Reliability Analysis Method Based on Beetle Swarm Optimization Algorithm

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Abstract

In this paper, a beetle swarm optimization (BSO) algorithm is proposed based on the beetle foraging principle. The performance of the proposed method is tested by numerical and engineering examples, and compared with the Monte Carlo method to verify the accuracy of the proposed method. Numerical experiments show that the BSO algorithm has good performance.

Keywords

Reliability analysis; Beetle Swarm Optimization Algorithm; Beetle Antennae Search Algorithm.

1. Introduction

The aerospace field is the first scientific research field to put forward the concept of reliability, and in the 1950s and 1960s, the earliest modern reliability calculation optimization method is successfully applied to the optimization design scheme of aircraft. The application of these methods is mainly reflected in the optimization design of engineering structure [1]. In the subsequent development, it has gradually become a hot research topic in the field of aerospace and other disciplines. Because of its wide application and not limited to the aerospace field, the rise of design optimization in other disciplines has also indirectly promoted the development of reliability computing. In the development of weapons and equipment, the United States began to fully implement the structural reliability program in the late 1960s. This initiative directly promoted the overall development of reliability engineering, and reliability engineering ushered in an era of rapid development. However, it is not only the United States that has realized the importance of reliability analysis. At the same time, countries such as France, Japan and the Soviet Union have also carried out reliability research one after another, making reliability research develop rapidly around the world.

According to the ratio of the number of failures to the total number of test samples, reliability can be obtained by testing a large number of individual products one by one. Luo and other scholars have carried out relevant research [2]. However, this is only applicable to some simple products and components. When faced with a large number of practical projects, this method is less available and will cause a lot of waste. Therefore, different methods of reliability have been gradually developed. With the increasing complexity of the mechanical structure, the demand for the accuracy of the analysis results in practical engineering has increased significantly, which makes it more difficult to optimize and call the high-confidence analysis model, which leads to too long optimization time and increased computational difficulty. In the face of such engineering problems, the traditional structural reliability analysis method cannot obtain satisfactory results or the economic cost and time cost required to obtain the results are unacceptable. Therefore, it is necessary to develop new structural reliability analysis theories and methods. Recently, the surrogate model method came into being [3-5]. In the later development of the surrogate model, a variety of methods have been developed, including polynomial response surface (RSM) [6-7], radial basis function (RBF) [8], neural network (NN)[9], support vector regression (SVR) [10], Kriging model [11], etc., which have become the key technologies in reliability analysis methods. The Kriging method and RBF method are also mixed and optimized.

2. Reliability Theory

2.1. Basic variables and response variables

The basic variable $x=\{x_1,x_2,\cdots,X_n\}$ has uncertainty, which determines the value of the system response. Geometric shape, material selection, load change and so on can be the basic variables. The response quantity is a function of the basic variables, which can be used to describe the behavior characteristics, strain and amplitude of the system. It can be marked as the response quantity, which is denoted as $r=\{r_1,r_2,\cdots,r_m\}$. There is an objective law between the response quantity and the basic variables. The essence of reliability analysis is to obtain the statistical law of response.

2.2. Limit state function and limit state equation

In order to describe the functional state of the system, it is defined as the limit state function or the functional function. It can be defined as the difference between the response and the threshold which is given as

$$g(x)=r_x-{r^*}_x \tag{1}$$

The equation $g(x)=r_x-{r^*}_x$ with the limit state function value of zero is the limit state equation, which can be regarded as the interface equation of the safety domain and the failure domain.

2.3. Failure domain and safe domain

When the system fails, it is said to be in the failure domain *F*, otherwise it is in the security domain *S*. When $F=\{s\,|\,g(x)\le 0\}$ is the failure domain, the corresponding $S=\{s\,|\,g(x)>0\}$ is the system security domain.

2.4. Failure probability and reliability

The probability of system failure P_f, that is, the probability of limit state function $g(x)\le 0$, can be expressed as

$$P_f=P\{F\}=P\{g(x)=r(x)-r^*\le 0\}=\int_F f_x(x,\theta_x)dx=\int_{g(x)\le 0} f_x(x,\theta_x)dx \tag{2}$$

On the contrary, the probability of $g(x)>0$ is called reliability P_r, expressed as

$$P_r=P\{S\}=P\{g(x)=r(x)-r^*>0\}=\int_S f_x(x,\theta_x)dx=\int_{g(x)>0} f_x(x,\theta_x)dx \tag{3}$$

And there is the following relationship between failure probability and reliability:

$$P_r+P_f=1 \tag{4}$$

3. Beetle swarm optimization algorithm

The Beetle Swarm Optimization (BSO) algorithm is optimized by the Beetle Antenna Search (BAS) algorithm. The performance of the BAS algorithm in dealing with high-dimensional functions is not very ideal, and the iterative results are very dependent on the initial position of the beetle. Inspired by the group optimization algorithm, we further improve the BAS algorithm and extend the individual to the group. This is the beetle swarm optimization (BSO) algorithm we will introduce.

