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Dynamic analysis of gear system considering tooth breakage fault

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Abstract

This article first derives an analytical expression for the position related meshing stiffness of healthy gears based on the potential energy method, and combines it with the characteristics of involute gear teeth to derive the expression for the meshing stiffness of gears with tooth breakage faults. An analysis and comparison were conducted on TVMS under different tooth breakage sizes. The TVMS results indicate that with the increase of tooth breakage size, the separation time of the gear pair is significantly advanced, and the meshing process is terminated prematurely. Finally, the influence of different tooth breakage sizes on the vibration response of gear systems was compared and studied. This study provides meaningful references for modeling, dynamic analysis, and fault diagnosis of gear transmission systems with tooth breakage faults.

Keywords

Gear system, tooth breakage, Dynamic analysis.

1. Introduction

Gear transmission system has the advantages of high efficiency, compact structure and long service life, and has been widely used in mechanical power transmission. Due to its complex structure, it usually operates under high speed and heavy load conditions, and the gear tooth failure frequently occurs, which will lead to serious equipment accidents. Understanding the dynamic characteristics of broken tooth faults is essential to reduce equipment downtime.

Among the common gear failure modes, tooth breakage is the most dangerous kind of failures modes, seriously affect the normal operation of the gear system. Tian et al.[1] solved the TVMS of spur gear pair considering tooth broken fault by the analytical method and the mesh stiffness became zero in the single tooth contact zone. Chaari et al.[2] and Ma et al.[3] discussed the effects of tooth fracture (also known as tooth breakage) on mesh stiffness, and the dynamic responses of tooth breakage were analyzed using the one-stage spur gear dynamic model. Therefore, the vibration responses of broken teeth and healthy teeth are compared in this paper. The necessity of considering broken teeth in spur gear transmission system is expounded.

This article is divided into four parts. This section provides an overview of the current status of simulation methods for gear fracture systems both domestically and internationally. In the following chapters, a simulation model was established. Then, in the third part, the TVMS and vibration signals under different degrees of tooth breakage faults were analyzed. The last section drew some conclusions.

2. Mesh stiffness calculation

2.1. Calculation of time-varying meshing stiffness of healthy gear teeth

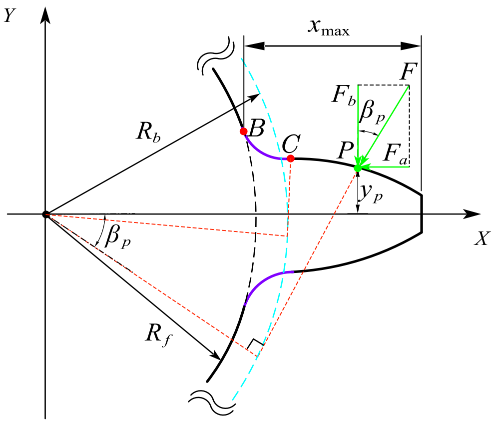

Fig 1 Cantilever beam model of healthy spur gears.

According to the elastic potential energy method in reference[4], considering the Hertz contact energy, bending potential energy, shear potential energy, and axial compression energy during gear meshing process, calculate the comprehensive meshing stiffness of gear. The potential energy stored in the meshing gear is calculated as follows:

$$U_a = \frac{F^2}{2k_a} = \int_{x_C}^{x_B} \frac{F_a^2}{2EA_{1x}} dx_1 + \int_{x_B}^{x_P} \frac{F_a^2}{2EA_{2x}} dx_2 \tag{1}$$

$$U_b = \frac{F^2}{2k_b} = \int_{x_C}^{x_B} \frac{\left[F_b(x_p - x_1) - F_a y_p\right]^2}{2EI_{1x}} dx_1 + \int_{x_B}^{x_P} \frac{\left[F_b(x_p - x_2) - F_a y_p\right]}{2EI_{2x}} dx_2 \tag{2}$$

$$U_s = \frac{F^2}{2k_s} = \int_{x_C}^{x_B} \frac{1.2F_b^2}{2GA_{1x}} dx_1 + \int_{x_B}^{x_P} \frac{1.2F_b^2}{2GA_{2x}} dx_2 \tag{3}$$

Yang and Sun[5] assumed that the Hertzian contact stiffness of the two meshing teeth in the meshing process is a constant, and the deduced expression of the Hertzian contact stiffness is given:

$$k_h = \frac{\pi E b}{4\left(1-\nu^2\right)} \tag{4}$$

Where ν is the Poisson's ratio of material of gear tooth and b represents tooth width.
Therefore, the total equivalent mesh stiffness of one tooth pair can be calculated as:

$$k_t = \frac{1}{\left(\frac{1}{k_h} + \frac{1}{k_{b1,1}} + \frac{1}{k_{a1,1}} + \frac{1}{k_{s1,1}} + \frac{1}{k_{b2,1}} + \frac{1}{k_{a2,1}} + \frac{1}{k_{s2,1}}\right)} \tag{5}$$

2.2. Calculation of time-varying meshing stiffness of gear with broken teeth

Suppose a tooth on the pinion gear begins to break from the top of the tooth. If the tooth remains intact, as it rotates to the meshing area, it will engage with one tooth of the mating gear. In the double tooth meshing area, two pairs of teeth engage simultaneously. In the single tooth meshing area, only one pair of teeth engages. However, when a tooth breakage fault occurs in the double tooth meshing area, only one pair of teeth engages due to the loss of contact between the teeth at the missing position; when a tooth breakage fault occurs in the single tooth meshing area, there is no tooth contact, resulting in zero meshing stiffness.

