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Time-varying reliability analysis based on Monte Carlo method

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Abstract

Since the parameters and loads of the structure change with time during service, it is difficult to measure the influence of time on the structural behavior by deterministic reliability analysis. Therefore, a time-varying reliability analysis based on Monte Carlo method is proposed. Firstly, the proposed method simplifies the original complex time-varying reliability problem by analyzing the time-varying factors of the structure. Then, multiple sample points are sampled near the MPP point of the random variable by Monte Carlo (MCS) sampling method. Finally, the sample points are substituted into the performance function, and the failure probability is obtained by comparing the total cumulative failure times with the total times. The effectiveness of the proposed method is proved by an engineering example.

Keywords

Failure probability; Monte Carlo method; Time-varying reliability analysis.

1. Introduction

In practical engineering, the general reliability research considers the influence of random factors such as performance degradation and random load on the structure under different working conditions. However, most factors tend to change over time. For example, metal rust leads to a decrease in resistance, and submarine metal corrosion leads to the changes of structure. All of these will reduce the reliability. Therefore, in order to ensure safety and reliability, it is necessary to study the time-varying reliability. As early as 1944, Rice [1] studied the crossing problem between dynamic response and prescribed threshold, and proposed the famous first-passage formula, which laid a foundation for the application of first-passage theory in structural time-varying reliability assessment. Siegert [2] provided a solution method for the first passage rate of the Markov process in the integral equation, and then many researchers have continuously improved and perfected the formula. Lv [3] used the PHI2 method based on the evolution of the spanning method to carry out the time-varying reliability analysis of the CRTS II base plate during fatigue. In recent years, time-varying reliability analysis has developed rapidly, and more research results have been obtained, such as: Wang [4] proposed the nested extremum response surface method and the quasi-static [5-6] method based on the time discretization method. However, when the above methods are used to analyze the structural reliability of nonlinear problems, they will become extremely difficult to replicate or have insufficient accuracy. In this case, the Monte Carlo method (MCS) can solve such problems well, and the time-varying reliability of the structure can be calculated by random sampling of a large number of object samples. Au [7] et al. proposed a stratified sampling method, which was improved on the basis of Monte Carlo method. Jing et al. [8] proposed a time-varying reliability solution method based on process discretization. This method discretizes the random process into several equivalent random variables, which simplifies the calculation process. Therefore, this paper uses Monte Carlo method to analyze the time-varying reliability of the structure, and verifies the feasibility of the proposed method through an engineering example.

2. Structural reliability analysis based on Monte Carlo method

The structural reliability is the probability that the structure completes the predetermined function within the specified time. Let a set of random variables $X = (x_1, x_2, ..., x_n)$ indicate the uncertain factors in the structure. Let the limit state function of the structure be Z=$g(X)$, when $g(X)$<0 represents the failure of the structure. $g(X)$=0 is in the limit state, and $g(X)$>0 represents that the structure is stable and reliable. If the random variables are independent of each other, the failure probability is:

$$p_f = P(Z \leq 0) = \int_{-\infty}^{0} f_Z(z)dz = F_Z(0) \tag{1}$$

In the formula: P is the probability operator; f_z is the probability distribution function; F_z is the cumulative distribution function of random variables. The Monte Carlo method is a large number of sampling through random variables, and the failure probability is obtained by statistics of the sampling results. The structural failure probability can be written as

$$p_f = \frac{\sum_{i=1}^{N} I[g(X)]}{N} \tag{2}$$

In the formula : $I[g(X)]$ is an index function:

$$I[g(X)] = I[g(x_1, x_2, ..., x_n)] = \begin{cases} 1, g(X) < 0 \\ 0, g(X) \geq 0 \end{cases} \tag{3}$$

N is the total number of sampling.

3. Time-varying reliability analysis

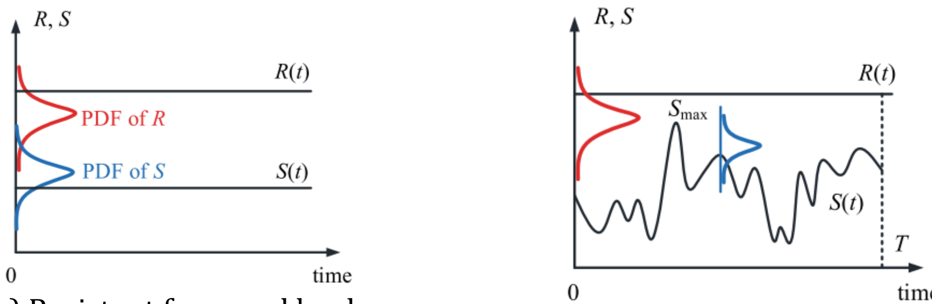
Structural reliability is defined as the probability that the structure completes the predetermined function under the expected service conditions within the service life. Time-varying reliability is to consider the influence of time factor. In general, the time-varying limit state equation of structure is defined as a linear or nonlinear equation composed of structural resistance random process $R(t)$ and load effect random process $s(t)$, and the expression is:

$$Z(t) = g(R(t), S(t)) \tag{4}$$

In the formula: t is the time variable; $R(t)$ is resistance; $s(t)$ is the load. The performance function is given as