In this algorithm, each beetle represents a potential solution to the optimization problem, and each beetle corresponds to a fitness value determined by the fitness function. Similar to particle swarm optimization, beetles also share information, but the distance and direction of beetles are determined by their speed and the intensity of information detected by their long tentacles.

In the mathematical form, we draw on the idea of particle swarm optimization. In S dimensional search space, there are n beetle populations, denoted by $X=(X_1, X_2, \ldots, X_n)$, where the i th beetle represents an S-dimensional vector $X_i=(X_{i1}, X_{i2}, \ldots, X_{is})^T$, represents the position of the i th beetle in the S-dimensional search space, and also represents the problem. According to the objective function, the fitness value of each beetle position can be calculated. The speed of the i th beetle is expressed as $V_i=(V_{i1}, V_{i2} \cdots, V_{iS})^T$. The individual limb of the beetle is denoted by $P_i=(P_{i1}, P_{i2} \cdots, P_{iS})^T$, and the population extreme value of the population is denoted by $P_g=(P_{g1}, P_{g2} \cdots, P_{gS})^T$. The mathematical model for simulating its behavior is given as

$$X_{is}^{k+1}=X_{is}^{k}+\lambda V_{is}^{k}+(1-\lambda)\xi_{is}^{k} \tag{5}$$

where $s=1,2,\cdots,S; i=1,2,\cdots,n$; k is the current number of iterations; V_{is} denotes the speed of the beetle; ξ_{is} is denotes the increase of the beetle 's position; λ is a normal number.

Then the speed formula is written as

$$V_{is}^{k+1}=\omega V_{is}^{k}+c_1 r_1(P_{is}^{k}-X_{is}^{k})+c_2 r_2(P_{gs}^{k}-X_{gs}^{k}) \tag{6}$$

where c_1 and c_2 are two positive constants; r_1 and r_2 are two random functions in the range of [0, 1]. ω is inertia weight. In the standard PSO algorithm, it is a fixed constant, but with the gradual improvement of the algorithm, many scholars have proposed strategies to change the inertia factor. In this paper, the strategy of reducing inertia weight is adopted. The formula is as follows:

$$\omega=\omega_{\max}-\frac{\omega_{\max}-\omega_{\min}}{K}*k \tag{7}$$

where $\omega_{\min}$ and $\omega_{\max}$ represent the minimum and maximum values, respectively. k and K are the current number of iterations and the maximum number of iterations. In this paper, the maximum value of ω is set to 0.9 and the minimum value is set to 0.4. The algorithm can search for a larger range at the beginning of evolution and find the optimal solution area as soon as possible. As the ω gradually decreases, the beetle 's speed decreases and then enters the local search.

The ξ function that defines the incremental function is calculated as

$$\xi_{is}^{k+1}=\delta^{k}*V_{is}^{k}*\mathrm{sign}(f(X_{rs}^{k})-f(X_{rs}^{k})) \tag{8}$$

In this step, we extend it to high dimension. δ denotes the step length. The search behaviors of the right antenna and the left antenna are expressed as

$$\begin{aligned} X_{rs}^{k+1}&=X_{rs}^{k}+V_{is}^{k}-d/2 \\ X_{rs}^{k+1}&=X_{rs}^{k}-V_{is}^{k}-d/2 \end{aligned} \tag{9}$$

The BSO algorithm first initializes a set of random solutions. In each iteration, the search agent updates its location based on its own search mechanism and the best solution currently available. The combination of these two parts can not only accelerate the iteration speed of the population, but also reduce the probability of the population falling into the local optimum, and is more stable in dealing with high-dimensional problems.

In theory, the BSO algorithm contains the ability of exploration and utilization, so it belongs to global optimization. In addition, the linear combination of speed and beetle search enhances the rapidity and accuracy of population optimization and makes the algorithm more stable. In the following, we will examine the performance of the proposed algorithm through a set of numerical examples and engineering examples.

