ISSN: 1813-4890

Stochastic Evolutionary Game Analysis of Electric Vehicle Charging and Discharging Strategies Based on Improved Replicator Dynamics

Qiang Gao[1, a], Xuepeng Tang[2, b], Tong Ye[1, c], Darun Lu[3, d], Li Jin[3, e]

[1]Wenzhou Polytechnic, Wenzhou 325035, China;

[2] Hefei Power Supply Company, Anhui Electric Power Company, Hefei 230022, China;

[3]Basa Electric Technology Co., Ltd, Wenzhou 325035, China;

[a]2020000119@wzpt.edu.cn, [b]474494369@qq.com, [c]2603870184@qq.com,

[d] 332609474@qq.com, [e] 786498072@qq.com

Abstract

This paper proposes a stochastic evolutionary game model for electric vehicle (EV) charging and discharging strategy selection based on an improved replicator dynamic equation. Firstly, a three-strategy game model for EV groups is established, considering charging strategy, discharging strategy, and neither-charging-nor-discharging strategy. Secondly, the traditional replicator dynamic equation is improved by introducing strategy influence factors and incentive coefficients to characterize the interdependence among different strategies. Furthermore, Gaussian white noise is incorporated into the evolutionary process to quantify the random disturbances caused by uncertain external factors such as weather changes and traffic conditions. The stability of the stochastic differential equations is analyzed using the moment exponential stability criterion. Finally, simulation results demonstrate that: (1) the introduction of incentive coefficients effectively captures the mutual influence among EV strategies; (2) larger incentive coefficients lead to faster convergence of the evolutionary game; (3) random disturbances introduce significant fluctuations in the strategy selection process, and stronger disturbances slow down the convergence speed. The proposed model provides theoretical support for formulating rational scheduling measures for electric vehicle grid integration under different scenarios.

Keywords

Electric vehicle, evolutionary game, stochastic disturbance, strategy stability.

1. Introduction

With the rapid development of new energy technologies and the increasing global emphasis on environmental protection, electric vehicles (EVs) have become an important component of modern transportation systems. The large-scale integration of EVs into power grids presents both opportunities and challenges for grid operation and management [1]. On one hand, EVs can serve as mobile energy storage units, providing grid support through vehicle-to-grid (V2G) technology; on the other hand, uncoordinated charging behaviors may cause significant load fluctuations and power quality issues [2]. Therefore, understanding and guiding EV users' charging and discharging behaviors has become a critical research topic in the field of smart grid.

Evolutionary game theory provides an effective framework for analyzing the dynamic decision-making processes of large populations under bounded rationality [3]. In recent years,

ISSN: 1813-4890

evolutionary game models have been widely applied to study EV charging strategies. However, most existing studies employ traditional replicator dynamic equations that assume individuals make decisions independently, neglecting the interdependence among different strategies in real-world scenarios [4]. In practice, the strategy choices of EV users are mutually influential: when more users choose to charge, grid load increases and electricity prices rise, which in turn affects the economic attractiveness of the charging strategy and prompts some users to switch to discharging or idle strategies. Moreover, the external environment introduces significant uncertainties into EV users' decision-making processes. Sudden weather changes, unexpected traffic congestion, and emergency events can alter users' travel plans and energy demands, leading to random fluctuations in strategy evolution [5]. Traditional deterministic evolutionary game models fail to capture these stochastic disturbances, resulting in incomplete descriptions of the actual evolution process.

To address these limitations, this paper makes the following contributions: (1) We establish a three-strategy game model for EV groups, incorporating charging costs, discharging benefits, and battery degradation factors; (2) We improve the traditional replicator dynamic equation by introducing strategy influence factors and incentive coefficients to characterize the interdependence among strategies; (3) We incorporate Gaussian white noise into the evolutionary process to model random disturbances from uncertain external factors; (4) We analyze the stability conditions of the stochastic differential equations and validate the theoretical results through comprehensive simulations.