Therefore, the total equivalent meshing stiffness of a broken tooth can be calculated as:

$$k_{tb}=\begin{cases}\dfrac{1}{\left(\dfrac{1}{k_h}+\dfrac{1}{k_{b1,1}}+\dfrac{1}{k_{a1,1}}+\dfrac{1}{k_{s1,1}}+\dfrac{1}{k_{b2,1}}+\dfrac{1}{k_{a2,1}}+\dfrac{1}{k_{s2,1}}\right)}\\ 0\end{cases} \tag{6}$$

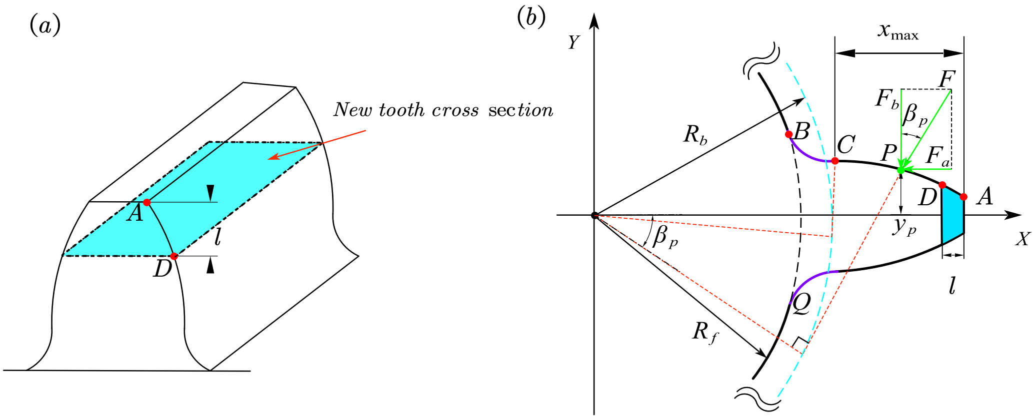

Fig 2 Cantilever beam model of spur gear with broken teeth

Fig 2 are the broken tooth model and the uniform cantilever beam model. In Fig 2 (a), the tooth fracture starts from the tip of the tooth, the cutting width is the tooth thickness and the tooth broken heights is l. Fig 2 (b) shows a cantilever beam model with a broken tooth. D is the starting point of the chip part. x_D is the distance from point D along the tooth height direction to the center of the circle, $x_{\max}$ is the distance from the root to the tip, which can be expressed as:

$$x_{\max}=R_b\left[\left(\alpha_\alpha+\alpha_2\right)\sin\alpha_\alpha+\cos\alpha_\alpha\right] \tag{7}$$

The length of any broken tooth can be represented as:

$$l=x_{\max}+x_C-x_D \tag{8}$$

Where α_α and α_2 can be expressed as:

$$\alpha_\alpha=\sqrt{\frac{R_a^2}{R_b^2}-1}-\alpha_2 \tag{9}$$

$$\alpha_2=\frac{\pi}{2Z}+\tan\alpha_0-\alpha_0 \tag{10}$$

The parameters of the gears used in this article are shown in Table 1.

Table 1 Parameters of gears

Parameter	Value
Number of teeth	23/23
Tooth width (mm)	10
Modulus of elasticity (GPa)	207
Initial pressure angle	20
Poisson's ratio	0.3
Input torque (N·m)	50

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3. Results and discussion

3.1. TVMS considering tooth broken fault

This article calculates the TVMS with a broken tooth rate of 0%, 20%, and 50%. The impact of TVMS with different broken tooth rates is shown in Fig 3.

