Dynamic analysis of a slider-crank mechanism with joint clearance

Jiadong Mu [a], Tieqiang Gang [b, *] and Zhihua Fu [c]

School of Physics and Mechanical & Electrical Engineering, Xiamen University, Xiamen 361005, China

[a]mjd1990@foxmail.com, [b]gangtq@xmu.edu.cn, [c]fzh1990@foxmail.com

Abstract. This paper is focused on the simulations of kinematic and dynamic behavior of a slider-crank mechanism with joint clearance. Simulation tests are carried out with commercial software ADAMS. Kinematic and dynamic characteristics of an ideal slider-crank mechanism are analyzed by the analytical method to verify the validity of the model built by ADAMS. The obtained simulation data are analyzed by spectral analysis to get the effect of joint clearance on the dynamic characteristics of the mechanism. It is shown that, for the presence of clearance, most of the dynamic characteristics are influenced greatly and the impact frequency of the journal and bearing with the sleeve is closely related to the joint clearance.

Keywords: Joint clearance, Slider-crank mechanism, ADAMS, Spectral analysis.

1. Introduction

For the assembly and mobility of components, clearance exists inevitably in the kinematic joints of mechanisms. It significantly affects the kinematic and dynamic characteristics of the mechanisms. Clearance acts as a main source of impact forces. These forces can not only enlarge vibration strength, but also reduce system reliability, stability, precision, efficiency and life. Impact forces can cause joints worn faster and losing efficacy. With the rapid development of the aerospace and high-speed system industry, requirements for precision and system reliability are rising [1]. Therefore, studies on clearance influence in mechanism system has become an advanced subject.

In this study, we take a slider-crank mechanism as an example to illustrate how joint clearance affects the kinematic and dynamic characteristics of the mechanism. For the simulation tests, we build the model by the software ADAMS, and analyze the simulation results with spectral analysis method. The results can provide theoretical basis for mechanical design and optimization.

2. Modeling of the mechanism with clearance

A slider-crank mechanism having one revolute joint with clearance is shown in Fig. 1. The revolute joint *2* is an ideal joint and the revolute joint *3* has a clearance. $\boldsymbol{l}_1, \boldsymbol{l}_2$ and $\boldsymbol{l}_3$ represent the vectors of the poles in the mechanism.

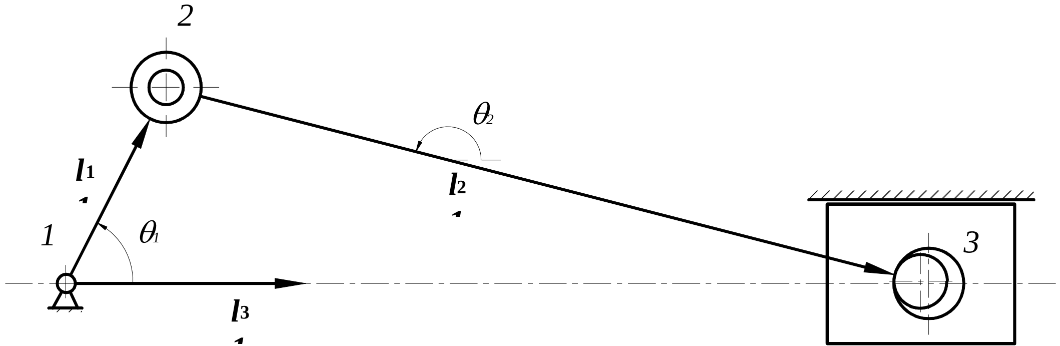

Fig.1 Slider-crank mechanism with joint clearance

3. Kinematics and dynamic analysis of slider-crank mechanism

As is shown in Fig. 1, l_1 and l_2 are the length of the crank and the link. The angle θ_1 and angular velocity ω_1 of the crank are given.

In this section, analytic method [2] is performed to analyze the kinematic and dynamic characteristics of the slider-crank mechanism without clearance, which is to verify the validity of the model built by ADAMS.

Firstly, the position equation of the mechanism can be established. Then take the first and second derivative of the position equation versus time, and the velocity and acceleration equation of the mechanism can be obtained. Solving these equations, we can get the expression of angular velocity and acceleration of the link, and the velocity and the acceleration of the slider.

In order to establish the displacement equation of the mechanism by vector method, the components should be represented by vectors, and a closed vector polygon should be made for the mechanism.