2. Model Establishment

2.1. Basic Assumptions and Game Model

To facilitate the analysis, we establish the following assumptions for the EV group charging and discharging strategy selection problem. The EV group consists of N electric vehicles. The strategy set for each EV is S = {charging strategy, discharging strategy, neither-charging-nor-discharging strategy}. The initial numbers of users selecting the three strategies are denoted as x1, x2, and x3, respectively, satisfying x1 + x2 + x3 = N. All EVs in the group are homogeneous, sharing identical battery technology, charging/discharging efficiency, and cost-benefit sharing coefficients. The game interactions occur only between pairs of EVs. The probabilities of selecting charging, discharging, and neither-charging-nor-discharging strategies are p1, p2, and p3, respectively, with p1 + p2 + p3 = 1. When both parties choose charging, EVs mainly consider charging costs; when both choose discharging, they focus on discharging benefits; when neither charges nor discharges, both costs and benefits are zero. When one charges and the other discharges, the price fluctuation is negligible, and no additional benefits or costs are considered.

When an EV selects the charging strategy, its charging cost Cc is determined by the electricity price and charging quantity. When selecting the discharging strategy, its discharging benefit Bd is determined by the feed-in tariff and discharging quantity [6]. If all EVs choose charging or discharging simultaneously, excessive charging/discharging causes grid load imbalance, prompting the grid to adjust electricity prices and generating additional benefits or costs for EVs. These benefits are denoted as Bc and Bd, respectively, with over-charging/discharging benefit sharing coefficients alpha and beta satisfying 0 < alpha, beta < 1. The payoff matrix for the three-strategy game model is shown in Table 1.

Table 1 Payoff matrix of the three-strategy game for EVs

		Electric Vehicle 1		
		Charging (α_1)	Discharge (α_2)	Not charging or discharging(α_3)
	Charging (α_1)	$a_1 = R_{ev} - \alpha E_c$	$b_1 = R_{ev}$	$c_1 = R_{ev} - E_c$

Electric Vehicle 2		$a_2 = R_{ev} - \alpha E_c$	$b_2 = E_{ev}$	$c_2 = 0$
	Discharge (α_2)	$d_1 = E_{ev}$ $d_2 = R_{ev}$	$e_1 = E_{ev} + \beta E_d$ $e_2 = E_{ev} + \beta E_d$	$f_1 = E_{ev} + E_d$ $f_2 = 0$
	Not charging or discharging (α_3)	$g_1 = 0$ $g_2 = R_{ev} - E_c$	$\hbar_1 = 0$ $\hbar_2 = E_{ev} + E_d$	$i_1 = 0$ $i_2 = 0$

In Table 1, $R_{ev}$and E_{ev}represent the cost and benefit incurred by an electric vehicle (EV) when it chooses the charging or discharging strategy, respectively. These are defined as follows:

$$\begin{cases} E_{ev} = (Q_d \cdot r_d - Q_d \cdot K_b) \cdot \rho^d \\ R_{ev} = \frac{(Q_c \cdot r_c + Q_c \cdot K_b)}{\rho^c} \end{cases} \tag{1}$$

where $\rho^d$and $\rho^c$are the discharging and charging efficiencies, respectively, typically taken as 90%.$K_b$denotes the unit energy loss of the EV battery, expressed as:

$$K_b = \frac{R_x}{L_c \cdot C^{i,EV} \cdot DoD} \tag{2}$$

Here:

$R_x$is the cost of a single new battery;

$L_c$is the average battery life (in number of cycles) under conventional usage conditions at a given depth of discharge;

$C^{i,EV}$ is the battery capacity of EV i;

$DoD$is the depth of discharge used in normal operation.

The quantities $Q_c$and $Q_d$represent the charging and discharging electricity amounts when EVs participate in the charging and discharging processes, respectively, and are given by:

$$\begin{cases} Q^c = \sum_{i=1}^{n} C^{EV} \cdot SOC_i \cdot f_i^c \cdot \eta_i^c \\ Q^d = \sum_{i=1}^{n} C^{EV} \cdot (1 - SOC_i) \cdot f_i^d \cdot \eta_i^d \end{cases} \tag{3}$$

where $C^{EV}$is the battery capacity of the EV, and $SOC_i$is the state of charge of EV i, which ranges from 20% to 90% to meet travel demands.The terms $f_i^c, f_i^d, \eta_i^c, \eta_i^d$denote the probability of EV response to electricity price and the probability of response to remaining charging/discharging time, respectively. Based on the literature, the influence of electricity price and dwell time on the response probability of EV charging and discharging can be determined [7].