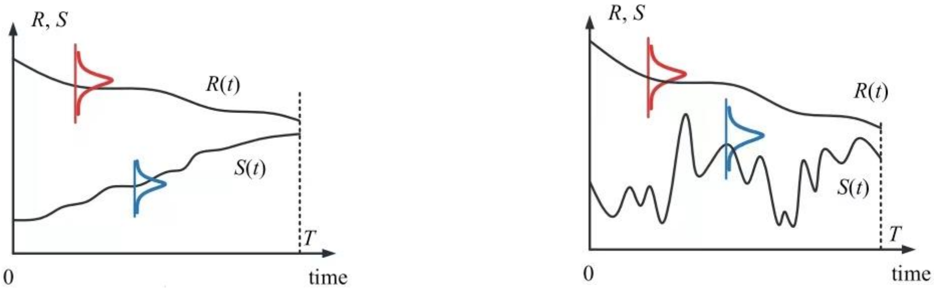
$$Z(t) = R(t) - S(t) \tag{5}$$

At the same time, the resistance and load change with time are given as Fig. 1



(a) Resistant force and load

(b) Resistance constant load random variation



(c) Resistance decreases year by year and Load increases year by year

(d) The resistance decreases year by year and the load changes randomly

Fig. 1 The process of resistance and load changing with time

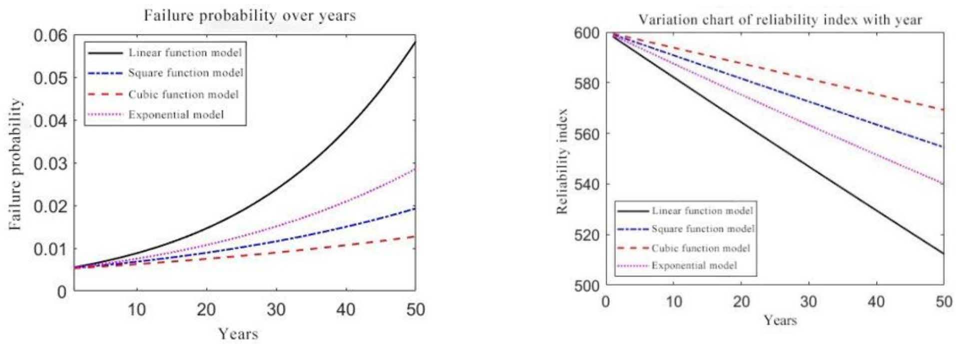
In Fig.1 (a), the resistance and load do not change with time, and the time-invariant model is considered; (b) The load will change randomly with time and the resistance is time-invariant; (c) The resistance decreases with time and the load increases with time; (d) The resistance decreases with time, while the load changes randomly.

For resistance attenuation, there are a variety of attenuation models, including common linear attenuation, exponential attenuation, logarithmic attenuation, etc. Assuming that the load is constant, the resistance attenuation model formula is given as:

$$k_1 = 1-\alpha^{\frac{1}{r}}(T), R(t) = R_0\left(1-k_1\frac{n}{T}\right)^r \begin{cases} R(t) = R_0(1-k_1\frac{n}{T}), r=1 \\ R(t) = R_0(1-k_1\frac{n}{T})^2, r=2 \\ R(t) = R_0(1-k_1\frac{n}{T})^3, r=3 \end{cases} \quad (6)$$

$$k_2 = -In\alpha(T), R(t) = R_0 e^{(-k_2\frac{n}{T})} \quad (7)$$

Eq. (6) is a power function attenuation model, and Eq. (7) is an exponential attenuation model. Among them, k_1 and k_2 are experimental engineering and empirical coefficients; T is the service life of the structure; n is the year of work; R_0 is the initial resistance. In the power function attenuation model, when $r=1$, it is a linear attenuation model; when $r=2$, it is a square attenuation model; when $r=3$, it is a cubic attenuation model. The failure probability and reliability index of different attenuation models with the increase of year are given as shown in Fig.2.



