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Lagrange stability analysis of memristive neural networks with mixed time-varying delays

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Abstract

In this paper, the problem on global exponential stability in Lagrange sense for memristive neural networks (MNNs) with both multiple delays and general activation functions is dealt with. Here, the nonsmooth analysis and control theory is adopted to handle MNNs with discontinuous right-hand side. And based on assuming that the activation functions are neither bounded nor monotonous or differentiable, several algebra criteria for the globally exponentially Lagrange stability of MNNs are obtained by virtue of constructing proper Lyapunov functional and using inequality techniques. Meanwhile, detailed estimations of the globally exponentially attractive sets are also given out. Finally, a numerical example is given to demonstrate the theoretical result.

Keywords

Memristive neural networks; Nonsmooth analysis; Global exponential stability in Lagrange sense; Mixed time-varying delays; Globally exponentially attractive set.

1. Introduction

Because their promising potential applications in areas such as image processing, pattern recognition, optimization and other areas, memristive neural networks(MNNs) have attracted increasing interest in scientific community [1-3]. Most importantly, these applications strongly depend on the dynamic behavior of the networks. As it is known, the integration and communication delays are unavoidably encountered both in biological and artificial neural networks, which may lead to poor performance such as oscillation, instability, chaos, etc. Hence, for the necessity of applications, the dynamical characteristics on the stability in Lyapunov sense of neural networks with time-varying delays and finite distributed delays have become a subject of intense research activities [4-5].

It is worth mentioning that Lyapunov stability refers to the stability of the equilibrium points which requires the existence of equilibrium points, while Lagrange stability refers to the stability of the total system, rather than the stability of equilibrium points. Moreover, the global stability in Lyapunov sense can be viewed as a special case of stability in Lagrange sense by regarding an equibrium point as an attractive set [6-7]. So it is necessary and rewarding to study Lagrange stability. Basically, the goal of the study on globally exponentially stability in Lagrange sense is to determine the globally exponentially attractive sets. Therefore, a considerable number of works studied the Lagrange stability for engineering systems, including neural networks with time-delays [6-8].

Especially, to our best knowledge, few authors have discussed the stability in Lagrange sense of MNNs with general activation functions. This has motivated the study of the Lagrange analysis of MNNs by using LMI technique, and the estimation of the Lagrange exponential convergence rate is investigated in this paper.

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Motivated by the above discussion, in this paper, we will mainly deal with the global exponential stability in Lagrange sense for MNNs with mixed time-varying delays as follows:

$$\begin{aligned}\frac{dx_i(t)}{dt} &= -d_i(x_i(t))x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t))g_j(x_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij}(x_i(t))g_j(x_j(t-\tau_j(t))) \\ &\quad + \sum_{j=1}^{n} c_{ij}(x_i(t))\int_{t-\sigma_j(t)}^{t} g_j(x_j(s))ds + U_i(t), \\ &\quad t \ge 0, \qquad i = 1, 2, \ldots n\end{aligned} \tag{1}$$

Where

$$d_i(x_i(t)) = \begin{cases} \hat{d}_i, & |x_i(t)| \le T_i, \\ \check{d}_i, & |x_i(t)| > T_i, \end{cases} \quad a_{ij}(x_i(t)) = \begin{cases} \hat{a}_{ij}, & |x_i(t)| \le T_i, \\ \check{a}_{ij}, & |x_i(t)| > T_i, \end{cases}$$

$$b_{ij}(x_i(t)) = \begin{cases} \hat{b}_{ij}, & |x_i(t)| \le T_i, \\ \check{b}_{ij}, & |x_i(t)| > T_i, \end{cases} \quad c_{ij}(x_i(t)) = \begin{cases} \hat{c}_{ij}, & |x_i(t)| \le T_i, \\ \check{c}_{ij}, & |x_i(t)| > T_i, \end{cases}$$

in which switching jumps $T_i > 0, \hat{d}_i > 0, \check{d}_i > 0,$ and $\hat{a}_{ij}, \check{a}_{ij}, \hat{b}_{ij}, \check{b}_{ij}, \hat{c}_{ij}, \check{c}_{ij}$ are all constant numbers. In system (1), $x_i(t)$ is the state of the i-th neuron at time t; $d_i(x_i(t))$ is the i-th neuron self-inhibitions at time t; $a_{ij}(x_i(t)), b_{ij}(x_i(t))$ and $c_{ij}(x_i(t))$ is the connection weight and the delayed connection weights, respectively. $g_j(x_j(t))$ denote the neuron activation function; $\tau_j(t), \sigma_j(t)$ correspond to the transmission delays and satisfy $0 \le \tau_i(t) \le \tau_i$, $0 \le \sigma_i(t) \le \sigma_i$. Here, τ_i and σ_i are constants. Let $\tau = \max_{1 \le i \le n}\{\tau_i\}$, and $\sigma = \max_{1 \le i \le n}\{\sigma_i\}$. Obviously, the memristive neural network (1) is a state-dependent switched system, which is the generalization of those for conventional neural networks.

