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Analysis of dynamic characteristics of gear systems with mass eccentricity

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Abstract

Consider that gear eccentricity is one of the most common errors in gear installation. Therefore, this should be an important consideration during dynamic simulation. Based on the establishment of a spur gear system dynamic model including mass eccentricity, the effects of eccentricity and initial phase Angle on the dynamic response of the system are considered in this paper. The results show that the larger the eccentricity, the larger the vibration amplitude of the system. Therefore, the eccentricity should be controlled within a reasonable range to prevent it from causing instability of the gear system. The initial eccentricity Angle of a single eccentric does not affect the minimum and maximum values of the DTE.

Keywords

Eccentricity, gear system dynamic model, dynamic response.

1. Introduction

The gear drive system is one of the most critical mechanical components and is widely used in a variety of applications. In recent years, many researchers have done a lot of analysis and research. Ozguven[1] reviews mathematical modeling methods for gear dynamics analysis and discusses and generally classifies these models. Ren[2] systematically analyzed the effect of flexural and torsional coupling vibration caused by gear meshing on the dynamic behavior of gear system, and analyzed the dynamic response of eccentricity to gear system. Liu [3]studied the dynamic behavior of a spur gear transmission system with external and internal dynamic excitation, and studied the spur gear model using a multi-degree-of-freedom system. Zhao[4] proposed a contact analysis method of tooth surface loading considering geometric eccentricity, and established a dynamic model of gear transmission system. Xiang [5]studied the 16DOF nonlinear dynamic model of the gear system considering the time-varying meshing stiffness and analyzed the influence of gear eccentricity. Yu[6] established a general dynamic model of a cylindrical gear rotor system considering local tooth profile errors and global installation errors. The effects of spiral Angle, number of teeth, profile error and eccentricity on dynamic coupling characteristics of gears were studied. Zhang[7] established a general three-dimensional dynamics model of helical gear pairs considering geometric eccentricity, taking into account gear meshing and bearing flexibility. The transmission error and gear geometric eccentricity are used as excitation sources for simulation.

2. Dynamic modeling

A simple dynamic model of a gear pair, which has been used by many researchers, consists of two discs representing gear inertia and a spring representing meshing stiffness. The dynamic analysis is carried out in the gear pair plane, and any out-of-plane motion is ignored. O1(x1, y1) is defined as the center of rotation coordinates of the driving gear. O2(x2, y2) is the rotation center coordinate of the driven gear; G1(xg1, yg1) is the centroid coordinate of the driving gear; G2(xg2, yg2) is the centroid coordinate of the driven gear. θ_1 and θ_2 are the torsional vibration angular displacements of the main and driven gears respectively. The eccentricities of the two gears are denoted by $\rho_1$andρ_2. Gear meshing is represented as a pair of rigid disks connected by a spring damper.

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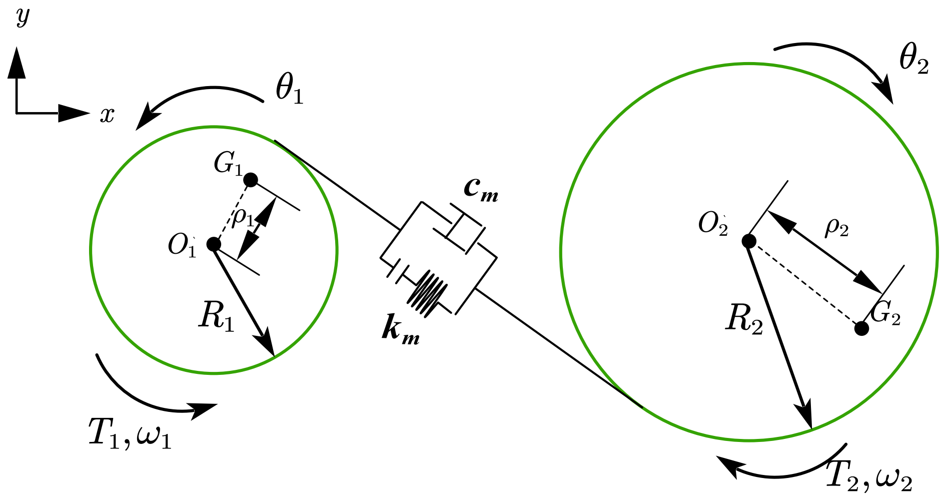