Based on the above assumptions and the payoff structure in Table 1, the expected payoffs for the three strategies are obtained as follows. When an EV selects the charging strategy (S1), the expected payoff is:

$$\begin{aligned} U_1 &= a_1 x + b_1 y + c_1 z \\ &= x(R_{ev} - \alpha E_c) + y R_{ev} + z(R_{ev} - E_c) \end{aligned} \tag{4}$$

When selecting the discharging strategy (S2), the expected payoff is:

$$\begin{aligned} U_2 &= d_1 x + e_1 y + f_1 z \\ &= x E_{ev} + y(E_{ev} + \beta E_d) + z(E_{ev} + E_d) \end{aligned} \tag{5}$$

When selecting the neither-charging-nor-discharging strategy (S3), the expected payoff is:

$$\begin{aligned} U_3 &= g_1 x + h_1 y + i_1 z \\ &= 0 \end{aligned} \tag{6}$$

Consequently, the average payoff of the EV group under the three-strategy selection is:

$$\bar{U} = [x U_1 + y U_2 + (1 - x - y) U_3] \tag{7}$$

2.2. Evolutionary Dynamics of EV Strategies with Influence Factors

In general, the evolutionary outcome of strategies in an electric vehicle (EV) population results from strategic interactions among individual EVs. The direction of strategy evolution depends on the choices made by each EV, which are primarily influenced by travel demand and economic benefits. In practice, an EV's strategy selection is also affected by the choices of other individuals. Specifically, each EV user can choose among three strategies: charging, discharging, or neither charging nor discharging.

In the initial stage of electricity pricing set by the grid, the price is typically low to meet the needs of various users. Consequently, most EV users prefer the charging strategy. However, as the demand for charging increases, grid power imbalance occurs. The grid then adjusts electricity prices to guide EV charging and discharging behavior. In response to price adjustments, EVs adjust their strategies while considering their own travel needs. Therefore, the internal distribution of strategies within the EV population influences individual strategy choices, and interdependencies exist among the strategies [8-9]. This section improves upon the classical replicator dynamic equation to capture the influence relationships among strategies.

Based on the evolutionary game model of EV strategy selection established in Section 1, let the number of EVs choosing the charging strategy be n_1, those choosing the discharging strategy be n_2, and those choosing neither be n_3. Then:

$$n_1 + n_2 + n_3 = n \tag{8}$$

As the strategy selection process evolves, the numbers of EVs adopting each strategy change over time, and eventually a dominant strategy evolves into a stable evolutionary strategy. Since payoff is a key driver of strategy updating, the rate of change $\dot{n}_i$in the number of EVs adopting strategy $i$is positively correlated with both the current number $n_i$and the fitness (expected payoff) $U_i$of that strategy:

$$\dot{n}_i \propto n_i \cdot U_i \tag{9}$$

To describe the influence among different strategies during the selection process, a strategy influence factor $\eta_i$is introduced. This factor represents the influence of strategy i, which is proportional to its payoff and determined by the intrinsic properties of the charging/discharging strategy. A larger $\eta_i$indicates a higher payoff from strategy i, implying that strategy $i$exerts a stronger influence on other strategies, and users are more inclined to choose it.Differentiating equation (8) yields the replicator dynamic equation for the charging strategy:

$$\dot{n}_1 = n_1(U_1 - \bar{U})\text{(extended form)} \tag{10}$$

Similarly, the replicator dynamic equations for the other two strategies are:

$$\dot{n}_2 = n_2(U_2 - \bar{U}), \dot{n}_3 = n_3(U_3 - \bar{U}) \tag{11}$$

When $\eta_1 = \eta_2 = \eta_3 = 1$, equations (10) and (11) reduce to the classical replicator dynamics.