In the next section, we describe some preliminaries, including some necessary notations, definitions, assumptions and lemmas. Our main results and their proofs for MNNs to global exponential stability in Lagrange sense and have globally exponentially attractive sets are given in Section 3. We present an illustrative example in Section 4 and finally give a summery in Section 5.

2. Preliminaries

For convenience, we first make the following preparations.

Throughout this paper, solutions of all the systems considered in the following are intended in Filippov's sense (see [9]). And the mark $[\cdot,\cdot]$ represents the interval. C[X,Y] is a class of continuous mapping set from the topological space X to the topological space Y. Especially, $C \triangleq [[-h, 0], R^n]$, where $h = \max\{\tau, \sigma\}$. For any initial function $\forall \varphi(s) \in C$, $s \in [t_0 - h, t_0]$, the solution of (1) that starts from the initial condition φ will be denoted by $x(t, t_0, \varphi)$ or simply $x(t)$ if no confusion should occur. For vector $v = (v_1, \ldots, v_n)^T \in R^n$, $\|v\|$ is said to be the Euclidean norm. The symbols R^n and $R^{n \times m}$ stand for the n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. M^T, M^{-1} and $\lambda(M)$ denote the matrix transpose, inverse and the eigenvalues of the square matrix M. $M > 0$ or $M < 0$ denotes that the matrix M is a symmetric and positive definite or negative definite matrix. Meanwhile, $M_1 < M_2$ indicates $M_1 - M_2 < 0$. Moreover, in

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symmetric block matrices, we use $*$ as an ellipsis for the terms that are introduced by symmetry.

Let $\bar{d}_i=\max\{\hat{d}_i,\check{d}_i\},\underline{d}_i=\min\{\hat{d}_i,\check{d}_i\},\bar{a}_{ij}=\max\{\hat{a}_{ij},\check{a}_{ij}\},\underline{a}_{ij}=\min\{\hat{a}_{ij},\check{a}_{ij}\}$,
$\bar{b}_{ij}=\max\{\hat{b}_{ij},\check{b}_{ij}\},\underline{b}_{ij}=\min\{\hat{b}_{ij},\check{b}_{ij}\},\bar{c}_{ij}=\max\{\hat{c}_{ij},\check{c}_{ij}\},\underline{c}_{ij}=\min\{\hat{c}_{ij},\check{c}_{ij}\}$.

And $co[\underline{\xi}_i,\bar{\xi}_i]$ denotes the convex hull of $[\underline{\xi}_i,\bar{\xi}_i]$, clearly, in this paper, we have $co[\underline{\xi}_i,\bar{\xi}_i]=[\underline{\xi}_i,\bar{\xi}_i]$. Then, we first make the following assumption for system (1).

Assumption 1 There exist two diagonal matrices $L=diag\{L_1,\cdots,L_n\}$ and $F=diag\{F_1,\cdots,F_n\}$ such that for any $x,y\in R$ and $x\neq y$, the neuron activation of system (1) satisfies: $L_i\leq\frac{g_i(x)-g_i(y)}{x-y}\leq F_i.$

Remark 1 It should be noted that the constants $L_i,\ F_i,\ i\in R$ in assumption 1 are allowed to be positive, negative or zero. So, the assumption 1 of this paper is weaker than the literatures [6-8, 10].

Now, as the literature [11], by applying the theories of set-valued maps and differential inclusions [9], from system (1), we have

$$\begin{aligned}\frac{dx_i(t)}{dt}\in&-[\underline{d}_i,\bar{d}_i]x_i(t)+\sum_{j=1}^{n}[\underline{a}_{ij},\bar{a}_{ij}]g_j(x_j(t))\\&+\sum_{j=1}^{n}[\underline{b}_{ij},\bar{b}_{ij}]g_j(x_j(t-\tau_j(t)))\\&+\sum_{j=1}^{n}[\underline{c}_{ij},\bar{c}_{ij}]\int_{t-\sigma_j(t)}^{t}g_j(x_j(s))ds+U_i(t),\\&for\ a.a.\ t\geq0,\end{aligned}\tag{2}$$

or equivalently, there exist $d_i(t)\in[\underline{d}_i,\bar{d}_i],a_{ij}(t)\in[\underline{a}_{ij},\bar{a}_{ij}],\ b_{ij}(t)\in[\underline{b}_{ij},\bar{b}_{ij}],c_{ij}(t)\in[\underline{c}_{ij},\bar{c}_{ij}]$ such that

$$\begin{aligned}\frac{dx_i(t)}{dt}=&-d_i(t)x_i(t)+\sum_{j=1}^{n}a_{ij}(t)g_j(x_j(t))+\sum_{j=1}^{n}b_{ij}(t)\\&\times g_j(x_j(t-\tau_j(t)))+\sum_{j=1}^{n}c_{ij}(t)\int_{t-\sigma_j(t)}^{t}g_j(x_j(s))ds\\&+U_i(t),\qquad t\geq0,\ \ i=1,\cdots n.\end{aligned}\tag{3}$$