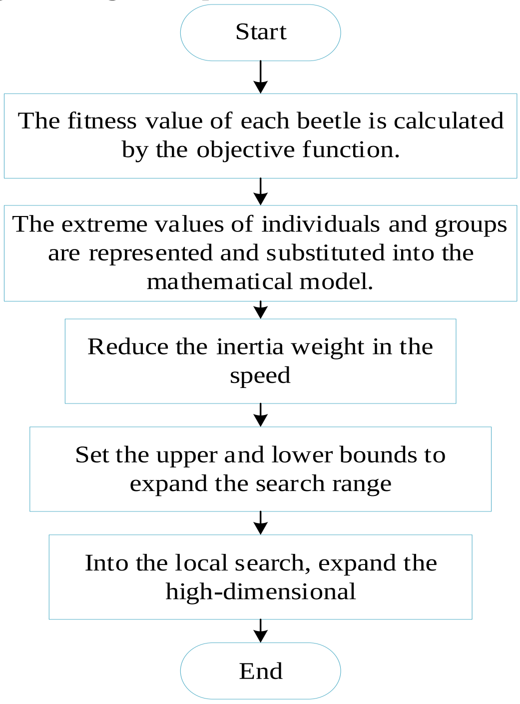

Figure 1. Flow chart of BSO algorithm

4. Numerical examples

Let's consider the following limit state equation

$$g(x) = 18.46 - 7.48 x_1 x_2^{-3} \quad (10)$$

where x_1 and x_2 and are subject to normal distribution, the mean μ are 10 and 2.5, the standard deviation σ is 2 and 0.375, respectively. Using the proposed algorithm in this paper, the initial population $sizepop = 200$, space dimension $\dim = 2$, inertia weight $c\_1 = 0.8$, self-learning factor $c\_2 = 0.5$,group learning factor $c\_3 = 0.5$.

The results are given as follows:

The objective of this method is to iterate for 100 times and converge for the 27th time. The MPP point is (12.6175,1.7242), reliability index $\beta = 2.4479$, failure probability $p_f = 0.0072$.

The MCS method is used to perform 1×10^7 sampling simulations on the objective function, similarly, $\beta = 2.3459$, $P_f = 0.0095$.

Table 1 The comparison of reliability index results

Methods	Reliability Index	Failure Probability	Number of function calls
MCS	2.3459	0.0095	—
The proposed method	2.4479	0.0072	27

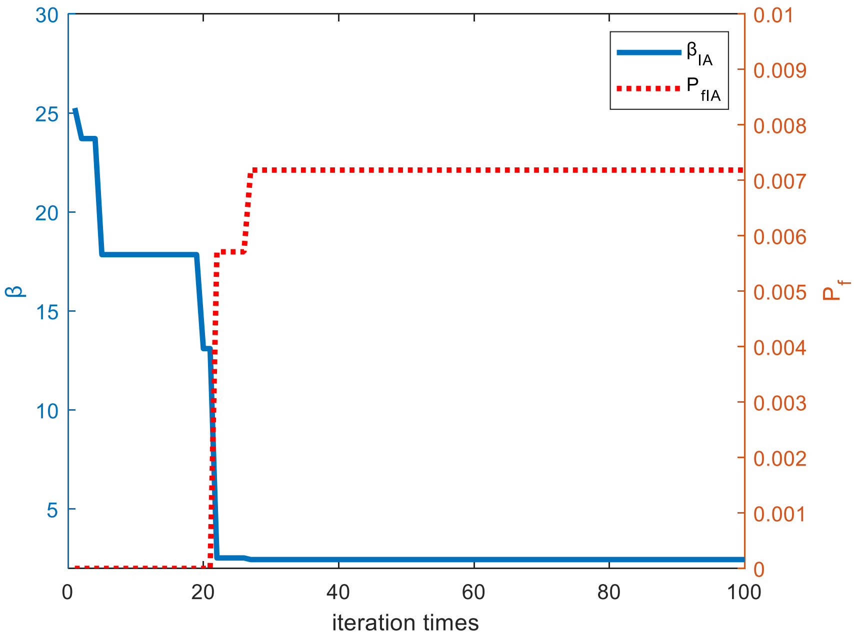

Figure 2 Iterative results of BSO algorithm.

The calculation results of the method in this paper are shown in Table 1 and Figure 2. According to Table 1 and Figure 2, it can be seen that the convergence speed of the method in this paper is faster; the reliability index and failure probability are close to the value of MCS method, which has high accuracy. The method in this paper is practical and reliable, and has high precision while ensuring the convergence speed, which verifies the feasibility of the method in this paper.