(a) (b)

Fig. 2 (a) is the failure probability varies with time and (b) is the reliability index changes with time

4. Time-varying reliability analysis of submarine steel

It is assumed that there are four reinforcing bars in the steel working on the seabed. The performance of the steel bar will gradually decline with the chloride ion penetration in the seawater. When the chloride ion penetrates into a certain threshold, the area of the steel bar will become smaller due to corrosion. The area change is given as

$$\Delta A_s = k\psi_0 \times i_{cor} \times (t - t_i), t > t_i \tag{8}$$

In the formula, A_S is the area of steel bar; ψ_0 is the diameter of steel bar; i_{cor} is chloride ion corrosion rate; t_i is the corrosion initiation time (chloride ion reaches the threshold time). The model diagram of the entire steel is given as shown in Fig. 3.

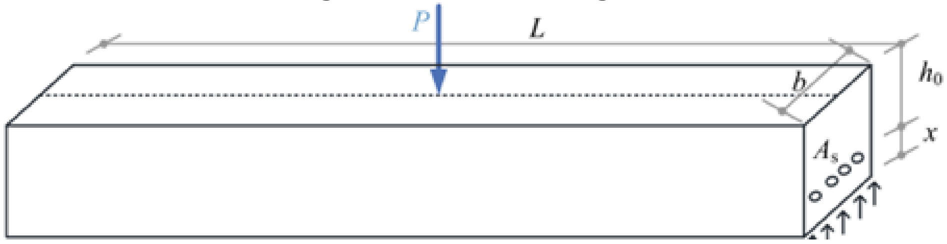


Fig. 3 steel model

The resistance change formula of the steel due to the change of the area of the steel bar is given as

$$\begin{aligned} M_u(t) &= f_y A_s (h_0 - \frac{f_y A_s}{2 f_c b}) \\ &= f_y (A_0 - \Delta A_s) \left[h_0 - \frac{f_y (A_0 - \Delta A_s)}{2 f_c b} \right] \end{aligned} \tag{9}$$

In the formula, f_y is tensile strength; f_c is the compressive strength of concrete; b is the width of the cross section; h_0 is the effective depth of the section; ΔA_s is the change area of steel bar. Suppose that in (9), $k = 0.0336$, i_{cor} obeys lognormal distribution. The average value is $\frac{0.67\mu A}{cm^2}$. The coefficient of variation COV is 0.58. The corrosion initiation time t obeys normal distribution, the average value is 30 years, and the coefficient of variation COV is 0.2. A total of 4 steel bars with a diameter of 18 mm are evenly placed at a depth of x from the surface. The compressive strength of concrete is $20MPa$, and the tensile strength of steel bar is $335MPa$. According to the geometric shape of the beam section, $b = 250mm$,$h_0 = 460mm$. The initial cross-sectional area of the four steel bars is given as:

$$A_0 = 4 \times \pi \times (\frac{18}{2})^2 = 1018mm^2 \tag{10}$$

Before the corrosion begins, the corrosion resistance (ultimate bending moment-bearing capacity) is calculated according to formula (9) as

$$M_u(t) = f_y A_s (h_0 - \frac{f_y A_s}{2 f_c b}) = 145.2KN \cdot m \tag{11}$$

When the Monte Carlo-based method is used to generate the sample trajectory of M_u over time, the process of each simulation run is given as follows:

(1) Based on the Monte Carlo method and the mean and standard deviation of random variables i_{cor} and t , 10^7 sample points are generated;

(2) Through a given time change, according to $\Delta A_s = 4 \times 18 \times 0.0366 \times max(0, t - t_i)$ to update the steel bar area change;

(3) Update the resistance value according to Formula (9).

The service time of a given structure is 100 years, and the time-varying resistance $M_u(t)$ changes with time as Fig.4. According to the diagram, it can be seen that based on 1000 samples, the resistance decreases by about 13% on average with time, and the maximum amplitude decreases by nearly 20%:

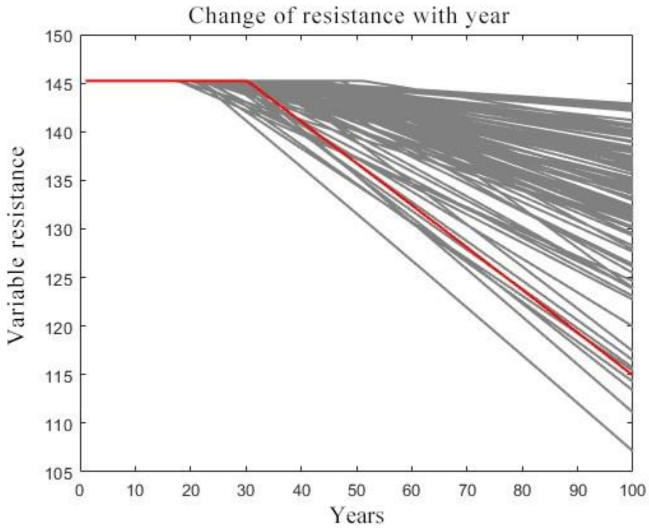


Fig. 4 Resistance changes with the year

In order to evaluate the reliability of the structure, a concentrated load $P = 160KN$ is set at the midpoint of the steel, which also satisfies the normal variable with a mean value of 160 and a standard deviation of 5. Because the beam is a simply supported beam, the initial bending moment generated by the concentrated load is given as