Now, let $\underline{D}=diag(\underline{d}_1,\cdots\underline{d}_n),\bar{D}=diag(\bar{d}_1,\cdots\bar{d}_n),\underline{A}=(\underline{a}_{ij})_{n\times n}$,
$\bar{A}=(\bar{a}_{ij})_{n\times n},\underline{B}=(\underline{b}_{ij})_{n\times n},\bar{B}=(\bar{b}_{ij})_{n\times n},\underline{C}=(\underline{c}_{ij})_{n\times n},\bar{C}=(\bar{c}_{ij})_{n\times n}$,

then, (2) and (3) can be rewritten as follows:

$$\begin{aligned}\frac{dx(t)}{dt}\in&-[\underline{D},\bar{D}]x(t)+[\underline{A},\bar{A}]g(x(t))+[\underline{B},\bar{B}]g(x(t-\tau(t)))\\&+[\underline{C},\bar{C}]\int_{t-\sigma(t)}^{t}g(x(s))ds+U(t),\qquad for\ a.a.\ t\geq0,\end{aligned}\tag{4}$$

or equivalently, there exist $D(t)\in[\underline{D},\bar{D}],A(t)\in[\underline{A},\bar{A}],\ B(t)\in[\underline{B},\bar{B}],C(t)\in[\underline{C},\bar{C}]$ such that

$$\frac{dx(t)}{dt} = -D(t)x(t) + A(t)g(x(t)) + B(t)g(x(t-\tau(t))) + C(t)\int_{t-\sigma(t)}^{t} g(x(s))ds + U(t), \qquad for\ a.a.\ t \geq 0, \tag{5}$$

where $x(t) = (x_1(t), \cdots x_n(t))^T$, $f(x(t)) = (f_1(x_1(t)), \cdots f_n(x_n(t)))^T$, $\tau(t) = (\tau_1(t), \cdots \tau_n(t))^T, \sigma(t) = (\sigma_1(t), \cdots \sigma_n(t))^T, U(t) = (U_1(t), \cdots U_n(t))^T$. For deriving the global exponential stability in Lagrange sense of system (1), the following definitions and lemmas are also needed.

Definition 1[11] If there exists a radially unbounded and positive definite Lyapunov function $V(x(t))$, and positive constants ζ and β such that for any solution $x(t)$ of system (1), when $V(x(t_0)) > \zeta,\ V(x(t)) > \zeta,\ t \geq t_0$ implies the inequality

$V(x(t)) - \zeta \leq (\bar{V}(x(t_0)) - \zeta)\exp\{-\beta(t-t_0)\}(\bar{V}(x(t_0)) \geq V(x(t_0))),\ t \geq t_0$ always holds.

Then, the system (1) is said to be globally exponentially stable in Lagrange sense. Moreover, the compact set $\Omega = \{x(t) \in R^n \mid V(x(t)) \leq \zeta\}$ is called a globally exponentially attractive set of system (1), where $V(x(t_0))$ is a constant.

Definition 2 MNNs (1) with globally exponentially attractive set is said to be globally exponentially stable in Lagrange sense.

Lemma 1[12] For any vectors $a, b \in R^n$, the inequality $\pm 2a^T b \leq a^T Y^{-1} a + b^T Y b$ holds, in which Y is any matrix with $Y > 0$.

Lemma 2[13] For any constant matrix $P \in R^{n\times n},\ P^T = P > 0, \gamma > 0$, vector function $\omega : [0,\gamma] \to R^n$ such that the integrations concerned are well defined, then

$$\left(\int_0^{\gamma} \omega(s)ds\right)^T P\left(\int_0^{\gamma} \omega(s)ds\right) \leq \gamma \int_0^{\gamma} \omega^T(s) P \omega(s) ds.$$

Lemma 3[14] The LMI $\begin{pmatrix} P & R \\ R^T & Q \end{pmatrix} < 0$ with $P^T = P, Q^T = Q$ is equivalent to one of the following conditions: $(1) Q < 0, P - RQ^{-1}R^T < 0;\ (2) P < 0, Q - R^T P^{-1} R < 0.$

Lemma 4[15] Assume there exist $r_1 > r_2 > 0$ and a nonnegative continuous quantity function $x(t)$, which satisfies $D^+ x(t) \leq -r_1 x(t) + r_2 \bar{x}(t)$, for all $t \in [t_0 - h, t_0]$, then $x(t) \leq \bar{x}(t_0)\exp(-\lambda(t-t_0))$ holds for $\forall t \geq t_0$, where $\bar{x}(t) = \sup_{t-h \leq s \leq t} x(s), h \geq 0$, and λ is the unique positive root of equation $\lambda = r_1 - r_2 e^{\lambda h}$.

Lemma 5[16] Given some constant matrices $A_1, A_2, A_3, B_1, B_2, B_3 \in R^{n\times n}$, and appropriate reversible matrices X, Y, Z, let

$$\Sigma_1 = \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} X^{-1} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix}^T + \begin{pmatrix} A_2 \\ B_2 \end{pmatrix} Y^{-1} \begin{pmatrix} A_2 \\ B_2 \end{pmatrix}^T + \begin{pmatrix} A_3 \\ B_3 \end{pmatrix} Z^{-1} \begin{pmatrix} A_3 \\ B_3 \end{pmatrix}^T,$$

$$\Sigma_2 = \begin{pmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{pmatrix} \begin{pmatrix} X^{-1} & 0 & 0 \\ 0 & Y^{-1} & 0 \\ 0 & 0 & Z^{-1} \end{pmatrix} \begin{pmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{pmatrix}^T.$$

Then $\Sigma_1 = \Sigma_2$.