5. Engineering Example

An example of a simply supported rectangular beam is considered. The length of the simply supported beam is l, the width of the section is b, the height is h, and a uniform load q is applied throughout its entire range. The strength is R, and all random variables obey the normal distribution. The choice of material is 45 steel. The schematic diagram of the structure is shown in Figure 3. The performance function is the difference between strength and load which is given as

$$Z = G(q, R, l, b, h) = R - \frac{0.75 \times q \times l^2}{b \times h^2} \quad (11)$$

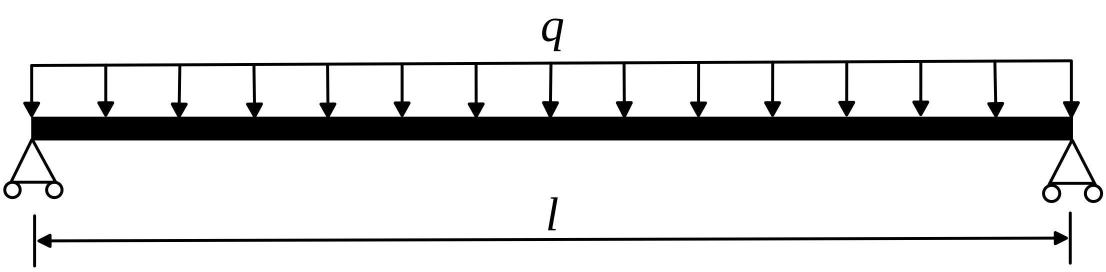

Figure 3 Simple beam structure diagram

Calculated by the method in this paper, the MPP points are (46.5609,2.6621,18.9880,16.6034, -32.1671), reliability index $\beta = 5.9172$, failure rate $P_f = 1.6 \times 10^{-9}$. The iteration diagram of the method in this paper is shown in Figure 4.

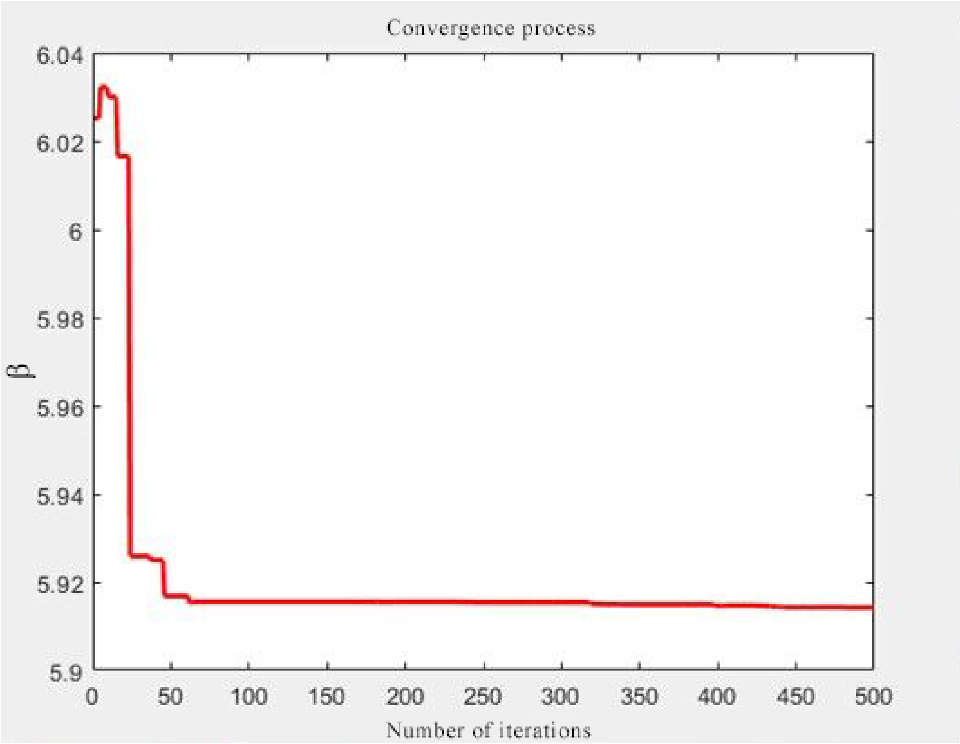

Fig. 4 The iterative curve of reliability index

It can be seen from Figure 4 that the reliability index curve tends to be gentle in the total number of 60 steps. Therefore, it can be seen that the algorithm in this paper has strong local search ability and can find the optimal solution in a large range. It has good convergence and speed for the optimization problem.

6. Conclusion

This paper proposes a beetle group optimization algorithm. The algorithm combines the foraging mechanism of beetles with the swarm optimization algorithm, and is applied to numerical and engineering examples. The results show that the BSO algorithm has good robustness and running speed. In addition, if improved, BSO can deal with multi-objective optimization problems more efficiently and stably. In the following work, we will further study the influence of different parameter settings on BSO.

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