Fig.1 Nonlinear dynamic model for the gear pair

The angular displacement of the gear is given by φ_1 and φ_2 and can be expressed as:

$$\begin{cases} \phi_1(t) = \omega_1 t + \theta_1(t) \\ \phi_2(t) = \omega_2 t + \theta_2(t) \end{cases} \tag{1}$$

where, θ_1 and $\theta_2$is the torsional angle displacement superimposed on the rigid-body rotation of gear. In order to meet the requirements of good lubrication, and to prevent tooth clamping due to tooth surface deformation and thermal expansion, there must be a certain tooth clearance between the non-working tooth surfaces. This part caused by tooth thickness deviation and installation center distance error is divided into normal backlash, also known as initial backlash or design backlash

$$b_0 = 5\text{E-}5 \tag{2}$$

The differential equation describing the torsional vibration and taking into account mass eccentricity is as follows

$$\begin{cases} m_1\left(\ddot{x}_1-\rho_1\ddot{\theta}_1\sin\phi_1-\rho_1\dot{\phi}_1^2\cos\phi_1\right)+c_{px}\dot{x}_1+k_{px}x_1+F_m\sin\alpha=0 \\ m_1\left(\ddot{y}_1+\rho_1\ddot{\theta}_1\cos\phi_1-\rho_1\dot{\phi}_1^2\sin\phi_1\right)+c_{py}\dot{y}_1+k_{py}y_1+F_m\cos\alpha=-m_1g \\ \left(I_1+m_1\rho_1^2\right)\ddot{\theta}_1-m_1\rho_1\left(\ddot{x}_1\sin\phi_1-\ddot{y}_1\cos\phi_1\right)+F_mR_1=T_1 \\ m_2\left(\ddot{x}_2-\rho_2\ddot{\theta}_2\sin\phi_2-\rho_2\dot{\phi}_2^2\cos\phi_2\right)+c_{gx}\dot{x}_2+k_{gx}x_2-F_m\sin\alpha=0 \\ m_2\left(\ddot{y}_2-\rho_2\ddot{\theta}_2\cos\phi_2+\rho_2\dot{\phi}_2^2\sin\phi_2\right)+c_{gy}\dot{y}_2+k_{gy}y_2-F_m\cos\alpha=-m_2g \\ \left(I_2+m_2\rho_2^2\right)\ddot{\theta}_2-m_2\rho_2\left(\ddot{x}_2\sin\phi_2+\ddot{y}_2\cos\phi_2\right)-F_mR_2=-T_2 \end{cases} \quad (3)$$

Similarly, the dynamic equation considering mass eccentricity can be described in matrix form, then the torsional vibration equation is expressed in matrix form, which can be expressed as:

$$I\ddot{\theta}=T \quad (4)$$

$$I=\begin{bmatrix} I_1+m_1\rho_1^2 & 0 \\ 0 & I_2+m_2\rho_2^2 \end{bmatrix} \quad (5)$$

$$\theta=\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \quad (6)$$

$$T=\begin{bmatrix} T_1-F_mR_1-m\ \rho_1\left(\ddot{x}_1\sin\phi_1-\ddot{y}_1\cos\phi_1\right) \\ -T_2+F_mR_2+m_2\rho_2\left(\ddot{x}_2\sin\phi_2+\ddot{y}_2\cos\phi_2\right) \end{bmatrix} \quad (7)$$