3. Main results

Theorem 1 Assume that assumption 1 holds, the MNNs system (1) is globally exponentially stability in Lagrange sense if there exist four positive diagonal matrices Q, R, S, T and two positive definite matrices $P, H \in R^{n\times n}$ such that the following LMIs (6) and (7) hold, where

$\Theta = P + Q(F-L) - P\underline{D} - \underline{D}P + 2LQ\overline{D} + W(R+\sigma^2 T)W,\ \ W = \mathrm{diag}\{w_1,\ldots,w_n\}, w_i = \max\{|L_i|,|F_i|\}, \forall i = 1,\ldots,n.$

Moreover, $\Omega = \{x \in R^n |\ x^T(t)Px(t) \le U^T HU/\varepsilon\}$ is a globally exponentially attractive set of the system (1), where $0 < \varepsilon \le 1$.

$$\begin{pmatrix} \Theta & P\overline{A} - LQ\underline{A} - \underline{D}Q & P\overline{B} - LQ\underline{B} & P\overline{C} - LQ\underline{C} & P - LQ \\ * & Q\overline{A} + \overline{A}^T Q - R & Q\overline{B} & Q\overline{C} & Q \\ * & * & -S & 0 & 0 \\ * & * & * & -T & 0 \\ * & * & * & * & -H \end{pmatrix} < 0, \tag{6}$$

$$WSW \le P. \tag{7}$$

Proof: We consider the following radial unbounded and positive definite Lyapunov functional with the given positive definite matrix P and positive diagonal matrix $Q = diag\{q_1,\cdots,q_n\}$,

$$V(x(t)) = x^T(t)Px(t) + 2\sum_{i=1}^{n} q_i \int_0^{x_i(t)} (g_i(s) - l_i s)ds. \tag{8}$$

Calculating the derivative of $V(x(t))$ along the positive semi-trajectory of (1), we can obtain

$$\begin{aligned} &\frac{dV(x(t))}{dt}\Big|_{(1)or(5)} \le 2x^T(t)P[-D(t)x(t) + A(t)g(x(t)) + B(t)g(x(t-\tau(t))) \\ &+C(t)\int_{t-\sigma(t)}^{t} g(x(s))ds + U] + 2(g(x(t)) - Lx(t))^T Q \times \\ &[-D(t)x(t) + A(t)g(x(t)) + B(t)g(x(t-\tau(t))) + C(t)\int_{t-\sigma(t)}^{t} g(x(s))ds + U] \\ &= 2(x^T(t)P + g^T(x(t))Q - x^T(t)LQ)(-D(t)x(t) + A(t)g(x(t))) \\ &+2(x^T(t)PB(t) + g^T(x(t))QB(t) - x^T(t)LQB(t))g(x(t-\tau(t))) \\ &+2(x^T(t)PC(t) + g^T(x(t))QC(t) - x^T(t)LQC(t))\int_{t-\sigma(t)}^{t} g(x(s))ds \\ &+2(x^T(t)P + g^T(x(t))Q - x^T(t)LQ)U(t). \end{aligned} \tag{9}$$

From assumption 1, for given positive diagonal matrix R we derive

$$\begin{aligned} &2(x^T(t)P + g^T(x(t))Q - x^T(t)LQ)(-D(t)x(t) + A(t)g(x(t))) \le \\ &2x^T(t)(-P\underline{D} + LQ\overline{D})x(t) + 2x^T(t)(P\overline{A} - LQ\underline{A} - Q\underline{D})g(x(t)) \\ &+2g^T(x(t))Q\overline{A}g(x(t)) + x^T(t)WRWx(t) - g^T(x(t))Rg(x(t)) = \\ &\begin{pmatrix} x(t) \\ g(x(t)) \end{pmatrix}^T \begin{pmatrix} -P\underline{D} - \underline{D}P + 2LQ\overline{D} + WRW & P\overline{A} - LQ\underline{A} - Q\underline{D} \\ * & Q\overline{A} + \overline{A}^T Q - R \end{pmatrix} \begin{pmatrix} x(t) \\ g(x(t)) \end{pmatrix}. \end{aligned} \tag{10}$$

By using assumption 1, Lemma 1 and Lemma 2, we know that there exist two positive diagonal matrices S, T and a positive definite matrix H such that the following inequalities hold.