As shown in Fig. 2, a Cartesian coordinate is established. l_1 is the length of crank and θ_1 is the angular; l_2 is the length of link and θ_2 is the angular; l_3 represents the vector from pivot to slider. Thus we have a closed vector polygon composed by the pole vectors. In this closed vector polygon, the sum of each vector must equal to zero, i.e.,

$$\boldsymbol{l_1} + \boldsymbol{l_2} = \boldsymbol{l_3} \tag{1}$$

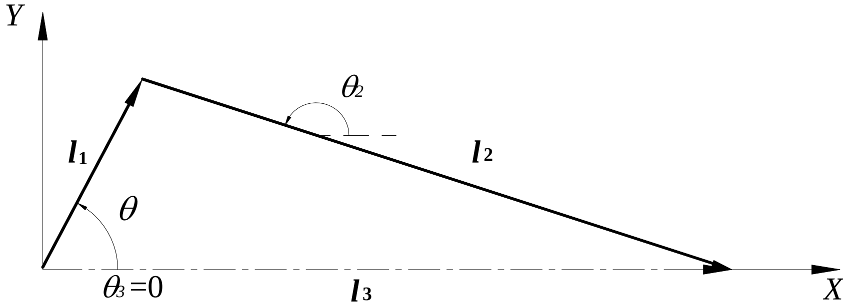

Fig. 2 Vector representation

Position Analysis. According to the law of cosines, the following equation can be given.

$$l_2^2 = l_1^2 + l_3^2 - 2l_1 l_3 \cos\theta_1 , \tag{2}$$

The position expression of the slider can be obtained as

$$l_3 = l_1 \cos\theta_1 + \sqrt{4l_1^2 \cos^2\theta_1 - l_1^2 + l_2^2} \ . \tag{3}$$

Velocity analysis. Rewrite the closed vector Eq. (1) in a plural vector form, we obtain

$$l_1 \cdot e^{i\theta_1} + l_2 \cdot e^{i\theta_2} = l_3 , \tag{4}$$

Taking the derivative of Eq (4) versus time and we have:

$$l_1 \cdot i \cdot \omega_1 \cdot e^{i\theta_1} + l_2 \cdot i \cdot \omega_2 \cdot e^{i\theta_2} = v_3 , \tag{5}$$

Using the Euler equation $e^{i\theta} = \cos\theta + i\sin\theta$, we can rewrite Eq. (5) as

$$l_1 \cdot i \cdot \omega_1 \cdot (\cos\theta_1 + i\sin\theta_1) + l_2 \cdot i \cdot \omega_2 \cdot (\cos\theta_2 + i\sin\theta_2) = v_3 , \tag{6}$$

Separate the real and imaginary parts of Eq.(6), we get:

$$l_1\omega_1 \cos\theta_1 + l_2\omega_2 \cos\theta_2 = 0 , \tag{7}$$

$$v_3 = -l_1\omega_1 \sin\theta_1 - l_2\omega_2 \sin\theta_2 . \tag{8}$$

By solving Eq. (7) and Eq. (8), we can get angular velocity of the link and velocity of the slider:

$$\omega_2 = \frac{-l_1\omega_1 \cos\theta_1}{l_2 \cos\theta_2} , \tag{9}$$

$$v_3 = -l_1\omega_1 \sin\theta_1 + \frac{l_1 \sin\theta_2 \cos\theta_1}{\cos\theta_2}\omega_1 . \tag{10}$$

Acceleration analysis. By taking the derivative of Eq. (7) and Eq. (8) versus *t*, we can get:

$$-l_1\omega_1^2 \sin\theta_1 + l_2\alpha_2 \cos\theta_2 - l_2\omega_2^2 \sin\theta_2 = 0 , \tag{11}$$

$$a_3 = -l_1\omega_1^2 \cos\theta_1 - l_2\alpha_2 \sin\theta_2 - l_2\omega_2^2 \cos\theta_2 , \tag{12}$$

Solving Eq. (11) and Eq. (12), angular acceleration of the link and acceleration of the slider are obtained, i.e.

$$\alpha_2 = \left(\omega_2^2 \sin\theta_2 + \frac{l_1}{l_2}\omega_1^2 \sin\theta_1 \right)\frac{1}{\cos\theta_2} ,$$

(13)

$$a_3 = -l_1\omega_1^2 \cos\theta_1 - \frac{l_2\omega_2^2 \sin^2\theta_2 + l_1\omega_1^2 \sin\theta_1 \sin\theta_2}{\cos\theta_2} - l_2\omega_2^2 \cos\theta_2 . \tag{14}$$

4. The random vibration analysis

In general, due to the time course of the vibration, random vibration is distinctly non-periodic functions. Through Fourier integral method, these continuous multi-frequency vibration components can be shown, and each has its corresponding frequency power or energy, their relationship is shown with a graph, the power spectral density (*PSD*). In the development process of random vibration theory, the establishment of the concept of the power spectral density has a very crucial significance because of its applicability for engineering analysis. For the mechanism with joint clearance described in this paper, the vibration caused by the clearance belongs to random vibration.