Then the translational vibration equation is described as a matrix form, which can be expressed as:

$$M\ddot{A}=F \quad (8)$$

$$M=\begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & m_1 & \\ & & & m_2 \end{bmatrix} \quad (9)$$

$$A=\begin{bmatrix} \ddot{x}_1-\rho_1\ddot{\theta}_1\sin\phi_1-\rho_1\dot{\phi}_1^2\cos\phi_1 \\ \ddot{x}_2-\rho_2\ddot{\theta}_2\sin\phi_2-\rho_2\dot{\phi}_2^2\cos\phi_2 \\ \ddot{y}_1+\rho_1\ddot{\theta}_1\cos\phi_1-\rho_1\dot{\phi}_1^2\sin\phi_1 \\ \ddot{y}_2-\rho_2\ddot{\theta}_2\cos\phi_2+\rho_2\dot{\phi}_2^2\sin\phi_2 \end{bmatrix} \quad (10)$$

$$F=\begin{bmatrix} -c_{px}\dot{x}_1-k_{px}x_1-F_m\sin\alpha \\ -c_{gx}\dot{x}_2-k_{gx}x_2+F_m\sin\alpha \\ -c_{py}\dot{y}_1-k_{py}y_1+F_m\cos\alpha-m_1g \\ -c_{gy}\dot{y}_2-k_{gy}y_2-F_m\cos\alpha-m_2g \end{bmatrix} \quad (11)$$

Where, $T_{1,2}$ is the input torque and braking torque of the gear, $I_{1,2}$ is the moment of inertia of the main wheel and the driven wheel respectively, and $m_{1,2}$ is the mass of the main wheel and the driven wheel.c_{px}, .c_{py},. c_{gx},. $c_{gy}$is the damping of the bearing and k_{px}, k_{py}, k_{gx}, $k_{gy}$is the stiffness of the bearing.

The relative displacement of the gear system δ_O along off-line-of-action ($OLOA$) due to translational motion, and the dynamic transmission error δ along LOA can be expressed as:

$$\delta_O=\left(x_1-x_2\right)\cos\alpha-\left(y_1-y_2\right)\sin\alpha \quad (12)$$

$$\delta\left(t\right)=\left(x_1-x_2\right)\sin\alpha+\left(y_1-y_2\right)\cos\alpha+R_1\theta_1-R_2\theta_2 \quad (13)$$

According to elastic theory, dynamic meshing force consists of elastic force and damping force along the direction of meshing line, which can be expressed as:

$$F_m = k_m(t) f(\delta) + c_m(t) \dot{f}(\delta) \tag{14}$$

Where: $c_m = 2\xi_m\sqrt{k_m I_1 I_2/(I_2 R_1^2 + I_1 R_2^2)}$ is time-varying meshing damping; ξ_m damping ratio (0.03~0.17); k_mτ - time-varying meshing stiffness of gears.

$$f(\delta, b_0) = \begin{cases} \delta - \mathrm{sign}(\delta) b_0 & |\delta| > b_0 \\ 0 & \text{else} \end{cases} \tag{15}$$

$$f_1(\delta, b_0) = \begin{cases} \dot{\delta} & |\delta| > b_0 \\ 0 & \text{else} \end{cases} \tag{16}$$

$$\dot{\delta}(t) = (\dot{x}_1 - \dot{x}_2)\sin\alpha + (\dot{y}_1 - \dot{y}_2)\cos\alpha + R_1\dot{\theta}_1 - R_2\dot{\theta}_2 \tag{17}$$

For simplicity, the rectangular wave time-varying meshing stiffness is used in this paper. The meshing stiffness of single tooth is 1.3E8K/m, and that of double tooth is 2.5E8K/m