$$
\begin{aligned}
&2(x^T(t)PB(t)+g^T(x(t))QB(t)-x^T(t)LQB(t))g(x(t-\tau(t)))\\
&\le (x^T(t)PB(t)+g^T(x(t))QB(t)-x^T(t)LQB(t))S^{-1}\\
&(x^T(t)PB(t)+g^T(x(t))QB(t)-x^T(t)LQB(t))^T\\
&+g^T(x(t-\tau(t)))Sg(x(t-\tau(t)))\le\\
&\begin{pmatrix} x(t)\\ g(x(t))\end{pmatrix}^T\begin{pmatrix} P\overline{B}-LQ\underline{B}\\ Q\overline{B}\end{pmatrix}S^{-1}\begin{pmatrix} P\overline{B}-LQ\underline{B}\\ Q\overline{B}\end{pmatrix}^T\begin{pmatrix} x(t)\\ g(x(t))\end{pmatrix}\\
&+x^T(t-\tau(t))WSWx(t-\tau(t)),
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
&2(x^T(t)P+g^T(x(t))Q-x^T(t)LQ)U(t)\le\\
&(x^T(t)P+g^T(x(t))Q-x^T(t)LQ)H^{-1}\\
&(x^T(t)P+g^T(x(t))Q-x^T(t)LQ)^T+U^T(t)HU(t)=\\
&\begin{pmatrix} x(t)\\ g(x(t))\end{pmatrix}^T\begin{pmatrix} P-LQ\\ Q\end{pmatrix}H^{-1}\begin{pmatrix} P-LQ\\ Q\end{pmatrix}^T\begin{pmatrix} x(t)\\ g(x(t))\end{pmatrix}\\
&+U^THU.
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&2(x^T(t)PC(t)+g^T(x(t))QC(t)-x^T(t)LQC(t))\int_{t-\sigma(t)}^{t}g(x(s))ds\\
&\le (x^T(t)PC(t)+g^T(x(t))QC(t)-x^T(t)LQC(t))T^{-1}\\
&(x^T(t)PC(t)+g^T(x(t))QC(t)-x^T(t)LQC(t))^T\\
&+(\int_{t-\sigma(t)}^{t}g(x(s))ds)^T T(\int_{t-\sigma(t)}^{t}g(x(s))ds)\\
&\le \sigma^2x^T(t)WTWx(t)+\\
&\begin{pmatrix} x(t)\\ g(x(t))\end{pmatrix}^T\begin{pmatrix} P\overline{C}-LQ\underline{C}\\ Q\overline{C}\end{pmatrix}T^{-1}\begin{pmatrix} P\overline{C}-LQ\underline{C}\\ Q\overline{C}\end{pmatrix}^T\begin{pmatrix} x(t)\\ g(x(t))\end{pmatrix}
\end{aligned}
\tag{13}
$$

where $U=(\max|U_1(t)|,\cdots\max|U_n(t)|)^T$.

Based on Lemma 5 and (9)-(13), we have

$$
\begin{aligned}
&\frac{dV(x(t))}{dt}\Big|_{(1)or(5)}\le x^T(t-\tau(t))WSWx(t-\tau(t))+U^T(t)HU(t)+\\
&\begin{pmatrix} x(t)\\ g(x(t))\end{pmatrix}^T\Pi\begin{pmatrix} x(t)\\ g(x(t))\end{pmatrix}+\begin{pmatrix} x(t)\\ g(x(t))\end{pmatrix}^T\Lambda\begin{pmatrix} x(t)\\ g(x(t))\end{pmatrix},
\end{aligned}
\tag{14}
$$

Where

$$\Pi=\begin{pmatrix}\Pi_1 & P\overline{A}-LQ\underline{A}-\underline{D}Q\\ * & Q\overline{A}+\overline{A}^TQ-R\end{pmatrix},$$

$$\Pi_1=-P\underline{D}-\underline{D}P+2LQ\overline{D}+W(R+\sigma^2T)W,$$

$$
\Lambda=\begin{pmatrix} P\overline{B}-LQ\underline{B} & P\overline{C}-LQ\underline{C} & P-LQ\\ Q\overline{B} & Q\overline{C} & Q\end{pmatrix}\begin{pmatrix} S^{-1} & 0 & 0\\ 0 & T^{-1} & 0\\ 0 & 0 & H^{-1}\end{pmatrix}
\begin{pmatrix} P\overline{B}-LQ\underline{B} & P\overline{C}-LQ\underline{C} & P-LQ\\ Q\overline{B} & Q\overline{C} & Q\end{pmatrix}^T.
$$

Following from (6), there exists $0<\varepsilon\le 1$ such that

$$\begin{pmatrix} \tilde{\Theta} & P\bar{A}-LQ\underline{A}-\underline{D}Q & P\bar{B}-LQ\underline{B} & P\bar{C}-LQ\underline{C} & P-LQ \\ * & Q\bar{A}+\bar{A}^TQ-R & Q\bar{B} & Q\bar{C} & Q \\ * & * & -S & 0 & 0 \\ * & * & * & -T & 0 \\ * & * & * & * & -H \end{pmatrix} < 0,$$

where $\tilde{\Theta}=(1+\varepsilon)(P+Q(F-L))-P\underline{D}-\underline{D}P+2LQ\bar{D}+W(R+\sigma^2T)W.$ In the light of Lemma 3, one gets

$$\Pi+\Lambda+\begin{pmatrix}(1+\varepsilon)(P+Q(F-L)) & 0 \\ 0 & 0\end{pmatrix}<0.$$