Autocorrelation function. In order to characterize a random vibration process caused by the joint clearance during the motion process of the slider-crank mechanism, defined by $X[t]$, a correlation function is used to study the interdependence of random variables at two different moments, i.e.

$$R_x(t_1, t_1+\tau) = E\left[X(t_1) X(t_1+\tau) \right] = \lim_{x\to\infty} \frac{1}{n} x_k(t_1) x_k(t_1+\tau) . \tag{15}$$

$R_x(t_1, t_1+\tau)$ Is autocorrelation function of the slider-crank motion process $X[t]$ at time t_1 and $t_1+\tau$ [3]? It is not only the function of τ , but also related to time t_1.

The power spectral density function. Correlation function shows the statistical properties of random vibration process, which is in the time difference domain; whereas the power spectral density shows statistical properties of random vibration process on each frequency component, which is in the frequency domain. In order to characterize the distribution of random vibration during the slider-crank mechanism motion process, we introduce a function of power spectral density, which is the Fourier transform of the autocorrelation function, i.e.

$$S_x(\omega) = \int_{-\infty}^{\infty} R_x(\tau) e^{-i\omega\tau} d\tau , \tag{16}$$

Its inverse transformation is

$$R_x(\tau) = \int_{-\infty}^{\infty} S_x(\omega) e^{i\omega\tau} d\omega . \tag{17}$$

$S_x(\omega)$ Is called power spectral density function [4], which represents the density of energy at each angular frequency in the random vibration process.

The function of power spectral density is an important parameter to describe the random vibration. It enables us to know which of the frequencies are major, and this analysis also contributes to vibration analysis.

Simulation data analysis. This paper mainly studies slider-crank mechanism with joint clearance and analyzes how the clearance affects the dynamic characteristics of the slider-crank mechanism. In the simulations, the clearance ranges from 0.1mm to 1.0mm by a step of 0.02mm, and the simulation results are analyzed by statistical methods.

Although there are some scholars who conducted dynamic analysis of mechanism with joint clearance, most of their works compared the dynamical characteristics of several mechanisms with different clearance to analyze how the clearance affects the mechanisms. This method is feasible, however, there is no quantitative assessment to judge the impact of the clearance accurately. While in this article, we use the statistical analysis of data to illustrate the impact on the mechanism with joint clearance, which is more convincing.

During the slider-crank mechanism motion process, due to the joint clearance, structural vibration is caused. This vibration is a random vibration, which can’t be described by certain functions. Thus, we must use probability and statistical method to present the characteristics of stochastic processes.

Data generated in random vibration are irregular, similar to the noise signal. So we use the spectral analysis method to analyze the simulation results, i.e., using power spectral density values to describe the dynamic characteristic of the mechanism quantitatively. Power spectral density can characterize the power and energy in the vibration process, through which we can describe the levels of vibration.

In the analysis process, we analyze the slider displacement error $e_1(t)$, the velocity error $e_2(t)$, and the link displacement error $e_3(t)$ by the spectral analysis method for each clearance, and get the auto-spectral density $S_{xc}(\omega_n)$ of n angular frequencies for each clearance. Here, c represents the clearance value and $c = 0.1, 0.12, 0.14, 0.16, 0.18 \cdots 1.0$. Then calculate the average of auto-power spectral density values for each clearance, i.e.

$$\overline{S_c} = \frac{S_{xc}(\omega_1)+S_{xc}(\omega_2)+S_{xc}(\omega_3)+\cdots+S_{xc}(\omega_n)}{n} . \tag{18}$$

5. Simulation and result

Simulation model and parameter selection. With the multi-body dynamics simulation software ADAMS [5], the model of slider-crank mechanism with joint clearance is built, as shown in Fig. 3. The model parameters and simulation parameters are shown in Table 1 and Table 2.