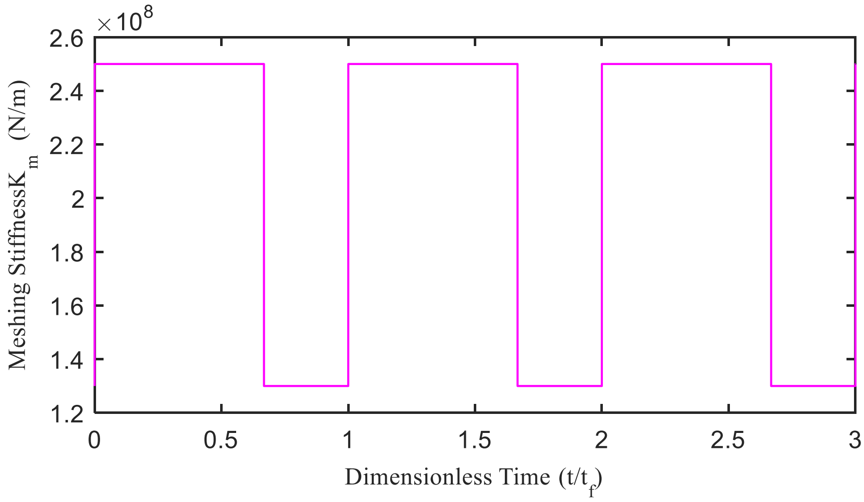

Fig.2. Meshing Stiffness

3. The influence of eccentricity on the system

In the manufacturing process of gear, due to processing or casting and other reasons, the center of mass is actually not the center of the gear, and mass eccentricity will occur. On the basis of considering constant bearing stiffness, The effects of different eccentricities on the dynamic response of gear systems are compared respectively. Parameters are shown in Table 1

Table 1 Three Scheme comparing

Gear Parameters	Pinion		Gear
Number of teeth	23		47
Mass(kg)	0.21		0.37
Module(mm)		2.5	
Pressure angle(°)		20	
Input torque(N·m)		50	
Bearing damping(N/(m/s))		1.0×10^3	
Bearing radial stiffness(N/m)		6.56×10^7	

The vibration response results of the system under different eccentricities ρ are shown in Fig. 3. This part uses the time process diagram of DTE to study the influence of eccentricity on the dynamic response of the gear system. According to the parameters in Table 1, the eccentricity of the pinion is set to 1mm and 3mm respectively, and compared with the healthy gear (that is, the eccentricity is 0), it can be found that when the eccentricity of the pinion is 0, the system DTE is a stable periodic motion. When mass eccentricity is considered, the imbalance of inertia force in a meshing period will cause the fluctuation of the system dynamic response due to the existence of external eccentric inertia force. The dynamic transmission error curve shows a large oscillation, but the change law of the two curves is the same. The amplitude and magnitude of DTE periodic fluctuation increase with the increase of eccentricity, but the fluctuation trend is basically the same. If the eccentricity is only on the pinion, then the number of rotation cycles saved should be multiples of the pinion.