Therefore, combining (3) and (10), we could derive that

$$\frac{dV(x(t))}{dt}\Big|_{(1)or(5)} \le -(1+\varepsilon)x^T(t)(P+Q(F-L))x(t) \\ +x^T(t-\tau(t))Px(t-\tau(t))+U^THU,\ t\ge t_0. \tag{15}$$

From assumption 1 and the formula (8), one has

$$V(x(t))\le x^T(t)(P+Q(F-L))x(t),\ t\ge t_0. \tag{16}$$

According to (15) and (16), we obtain $\forall t\ge t_0$,

$$\frac{dV(x(t))}{dt}\Big|_{(1)or(5)} \le -(1+\varepsilon)V(x(t))+\bar{V}(x(t))+U^THU, \tag{17}$$

Where $\bar{V}(x(t))=\sup_{t-h\le s\le t}V(s).$

On the basis of (17), when $V(x(t))>\eta,\bar{V}(x(t))>\eta,$ one gets

$$\frac{d(V(x(t))-\eta)}{dt}\Big|_{(1)or(5)} \\ \le -(1+\varepsilon)(V(x(t))-\eta)+(\bar{V}(x(t))-\eta),\ t\ge t_0,$$

Where $\eta=U^THU/\varepsilon.$ According to Lemma 4, we deduce $(V(x(t))-\eta)\le(\bar{V}(x(t))-\eta)\exp(-\lambda t),$ where λ is the unique positive root of the equation $\lambda=(1+\varepsilon)-e^{\lambda h}$. In terms of Definition 1, and noticed $V(x(t))\ge x^T(t)Px(t),$ it is deduced that $\Omega=\{x\in R^n|\ x^T(t)Px(t)\le U^THU/\varepsilon\}$ is a globally exponentially attractive set of the system (1). Hence, the MNNs system (1) is globally exponentially stable in Lagrange sense via the Definition 2. So, the proof of Theorem 1 is completed.

Corollary 1 Assume that assumptions 1 holds and let σ(t)=0. The MNNs (1) is globally exponentially stable in Lagrange sense if there exist three positive diagonal matrices Q, R, S and two positive definite matrices $P,H\in R^{n\times n}$ such that the following LMIs hold:

$$\begin{pmatrix} \Xi & P\bar{A}-LQ\underline{A}-\underline{D}Q & P\bar{B}-LQ\underline{B} & P-LQ \\ * & Q\bar{A}+\bar{A}^TQ-R & Q\bar{B} & Q \\ * & * & -S & 0 \\ * & * & * & -H \end{pmatrix}<0, \tag{18}$$

$$WSW \le P. \tag{19}$$

$\Xi = P + Q(F-L) - P\underline{D} - \underline{D}P + 2LQ\bar{D} + WRW$. Similarly, $W = \mathrm{diag}\{w_1, \ldots, w_n\}, w_i = \max\{|L_i|, |F_i|\}, \quad \forall i=1,\ldots,n.$ What's more, $\Omega = \{x \in R^n | \; x^T(t)Px(t) \le U^T HU / \varepsilon\}$ is a globally exponentially attractive set of (1), where $0 < \varepsilon \le 1$.

Proof: The course of proof is almost parallel to that of Theorem 1, except for getting rid of the inequality (13) in the theorem 1. So the process of the proof is omitted in here.

Remark 2 In this paper, we adopt nonsmooth analysis and control theory to handle memristive neural networks. And compared with the results on globally exponentially stable in Lagrange sense of the neural networks with continuous right-hand side [6-7, 12, 16], our results for globally exponentially stable in Lagrange sense of the neural networks are right-hand side with coefficients discontinuous, so the results of this paper extend the earlier publications.

Remark 3 For the memristive neural networks, some sufficient conditions were obtained for periodic dynamic behaviors [2-3], exponential dissipativity [8], exponential synchronization [1, 11]. Compared with the above results, the main results of this paper are obtained for global exponential stability in Lagrange sense and the existence of globally exponentially attractive sets, which complement the earlier publications.

4. Illustrative example

Example: Considering the following three-neuron MNNs system with time-varying and finite distribute delays:

$$\begin{aligned}\frac{dx_i(t)}{dt} &= -d_i(x_i(t))x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t))g_j(x_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}(x_i(t))g_j(x_j(t-\tau_j(t))) \\ &+ \sum_{j=1}^{n} c_{ij}(x_i(t))\int_{t-\sigma_j(t)}^{t} g_j(x_j(s))ds + U_i(t), \\ t &\ge 0, \qquad i = 1,2,3\end{aligned} \tag{20}$$

where $\sigma(t) = 2\left|\cos(t)\right|, \tau(t) = 0.8\,|\sin(t)|,$

$$c_{32}(x_3(t)) = \begin{cases} 0.88, & |x_3(t)| \le 1, \\ 0.36, & |x_3(t)| > 1, \end{cases} c_{33}(x_3(t)) = \begin{cases} 1.55, & |x_3(t)| \le 1, \\ 0.75, & |x_3(t)| > 1, \end{cases}$$