$$M_p = \frac{PL}{4} = \frac{1}{4} \times 16 \times 3 = 12KN \cdot m \tag{12}$$

Therefore, the limit state function of the structure is $g(X) = M_u(t) - M_p$. When $g(X) > 0$ represents the reliability of the structure, the Monte Carlo method is also used to sample the concentrated load. At the same time, a certain resistance sample is randomly selected and substituted into the limit state function to evaluate the structural reliability. Finally, the failure probability and reliability index with time are obtained as Fig. 5.

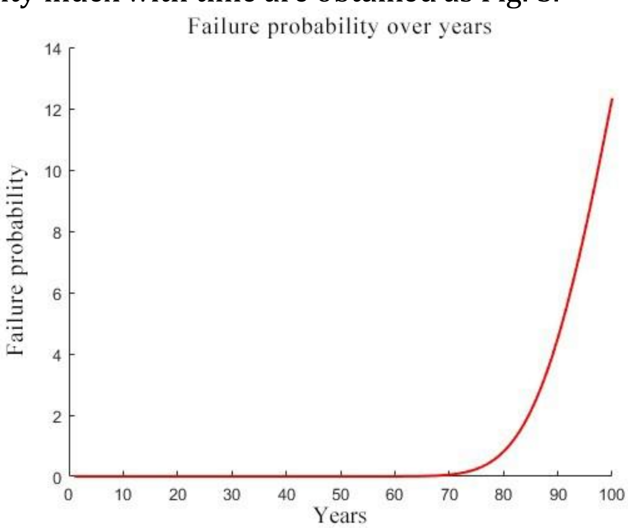


Fig. 5 The failure probability varies with time

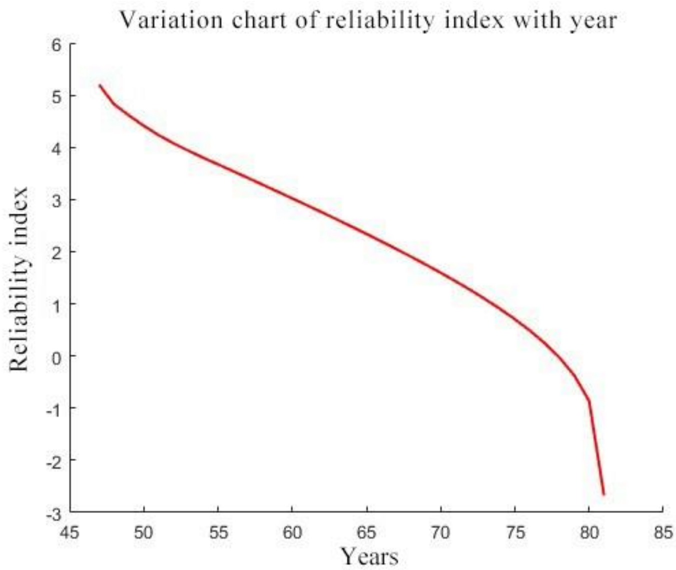


Fig. 6 The reliability index changes with time

It can be seen from Fig. 6 that the failure probability begins to increase in the past 65 years. Because the chloride ion reaches the threshold in this year, the steel bar area becomes smaller, which leads to the deterioration of the structural performance, thus the failure probability increases and the reliability index decreases. However, the maximum failure probability of the structure in 100 years is only 1.6×10^{-3}, so the structure is still relatively reliable.

5. Conclusion

In this paper, the time-varying reliability analysis of submarine structural steel is carried out by using Monte Carlo method. After calculation, the service time of the structure within 100 years is specified. The failure probability begins to increase when the chloride ion penetrates into the steel bar to reach the threshold, and the reliability decreases continuously. However, because the maximum failure probability is still within the target range, the structure is still relatively reliable. The proposed method in this paper avoids some mathematical problems in reliability analysis, and simplifies the complex time-varying reliability analysis problem into the relationship between chloride ion reaching the threshold and time, and it has a good ability to solve the problem.

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