Define:

$$\lambda_{21} = \frac{\eta_2}{\eta_1}, \lambda_{31} = \frac{\eta_3}{\eta_1}, \lambda_{32} = \frac{\eta_3}{\eta_2} \tag{12}$$

where $\lambda_{21}$is the incentive coefficient between charging and discharging strategies. If $\lambda_{21} > 1$, the payoff from discharging exceeds that from charging, meaning discharging becomes the dominant and eventually stable strategy.$\lambda_{31}$is the incentive coefficient between charging and the "neither" strategy. If $\lambda_{31} > 1$, considering both charging/discharging benefits and the benefits from vehicle operation, the "neither" strategy yields higher payoff than charging, making it dominant.

Thus, the improved replicator dynamic equations for the three strategies are:

$$\begin{cases} \dot{x} = x(U_1 - \bar{U}) \cdot f(\lambda_{21}, \lambda_{31}) \\ \dot{y} = y(U_2 - \bar{U}) \cdot f(\lambda_{12}, \lambda_{32}) \\ \dot{z} = z(U_3 - \bar{U}) \cdot f(\lambda_{13}, \lambda_{23}) \end{cases} \tag{13}$$

The differential equation system (12) describes the evolutionary process of EV strategy selection, accounting for the influence of each strategy and the incentive relationships among them. Under initial proportions of each strategy, the fractions of EVs choosing different strategies change over time and eventually evolve to a stable state. By setting $F(x) = 0, F(y) = 0, F(z) = 0$and solving the improved replicator dynamic equations (13), the equilibrium points of the evolutionary game can be obtained, enabling analysis and prediction of strategy selection in the EV population.

2.3. Stochastic Evolutionary Game Process

The improved replicator dynamic equations can describe the deviations in strategy selection caused by the influence of strategies on user decision-making and the interdependencies among strategies. However, real-world scenarios are more complex and also require consideration of various external factors that affect individual decisions [10-11]. During the strategy selection process, individuals may possess different levels of information or may be subject to unexpected external disturbances, which can influence their choices and lead to stochastic mutations in the evolutionary process.

Therefore, in the context of EV charging and discharging strategy selection, the social environment may give rise to many random events that impact users' decision-making processes. For example: Sudden weather changes may alter an EV user's travel plans, prompting a re-evaluation of charging or discharging behavior based on updated travel needs. Uncertainty in traffic conditions and unexpected traffic events may change road congestion levels, thereby reducing the driving range of EVs and ultimately affecting users' charging or discharging decisions. Numerous stochastic events can introduce random disturbances to the collective behavior of EV populations, leading to random mutations in the evolutionary process. Moreover, these random disturbances are often difficult to quantify. Consequently, it is necessary to account for the impact of stochastic factors on users' strategy selection during the evolutionary process.

To better describe the random disturbances introduced by uncertain factors into the evolutionary game process, based on stochastic analysis theory, Gaussian white noise is introduced into the game model to capture the disturbances caused by various uncertain factors. The combined effect of these stochastic factors is quantified as a random disturbance intensity σ, which is a positive constant. A larger σindicates a greater influence of these factors, while a smaller σindicates a weaker influence. When $\sigma = 0$, the aforementioned stochastic factors are neglected.

Thus, equation (14) is modified as follows:

$$\begin{cases} dx = x(U_1 - \bar{U})dt + \sigma x\, dW_t \\ dy = y(U_2 - \bar{U})dt + \sigma y\, dW_t \\ dz = z(U_3 - \bar{U})dt + \sigma z\, dW_t \end{cases} \tag{14}$$

where dW_t represents Gaussian white noise, and W_t follows a standard one-dimensional Brownian motion. This formulation effectively captures the influence of stochastic disturbance factors on collective game behavior. When the step size satisfies certain conditions, $dW_t \sim \mathcal{N}(0, dt)$. The improved system is a three-dimensional stochastic differential equation that describes the evolutionary replicator dynamics of the EV population under random disturbances.

3. Simulation Analysis

To verify the effectiveness of the proposed model, comprehensive simulation experiments are conducted under different scenarios. The simulation parameters are set as follows: total EV number N = 1000, battery capacity C= 60 kWh, charging efficiency c = 0.9, discharging efficiency d = 0.9, battery cost C_{bat} = 50000 RMB, battery cycle life L = 2000 cycles, depth of discharge DOD = 0.8, electricity price pc = 0.6 RMB/kWh, feed-in tariff pd = 0.8 RMB/kWh, SOC range [0.2, 0.9].