$$\begin{aligned} U &= (\max\left|u_1(t)\right|, \max\left|u_2(t)\right|, \max\left|u_3(t)\right|)^T \\ &= (0.8862 \;\; 0.6455 \;\; 0.9312)^T, \end{aligned}$$

$$d_1(x_1(t)) = \begin{cases} 2.5, & |x_1(t)| \le 1, \\ 1, & |x_1(t)| > 1, \end{cases} \quad d_2(x_2(t)) = \begin{cases} 3, & |x_2(t)| \le 1, \\ 3.2, & |x_2(t)| > 1, \end{cases}$$

$$d_3(x_3(t)) = \begin{cases} 2.3, & |x_3(t)| \le 1, \\ 2, & |x_3(t)| > 1, \end{cases} \quad a_{11}(x_1(t)) = \begin{cases} 8, & |x_1(t)| \le 1, \\ 4, & |x_1(t)| > 1, \end{cases}$$

$$a_{12}(x_1(t)) = \begin{cases} -2, & |x_1(t)| \le 1, \\ -2.5, & |x_1(t)| > 1, \end{cases} \quad a_{13}(x_1(t)) = \begin{cases} 2, & |x_1(t)| \le 1, \\ 5, & |x_1(t)| > 1, \end{cases}$$

$$a_{21}(x_2(t)) = \begin{cases} -8, & |x_2(t)| \le 1, \\ -9, & |x_2(t)| > 1, \end{cases} \quad a_{22}(x_2(t)) = \begin{cases} 3, & |x_2(t)| \le 1, \\ 6, & |x_2(t)| > 1, \end{cases}$$

$$a_{23}(x_2(t)) = \begin{cases} 9, & |x_2(t)| \le 1, \\ -0.5, & |x_2(t)| > 1, \end{cases} \quad a_{31}(x_3(t)) = \begin{cases} 2, & |x_3(t)| \le 1, \\ 4, & |x_3(t)| > 1, \end{cases}$$

$$a_{32}(x_3(t)) = \begin{cases} 2, & |x_3(t)| \le 1, \\ 0.5, & |x_3(t)| > 1, \end{cases} \quad a_{33}(x_3(t)) = \begin{cases} -5, & |x_3(t)| \le 1, \\ -6, & |x_3(t)| > 1, \end{cases}$$

$$b_{11}(x_1(t)) = \begin{cases} 2, & |x_1(t)| \le 1, \\ 1, & |x_1(t)| > 1, \end{cases} \quad b_{12}(x_1(t)) = \begin{cases} -5, & |x_1(t)| \le 1, \\ -6.5, & |x_1(t)| > 1, \end{cases}$$

$$b_{13}(x_1(t)) = \begin{cases} 1, & |x_1(t)| \le 1, \\ 0.2, & |x_1(t)| > 1, \end{cases} \quad b_{21}(x_2(t)) = \begin{cases} 4, & |x_2(t)| \le 1, \\ 1, & |x_2(t)| > 1, \end{cases}$$

$$b_{22}(x_2(t)) = \begin{cases} 8, & |x_2(t)| \le 1, \\ 2, & |x_2(t)| > 1, \end{cases} \quad b_{23}(x_2(t)) = \begin{cases} 3, & |x_2(t)| \le 1, \\ 5, & |x_2(t)| > 1, \end{cases}$$

$$b_{31}(x_3(t)) = \begin{cases} 7, & |x_3(t)| \le 1, \\ 4, & |x_3(t)| > 1, \end{cases} \quad b_{32}(x_3(t)) = \begin{cases} -6, & |x_3(t)| \le 1, \\ -8, & |x_3(t)| > 1, \end{cases}$$

$$b_{33}(x_3(t)) = \begin{cases} -7, & |x_3(t)| \le 1, \\ -9, & |x_3(t)| > 1, \end{cases} \quad c_{11}(x_1(t)) = \begin{cases} 1.32, & |x_1(t)| \le 1, \\ -1.03, & |x_1(t)| > 1, \end{cases}$$

$$c_{12}(x_1(t)) = \begin{cases} 0.58, & |x_1(t)| \le 1, \\ 0.34, & |x_1(t)| > 1, \end{cases} \quad c_{13}(x_1(t)) = \begin{cases} -0.55, & |x_1(t)| \le 1, \\ 0.85, & |x_1(t)| > 1, \end{cases}$$

$$c_{21}(x_2(t)) = \begin{cases} 1.01, & |x_2(t)| \le 1, \\ -0.1, & |x_2(t)| > 1, \end{cases} \quad c_{22}(x_2(t)) = \begin{cases} 0.35, & |x_2(t)| \le 1, \\ 0.13, & |x_2(t)| > 1, \end{cases}$$

$$c_{23}(x_2(t)) = \begin{cases} 0.58, & |x_2(t)| \le 1, \\ 0.28, & |x_2(t)| > 1, \end{cases} \quad c_{31}(x_3(t)) = \begin{cases} 0.13, & |x_3(t)| \le 1, \\ 0.02, & |x_3(t)| > 1, \end{cases}$$