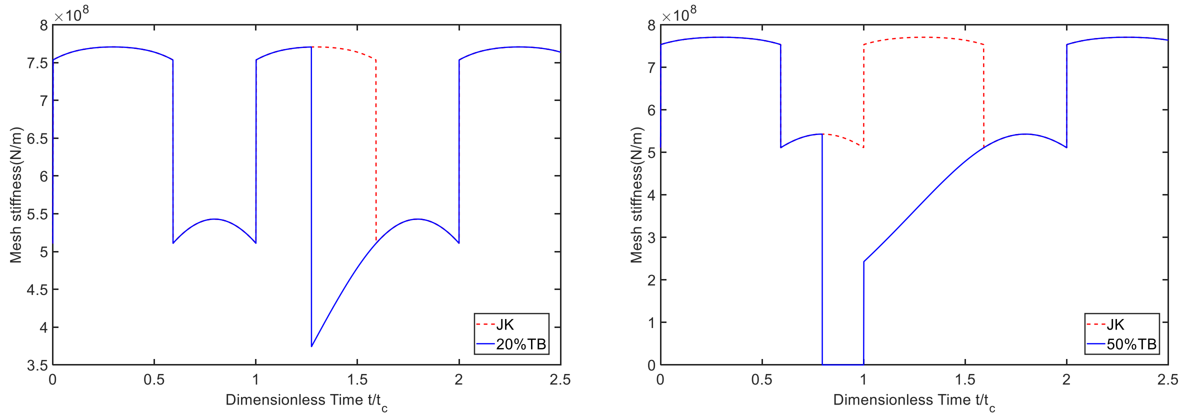

Fig 3 The comparisons of the TVMS considering various tooth broken lengths

If the height of the broken tooth only damages the second double tooth area (such as TB20%), the decrease in time-varying meshing stiffness caused by the broken tooth fault is only reflected in the second double tooth area, while the single tooth area and the first double tooth area remain unchanged. In the second double tooth region, due to tooth breakage fault, only one pair of teeth participate in meshing, and the load-bearing capacity of the gear pair is weakened. As the height of the broken teeth increases, the single tooth area is damaged (such as TB50%), and the gear meshing state undergoes a qualitative change. No load transfer errors have started to occur. If the gear pair with tooth breakage fault rotates strictly according to the transmission ratio, it will be affected by the no-load transmission error, and the abnormal meshing state of "disengagement" will occur in the single tooth area. Under the influence of tooth breakage fault, the time-varying mesh stiffness in the single tooth area is 0.

3.2. Dynamic response of the gear system

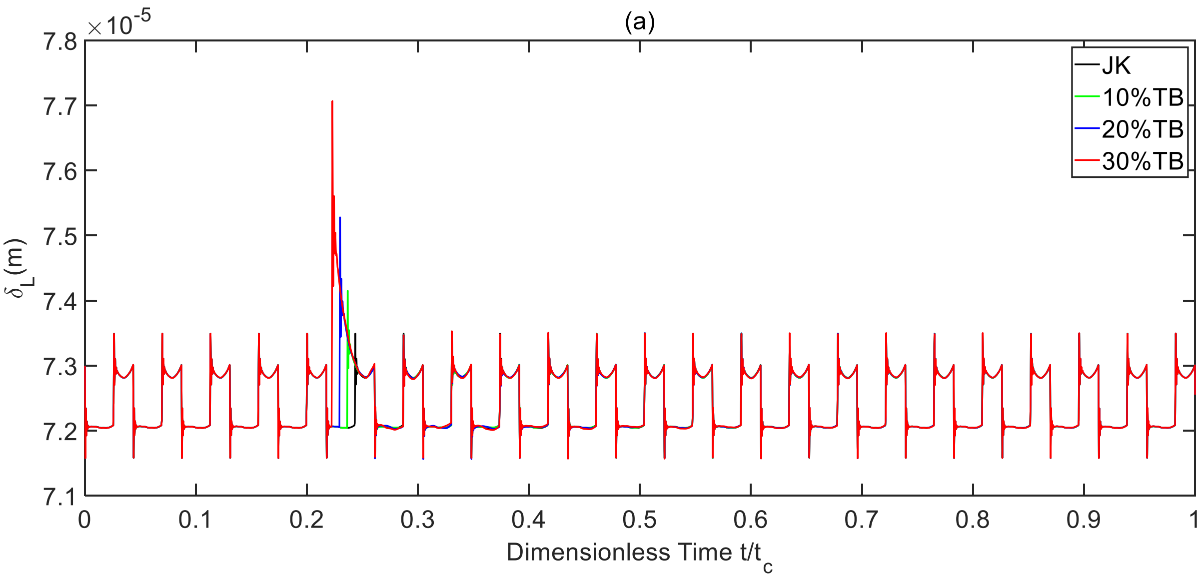

Fig 4 Vibration responses of gear pairs with a tooth broken fault

The vibration response of the gear pair under different tooth breakage sizes is illustrated in Fig 4. It can be observed from the figure that when all teeth are in a healthy state, the amplitude of the vibration response remains constant. However, when a fault occurs due to broken teeth on

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the pinion gear, periodic pulses are generated, which correlate with the rotation period of the pinion. Significant impacts occur during tooth contact at the breakage point, leading to a marked increase in vibration response; as the gears pass through the broken tooth region, this response gradually stabilizes. Furthermore, it is noted that the magnitude of periodic pulses induced by broken teeth increases with an increase in breakage length.

4. Conclusion

This article uses the energy method to establish a stiffness model for gear tooth breakage faults and solves the stiffness excitation during gear tooth breakage faults. Studied the meshing stiffness and vibration response of gears under different fault sizes. The conclusion of this article is as follows:

(1) Broken teeth reduce the meshing stiffness of gears and increase dynamic transmission errors. Significant dynamic response oscillations were generated in the broken tooth area.

(2) With the size of the fault increases, the movement of the stiffness of the faulty gear pair shows an earlier trend. When the fault size is large, the stiffness of the faulty gear pair may be 0, causing the gear system to be in an unstable state.

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