3.1. Scenario 1: Low Economic Sensitivity

First, the premise for EVs participating in grid dispatch is satisfying their own travel demands. Some users have very low sensitivity to economic benefits, and more users select charging/discharging behaviors based on whether the current SOC meets the next travel demand. At this time, the economic benefits from different strategy choices have limited impact on these users. Without considering additional influences from other users, the three strategies have identical influence factors, i.e., gamma_1 = gamma_2 = gamma_3 = 1. This corresponds to the traditional replicator dynamic equation. The evolutionary trend of EV strategy selection is shown in Fig. 1.

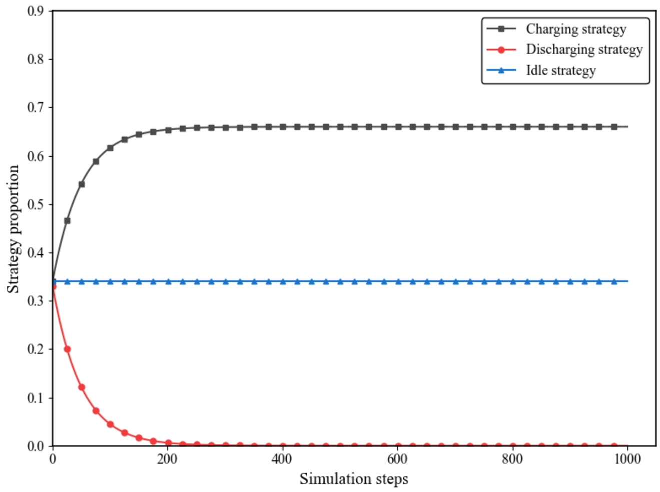

Fig. 1 Evolutionary trend without inter-strategy incentive relationships

When users have low sensitivity to charging/discharging benefits, under the given initial conditions, although discharging benefits exceed charging costs, users select charging/discharging strategies based on travel demands. At this time, the proportion of users selecting the charging strategy exceeds that of the discharging strategy. For the neither-charging-nor-discharging strategy, considering users' low sensitivity to this strategy, the selection proportion does not exhibit significant fluctuations during evolution but maintains the initial selection probability until evolution ends. From the evolutionary results, the charging strategy dominates over the other two strategies, and the discharging strategy eventually evolves to zero.

3.2. Scenario 2: High Economic Sensitivity

(1) Case 1: Charging strategy dominance

Considering users with high sensitivity to economic benefits, users combine their travel demands with charging/discharging economic benefits, resulting in different strategy influences among different users. In the initial stage of grid pricing, electricity prices are relatively low to meet various user demands. At this time, EVs considering their own needs and charging/discharging cost-benefits lead to increased users selecting the charging strategy. Thus, the charging strategy exerts greater influence on user decisions, i.e., gamma_1 > gamma_2 > gamma_3. Taking lambda_12 = 2.0, lambda_13 = 3.0, the evolutionary trend of EV strategy selection is shown in Fig. 2. The final evolutionary result shows that the charging strategy dominates over the other two strategies, and users tend to select the charging strategy. After reaching stability, the proportion of users selecting the charging strategy is larger than that without considering inter-strategy incentive relationships.

