional Markov jump system to describe the behavior of many real world dynamic systems[2].

The study of semi-Markov jump systems is very important for understanding and controlling complex systems with stochastic and temporal properties[3]. Because of its powerful modeling capabilities, SMJSs is used in many fields to improve system performance, reliability and efficiency. Especially when the randomness and delay in the system need to be accurately modeled and analyzed, the semi-Markov model shows its unique advantages. Studying the stability, control strategy and optimization algorithm of such systems can significantly improve the safety of engineering design, and also provide a new perspective for dealing with high complexity problems[4].

The study of semi-Markov jump systems originated from the study of Markov and semi-Markov processes in the middle of the 20th century[5]. Initially, researchers focused on the fundamental and statistical properties of these random processes. Over time, researchers have begun to apply these random processes to the modeling of real systems, especially in situations where state transitions are clearly time-dependent. Since semi-Markov processes were introduced into the study of jump systems, the theory and application of SMJSs have gradually expanded, forming a multidisciplinary research field[6].

In the study of semi-Markov jump systems, some basic problems are constantly proposed and explored. (1) System identification: how to accurately identify and estimate the parameters of

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semi-Markov jump systems from data, including the jump rate, transition probability and dwell time distribution[7]. (2) Stability analysis: how to judge the stability of the system under various random influences, including probabilistic stability, moment stability and almost certain stability[8-9]. (3) Control and optimization: how to design control strategies to optimize the performance of the system to achieve stable control, robust control and adaptive control[10-11]. (4) Performance evaluation: How to evaluate the performance of the system under different working conditions, involving fault detection, performance optimization and other aspects[12].

2. The main results already achieved

2.1. Advances in semi-Markov jump systems

Stability is the core problem in the study of semi-Markov jump systems. In control theory, stability usually refers to the ability of a disturbed system to return to an equilibrium state over time or to remain within a bounded region. For a system with a semi-Markov jump, the definition of stability can be divided into the following categories: (1) Probabilistic stability: A system is considered to be probabilistically stable if the state of the system, starting from any initial distribution, converges over time to a steady-state distribution with the probability of 1. (2) Moment stability: A system is considered to have moment stability if a moment of its state (such as a first or second order moment) is bounded or tends to zero with time. (3) The system is considered to be almost necessarily stable if the state of the system is almost necessarily (i.e., with probability 1) towards an equilibrium point or bounded region [5].

The stability analysis of semi-Markov jump systems usually involves complex mathematical tools and methods, mainly including: ① Random Lyapunov function: By constructing an appropriate non-negative function, the expected value of the function decreases along the trajectory of the system, thus proving the stability of the system[8]. ② Probabilistic analysis: Use probabilistic and statistical methods to directly analyze the state transition probability and residence time distribution of the system, so as to evaluate stability[13]. ③ Moment equation method: Establish the dynamic equation of the system state moment, and study the moment stability of the system state by analyzing these equations[14].

In the field of stability research of semi-Markov jump systems, scholars have made a series of important achievements. Reference [4] studies the stability analysis and stabilization of random switching systems under a class of switching signals, in which the switching signals are assumed to be semi-Markov switching. The almost inevitable stability, mean square stability (first moment stability) and probabilistic stability of switching systems are considered, and the corresponding sufficient conditions are given. Reference [5] studies the asymptotic stability of semi-Markov switching stochastic systems. Based on the multiple Lyapunov function method and the semi-Markov process structure, the sufficient conditions for the stochastic asymptotic stability of a semi-Markov switching random system without bound transition rate constraint are given. The generalized moment stability of nonlinear systems with semi-Markov jumps is discussed in reference [6]. Based on this, the generalized moment stability of stochastic nonlinear systems with semi-Markov jumps is further derived in reference [15]. It is pointed out in reference [16] that the additional restriction of residence time is not considered in previous literatures when analyzing the stability of semi-Markov hopping linear systems. To solve this problem, literature [17] proposes a new stochastic analysis method under the assumption of modular correlation linear comparability, and studies the exponential stability of semi-Markov stochastic nonlinear systems. It should be pointed out that the semi-Markov kernel method is an effective method to solve the stability and stabilization problem of the semi-Markov jump system because it can synthesize the statistical information of mode jump and dwell time [6]. In order to obtain numerically testable stability and stabilization conditions,

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[18-19] a bounded dwell time was introduced, and then a semi-Markov kernel method was adopted in literature [20-21] to solve the stability problem under unbounded dwell time.

The control problem of random switching system is one of the most important research fields in recent decades. As a kind of important random switching system, the control problem of semi-Markov jump system has been widely concerned by scholars in recent years. Reference [22] studies the anti-interference observer control of fuzzy chaotic semi-Markov jump systems with multi-disturbance hybrid actuator failure. Reference [23] studies the model-based fuzzy l2-l∞ filtering problem for a class of discrete semi-Markov jump nonlinear systems. The problem of asynchronous generalized H2 control for a class of continuous discrete state semi-Markov jump linear systems is studied in reference [24]. The sliding mode control is considered to be an effective control method because of its strong robustness to model uncertainties, parameter changes and external disturbances. It is worth mentioning that sliding mode control has been successfully applied to various practical systems, such as aircraft navigation and control, power system stabilizers, etc. Therefore, the problem of sliding mode control design receives more and more attention. In reference [25], the state estimation and sliding mode control of phase type semi-Markov jump systems are studied. In literature [26], the sliding mode control problem of a class of random switching systems with semi-Markov processes is studied by using the adaptive event triggering mechanism. Reference [27] uses the output feedback method of integral resident time distribution function to deal with sliding mode control of continuous-time semi-Markov jump systems.