And then, the parameters are given as follows:

$$\underline{D} = diag\{1,3,2\}, \bar{D} = diag\{3.5, 6.2, 4.3\},$$

$$\bar{A} = \begin{pmatrix} 8 & -2 & 5 \\ -8 & 6 & 9 \\ 4 & 2 & -5 \end{pmatrix}, \underline{A} = \begin{pmatrix} 4 & -2.5 & 2 \\ -9 & 3 & 4.5 \\ 2 & 0.5 & -6 \end{pmatrix},$$

$$\bar{B} = \begin{pmatrix} 2 & -5 & 1 \\ 4 & 8 & 5 \\ 7 & -6 & -7 \end{pmatrix}, \underline{B} = \begin{pmatrix} 1 & -7 & 0 \\ 1 & 2 & 3 \\ 4 & -8 & -9 \end{pmatrix}$$

$$\bar{C} = \begin{pmatrix} 1.32 & 0.58 & 0.85 \\ 1.01 & 0.35 & 0.58 \\ 0.13 & 0.88 & 1.55 \end{pmatrix}, \underline{C} = \begin{pmatrix} -1.32 & 0.34 & -0.55 \\ -0.10 & 0.13 & 0.28 \\ 0.02 & 0.36 & 0.75 \end{pmatrix}.$$

Case 1: When the activation function is selected as $g(x(t))=\frac{1}{16}(x(t)+\tanh(x(t))),$ it is obviously that the activation function $g(\cdot)$ satisfies the assumption 1 with $L=0,$ $F=W=diag\{0.125,0.125,0.125\}$.

Then by using the Matlab LMI Control Toolbox, the solutions to the LMIs in (6) and (7) are derived as follows:

$$P=\begin{pmatrix}0.6819 & 0.1166 & -0.2544\\ 0.1166 & 0.7451 & 0.2085\\ -0.2544 & 0.2085 & 0.8519\end{pmatrix},$$
$$Q=diag\{0.0458,0.0777,0.3611\},$$
$$R=diag\{25.3785,19.1418,21.9942\},$$
$$S=diag\{20.9788,22.8649,19.1807\},$$
$$T=diag\{6.8177,9.4480,8.0978\},$$
$$H=\begin{pmatrix}13.1677 & 0.0083 & 0.0208\\ 0.0083 & 13.0189 & 0.0522\\ 0.0208 & 0.0522 & 13.1625\end{pmatrix}.$$

Calculating the eigenvalues of H, we get the eigenvalues are 13.0019, 13.1505, 13.1967. Therefore, following from Theorem 1, we gain that the MNNs system (20) is globally exponentially stable in Lagrange sense. Moreover, the compact set $\Omega=\{x\in R^n|\ x^T(t)Px(t)\le 27.1337/\varepsilon\}$ is a globally expo-nenttially attractive set of (20).

Case 2: On the other hand, when considered the function $g(x(t))=\frac{1}{16}(|x+1|+|x-1|),$ the activation function $g(\cdot)$ satisfies the assumption 1 with

$$L=diag\{-0.125,-0.125,-0.125\},$$
$$F=W=diag\{0.125,0.125,0.125\}.$$

Analogously, the solutions to the LMIs in (6) and (7) are derived as follows:

$$P=\begin{pmatrix}5.2500 & 0.9791 & -2.1937\\ 0.9791 & 5.8322 & 1.1810\\ -2.1937 & 1.1810 & 7.4673\end{pmatrix},$$
$$Q=diag\{1.9109,1.6459,4.1828\},$$
$$R=diag\{260.7605,190.7273,194.4384\},$$
$$S=diag\{153.7660,183.9017,138.0816\},$$
$$T=diag\{51.0309,77.4338,75.3789\},$$
$$H=\begin{pmatrix}104.9556 & -0.1554 & 0.2436\\ -0.1554 & 103.6230 & 0.5247\\ 0.2436 & 0.5247 & 104.1278\end{pmatrix}.$$

Calculating the eigenvalues of H, we get the eigenvalues are 103.2528, 104.4315, 105.0220. Therefore, following from Theorem 1, we gain that the MNNs system (20) is globally exponentially stable in Lagrange sense, and the compact set $\Omega=\{x\in R^n|\ x^T(t)Px(t)\le 215.7497/\varepsilon\}$ is a globally exponentially attractive set of (20).

Remark 4 The activation functions in Case 2 of this example don't meet with the conditions of [10], which mean that the conclusion in [10] can't be applied to ensure the stability in Lagrange sense of system (20). Hence, our results in this paper are less conservative than those in [10].

5. Conclusion

In this paper, we have analyzed the Lagrange stability problem for memristive neural networks with general activation functions and mixed delays. The mixed delays considered here are time-varying and finite distributed delays. Under the framework of Filippov's solution, and by utilizing a new Lyapunov-Krasovskii functional, the Halanay inequality and Linear matrix inequality technique, a set of novel sufficient conditions are obtained to ensure the globally exponentially stability in Lagrange sense of MNNs. Obviously, our approaches are different from those of the pre-existing. Moreover, our results show that the globally exponentially attractive set does contribute to the Lagrange stability of the considered system. Finally, a numerical example is given to show the effectiveness of the theoretical result.

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