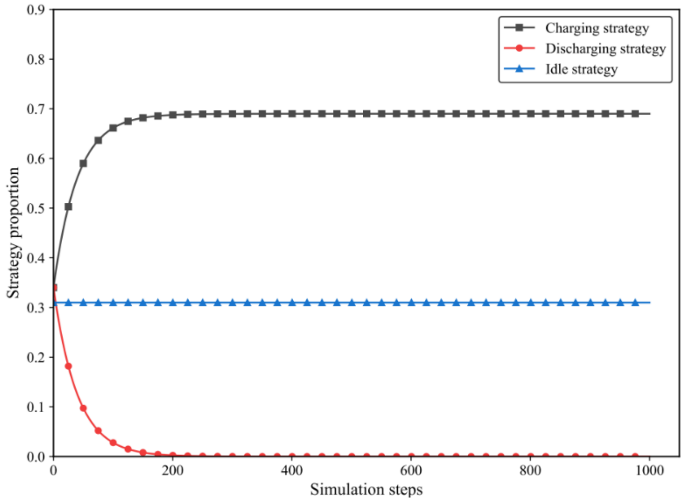

Fig. 2 Evolutionary trend when lambda_12 = 2.0

(2) Case 2: Discharging strategy influence increase

As the number of EVs selecting charging gradually increases, power deficits occur on the grid side. To maintain power balance, the grid raises electricity prices to dispatch EVs. As electricity prices increase, charging costs gradually rise while discharging benefits also increase. Thus, the discharging strategy exerts greater influence on user decisions, i.e., gamma_2 > gamma_1 > gamma_3. Taking lambda_12 = 0.5, the evolutionary trend of EV strategy selection is shown in Fig. 3. The final evolutionary result shows that the charging strategy still dominates over the other two strategies, and users tend to select the charging strategy.

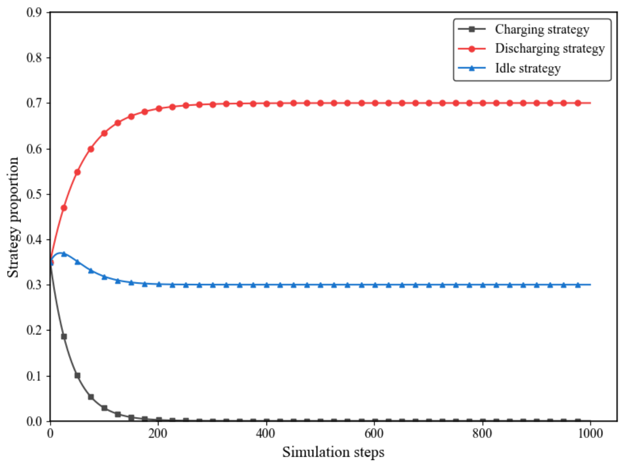

Fig. 3 Evolutionary trend when lambda_12 = 0.5

Finally, to analyze the influence of incentive factors on convergence speed, taking the charging strategy dominance as an example, different incentive coefficients are set to compare their effects on strategy game convergence speed. Taking lambda_12 = 3.3, lambda_13 = 4.0, the charging strategy's influence is stronger compared to Case 1. The evolutionary trends under different incentive coefficients are shown in Fig. 4.

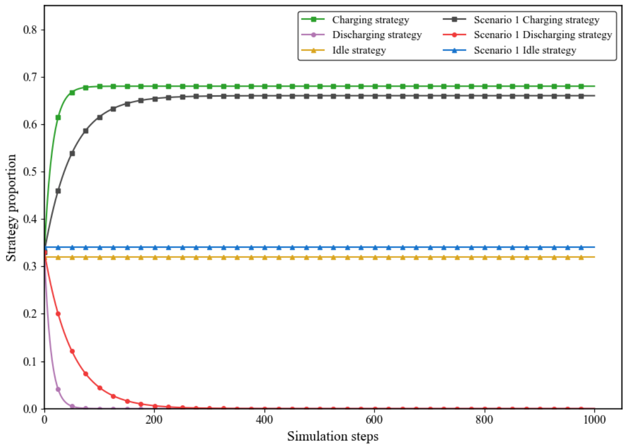

Fig. 4 Evolutionary trends under different incentive coefficients

Compared with Scenario 1, when the incentive coefficient of the charging strategy relative to the discharging strategy increases from 2.0 to 3.3, the evolutionary trend shows that EVs' selection of the charging strategy increases with higher incentive coefficients. This indicates that larger incentive coefficients lead to stronger inter-strategy relationships and higher user preference for the corresponding strategy. Furthermore, from the convergence speed perspective, larger incentive coefficients result in faster game convergence.

Based on the above scenario analysis results, the following conclusions can be drawn regarding the interdependence of charging/discharging strategies within the EV group: (1) Strategies with stronger influence effects eventually evolve into stable strategies; (2) The introduction of incentive relationships affects the convergence speed of evolutionary game equilibrium strategies, with stronger influence leading to faster convergence; (3) Random changes in the social environment affect users' selection processes, with random disturbances introducing significant fluctuations in group behavior strategy selection and slowing down the convergence speed to equilibrium.