2.2. Applications in complex networks and multi-agent systems

A complex network is a network consisting of a large number of nodes and the edges that connect them. These networks are characterized by a complex and dynamic structure, in which each node is a basic entity with a specific system. The theory of semi-Markov jump systems can provide a deep understanding and effective control of network dynamics. Since complex networks can describe many real-world systems, such as the World Wide Web, epidemic transmission networks, power grids, and cellular neural networks, this research direction has attracted a lot of attention. The problem of fault-tolerant synchronization control for complex dynamic networks with semi-Markov jump topology is studied in reference [28]. Reference [29] studies the dissipative synchronization problem of complex dynamic networks with semi-Markov switching topology. Reference [30] studies the finite-time H∞ synchronization problem of complex networks with time-varying delay and semi-Markov jump topology. Multi-agent system is a kind of system which is closely related to complex network. They are two related but distinct concepts that differ in their object of study, focus, and field of application. Complex networks focus on the structural properties of networks (such as small-world properties, scale-free properties) and the dynamic processes on the network (such as propagation, synchronization). The multi-agent system is composed of a group of interacting agents, and focuses on the autonomy, distribution and cooperation and competition between agents on this basis. Each agent in a multi-agent system can have its own goals, decision-making capabilities, and action strategies, and they can be software agents, robots, or humans. In the research of multi-agent systems, people focus on how to design protocols and algorithms to achieve effective cooperation and coordination among agents.

As a key problem in the research of multi-agent systems, the consistency problem has always been the focus of scholars. The basic goal of consistency is to design a suitable consistency control protocol, which can realize the convergence of all agents to a common value only through local information exchange with neighboring agents. In recent years, the consistency problem of semi-Markov hopping multi-agent systems and multi-agent systems with semi-Markov switching topology has received a lot of attention. Reference [31] studies the reliable leader-follower consistency of discrete-time semi-Markov hopping multi-agent systems.

Reference [32] studies the consistency of semi-Markov hopping multi-agent systems with random mismatched topology. Reference [33] studies the extended dissipative finite time distributed time-varying delay active fault-tolerant consistency control for semi-Markov jump nonlinear multi-agent systems. Reference [34] studies the mixed event leader-following consistency problem of nonlinear multi-agent systems with semi-Markov jump parameters. Reference [35] studies the leader-following consistency of semi-Markov jump nonlinear multi-agent systems under hybrid network attacks. Reference [36] studies the adaptive event triggering and biquantization consistency problems of leader-attendant multi-agent systems with semi-Markov jump parameters. It should be pointed out that the above literature studies the consistency problem of the equation of state with jump parameters of multi-agent systems. In fact, the communication topology between agents often changes during their movement. The way to deal with this time-varying topology in the existing literature is to model it as a random switching topology. However, semi-Markov processes have attracted much attention in recent years because they can describe a wider range of random time-varying topologies. Reference [37] studies the consistency control problem of nonlinear multi-agent systems with semi-Markov switching topology and incremental quadratic constraints. Reference [38] studies the pulse consistency of random multi-agent systems in semi-Markov switching topology and its application.

In literature [39], the event-triggered adaptive consistent tracking control for non-affine multi-agent systems is studied. Reference [40] studies the $H\infty$ consistency problem of multi-agent systems with semi-Markov switching topology and mode-dependent delay. Reference [41] studies the finite-time fault-tolerant consistency of pilot-following multi-agent systems in semi-Markov interactive topology. Reference [42] studies the proportional consistency of multi-agent systems with semi-Markov switching topology from the perspective of probability. It should be noted that literature [39-42] only considers the communication topology as semi-Markov switching. In the real multi-agent system, randomness not only appears in the communication topology, but also in the system dynamics equation. In this paper [43], the observation consistency problem based on adaptive event triggering mechanism for multi-agent systems with semi-Markov switching topology and semi-Markov dynamics is also considered.

3. Future research

In the future work, the semi-Markov jump system still has the following problems worthy of further study:

(1) Adaptive fault-tolerant cooperative control of nonlinear semi-Markov hopping multi-agent systems in fading channels;

(2) The consistency and cooperative control of heterogeneous semi-Markov hopping multi-agent systems under network attack are discussed;

(3) The finite time fault-tolerant consistency problem of multi-agent systems with semi-Markov jump dynamics and multi-type actuator faults is discussed.

(4) Considering the multi-agent system cooperative control problem under the influence of multiple factors such as noise, event triggering, random communication and network attack;

(5) The quantitative consistency, finite time consistency and fault-tolerant control problems of generalized semi-Markov jump multi-agent systems are studied.

(6) Study the consistency of hidden semi-Markov hopping multi-agent systems and hidden semi-Markov switching topology;

(7) Considering the pulse control, asynchronous control and filtering of semi-Markov jump fuzzy systems under state delay and spoofing attacks;

(8) Consider the stability, stabilization, quantization control, dissipation control and event-triggered control of two-dimensional semi-Markov jump systems;

(9) Consider the application of semi-Markov jump systems to practical problems such as financial systems, power network systems and sensor networks.

4. Conclusion

In this paper, we briefly review the literature on semi-Markov jump systems and their applications in complex networks and multi-agent systems in recent decades. However, it is not difficult to see that the stability, stabilization, control, filtering of semi-Markov jump systems, complex network synchronization with semi-Markov switching topology, multi-agent system consistency and other problems have been widely concerned by scholars. Relevant research results have been published in major journals in the field of Automatic Control, such as Automatica, IEEE Transactions on Automatic Control, and Systems & Control Letters. It also shows that the use of semi-Markov process to describe the randomness of the system has been recognized by scholars in this field. The relevant theoretical research is bound to be further developed in the future, which will also promote the application of the theoretical achievements of semi-Markov jump systems in practice.

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