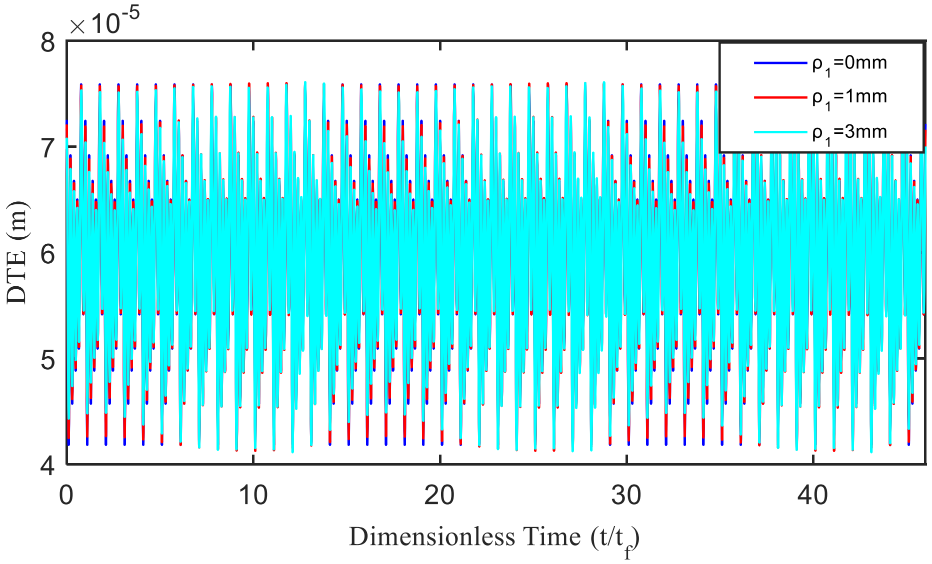

Fig. 3 Pinion with different eccentric DTE time-domain vibration response

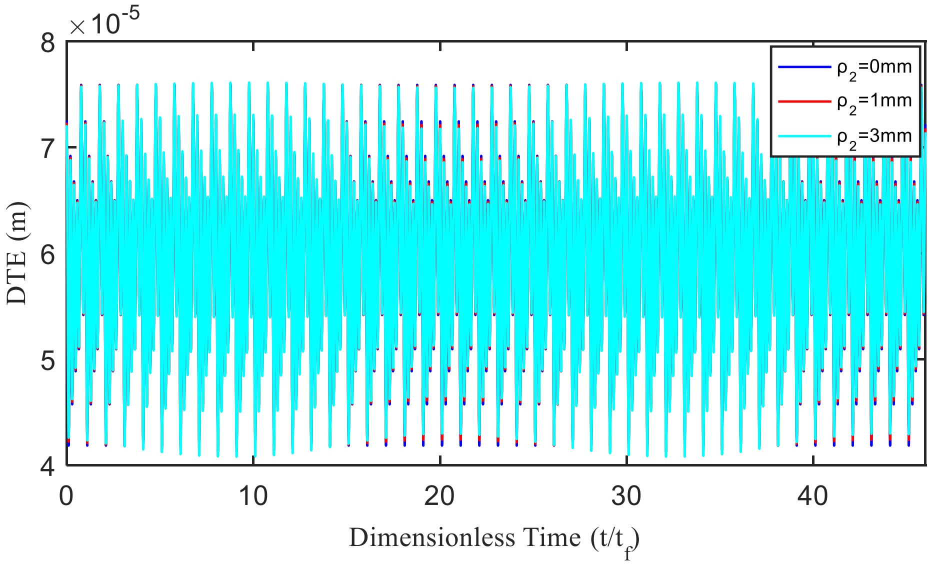

Fig. 4 Gear with different eccentric DTE time-domain vibration response

Fig 4 shows the vibration response results of the large gear with different eccentricities only. The eccentricity of the large gear is set to 1mm and 3mm respectively, and compared with the healthy gear (that is, the eccentricity is 0), it can also be found that when the eccentricity of the large gear is 0, the system DTE is stable periodic motion. When mass eccentricity is considered, the dynamic response of the same system will show a certain fluctuation, the greater the eccentricity, the greater the fluctuation of transmission error, but the fluctuation trend is basically the same. If the eccentricity is only on the big gear, the number of rotation cycles saved should be several times that of the big gear.

The eccentricity of the driving wheel is set to be 3 mm, and the initial eccentricity Angle is set to be 0°, 60°, 120°, 180°, 240° and 300°. In order to compare the differences of systems under different initial eccentricity angles, the maximum and minimum DTE values of the system under different initial eccentricity angles are respectively given in Fig. 5. It can be seen from the figure that the maximum and minimum DTE values of the system are equal under different initial eccentricity angles.

On the contrary, as shown in Fig 6, the eccentricity of the driven wheel is set to 3 mm, and the initial eccentricity Angle is also set to 0°, 60°, 120°, 180°, 240° and 300°. Figure 5 shows the DTE values of the system under different initial eccentricity angles, and the initial eccentricity Angle does not affect the minimum and maximum DTE values