4. Conclusion

This paper proposes a stochastic evolutionary game model for EV charging and discharging strategy selection based on an improved replicator dynamic equation. The main contributions and conclusions are summarized as follows:

(1) A three-strategy game model is established for EV groups, incorporating charging costs, discharging benefits, battery degradation, and grid price response mechanisms. The model considers the actual economic and technical characteristics of EV participation in grid dispatch.

(2) The traditional replicator dynamic equation is improved by introducing strategy influence factors and incentive coefficients, effectively characterizing the interdependence among different charging/discharging strategies. The introduction of incentive coefficients enhances the accuracy of strategy evolution descriptions and accelerates game convergence.

(3) Gaussian white noise is incorporated into the evolutionary process to model random disturbances from uncertain external factors such as weather changes and traffic conditions. The stability conditions of the stochastic differential equations are analyzed using the moment exponential stability criterion, providing theoretical guarantees for the stability of evolutionary results.

(4) Comprehensive simulation experiments validate the theoretical results. The results show that stronger strategy influence leads to higher user preference and faster convergence, while random disturbances introduce fluctuations and slow down convergence.

ISSN: 1813-4890

Future research directions include: (1) extending the model to heterogeneous EV populations with different battery technologies and usage patterns; (2) incorporating dynamic pricing mechanisms and demand response programs; (3) developing multi-objective optimization frameworks considering both user benefits and grid stability.

Acknowledgements

This work was supported by the General Project of Zhejiang Provincial Department of Education (Grant No. Y202250693) and the Wenzhou Municipal Science and Technology Bureau Project "Research on Game Theory-Based Vehicle-Grid Bidirectional Interaction Strategy" (Grant No. S20240018).

References

[1] DONG L C, CHEN M Y, LI Z, et al. Ordered charging and discharging control strategy of EVs based on V2G[J]. Journal of Chongqing University,2019,(01):1-15.

[2] Tundys B, Wiśniewski T. Smart Mobility for Smart Cities—Electromobility Solution Analysis and Development Directions[J]. Energies, 2023, 16(4): 1958.

[3] CAI G W, JIANG Y Q, HUANG N T, et al. Large-scale electric vehicles charging and discharging optimization scheduling based on multi-agent two-level game under electricity demand response mechanism[J]. Proceedings of the CSEE,2023,43(01):85-99.

[4] CHEN L P, PAN Z N, YU T, et al. Real-time optimal dispatch for large-scale electric vehicles based on dynamic non-cooperative game theory[J]. Automation of Electric Power Systems, 2019, 43(24):32-40+66.

[5] CHENG S, CHEN Z M, XU K Y, et al. An orderly charging and discharging method for electric vehicles based on a cooperative game and dynamic time-of-use price[J]. Power System Protection and Control,2020,48(21):15-21.

[6] LIU Dongqi, ZHANG Xi, QIAN Yiheng. Evolutionary game coordination strategy of electric vehicle cluster charging and discharging[J]. Power System Protection and Control,2023,51(16):84-93.

[7] Yu Y, Li G, Li Z. A game theoretical pricing mechanism for multi-microgrid energy trading considering electric vehicles uncertainty[J]. IEEE Access, 2020, 8: 156519-156529.

[8] Li J, Ren H, Wang M. How to escape the dilemma of charging infrastructure construction? A multi-sectorial stochastic evolutionary game model[J]. Energy, 2021, 231: 120807.

[9] Sun P J. The optimal privacy strategy of cloud service based on evolutionary game[J]. Cluster Computing, 2022, 25(1): 13-31.

[10] XU S H, HU S L, HAN C F. Stochastic evolutionary game model of NIMBY events based on improved repetitive dynamic equations[J]. Mathematics in Practice and Theory,2021,51(15):1-12.

[11] Yang Z, Shi Y, Li Y. Analysis of intellectual property cooperation behavior and its simulation under two types of scenarios using evolutionary game theory[J]. Computers & Industrial Engineering, 2018, 125: 739-750.