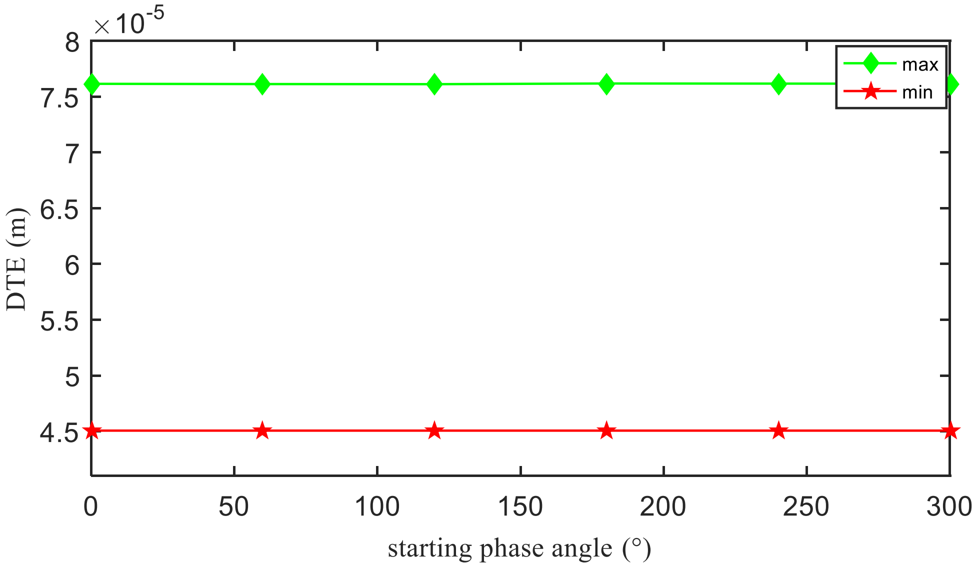

Fig. 5 Pinion eccentricity DTE with different initial phase angles

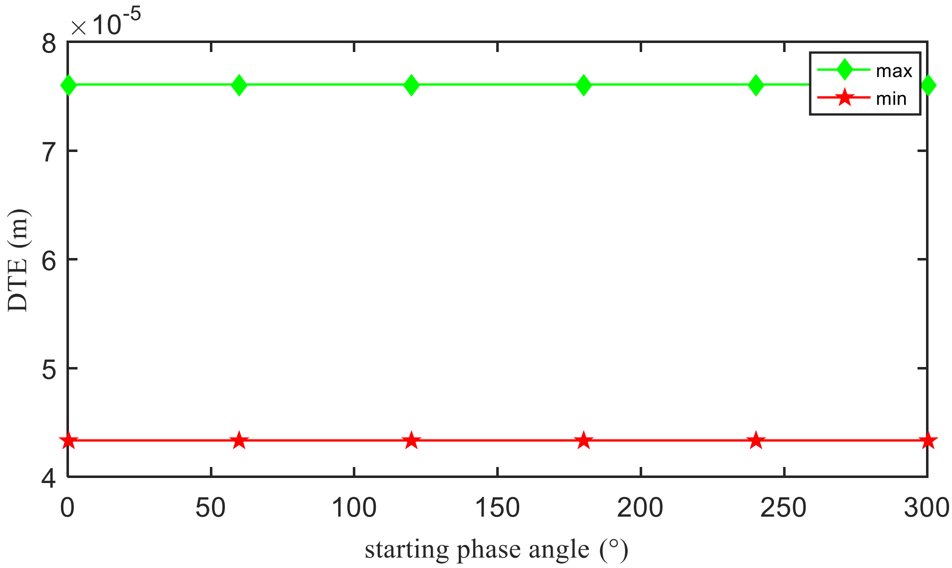

Fig. 6 Gear eccentricity DTE with different initial phase angles

4. Conclusion

The effect of eccentricity on the gear system is mainly considered. A typical dynamic model of the most commonly used six degrees of freedom is established, the dynamic equations of the system are given according to Newton's second law, and the effect of gear eccentricity on the system is discussed using the given rectangular wave single and double tooth stiffnesses. The results show that the system DTE is a stable periodic motion when the eccentricity of the pinion gear is zero. When mass eccentricity is considered, the imbalance of inertial forces in the meshing cycle causes fluctuations in the dynamic response of the system due to the presence of external eccentric inertial forces. The amplitude and magnitude of the DTE cycle fluctuations increase with the increase of the eccentricity, but the fluctuation trend is basically the same. The initial eccentricity Angle of a single eccentric does not affect the minimum and maximum values of the DTE.

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