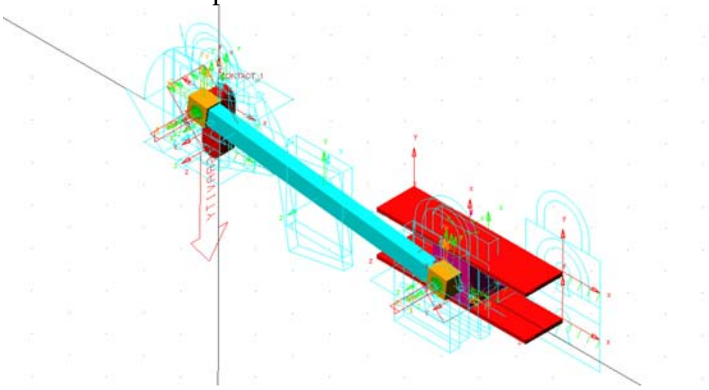

Fig. 3 Slider-crank Mechanism model built under ADAMS

Table 1 Modeling parameter

	Length[mm]	Cross-sectional area[mm^2]	Mass[kg]
l_1	100	25×20	0.24
l_2	250	20×15	0.70
Slider	50	50×50	0.90

Table 2 Simulation parameter

Crank speed[rad/s]	Stiffness	Damping[N*sec/mm]	Force Exponent	Step[s]
30	1.0E+0.005	15.0	1.4	1/2000

Contact parameter directions:

Stiffness refers to the material stiffness. In general, the greater the stiffness is, the more difficult the integral is.

Force Exponent is used to calculate the contribution exponent of material stiffness in instantaneous normal force, usually take 1.5 or greater. Its value ranges from 1.3 to 1.5 for metal.

Damping defines damping properties of the contact material. In general, its value is 0.1~1 percent of stiffness values.

Result analysis. As shown in Fig. 4 and Fig. 5, the data points represent the average value of power spectral density of link displacement and slider displacement for each clearance. These data points in graphs can be fitted well by exponential function curves. It indicates that when the slider-crank mechanism is in motion, with the increasing of joint clearance, the influence of clearance on the slider and link displacement presents exponential growth. When the joint clearance is small, mainly due to the presence of clearance, the journal will collide with the sleeve and this affects the displacement of slider and link. When the joint clearance is large, the collisions influence the dynamic characteristic, and the presence of joint clearance also to some extent changes the geometric parameters of the mechanism. Therefore, with the joint clearance increasing, due to the above two factors, the effect of clearance on displacement of slider and link will become more and more obvious.

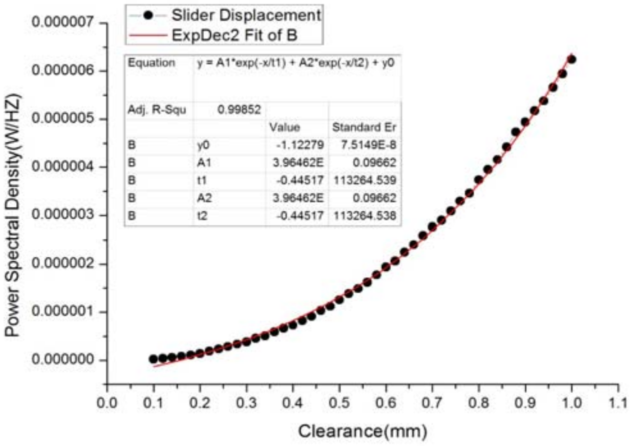

Fig. 4 PSD statistic analysis of slider displacement

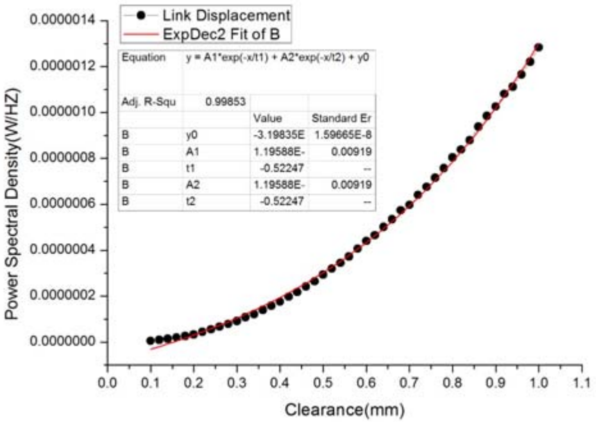

Fig. 5 PSD statistic analysis of link displacement

As shown in Fig. 6, the data points represent the average value of power spectral density of slider velocity for each clearance. There are some minor fluctuations in the curve, but the general trend is quite obvious. With the increasing of joint clearance, the impact of clearance on slider velocity becomes bigger.

Fig. 7 presents the power spectral density curve of slider velocity of the mechanism with joint clearances of 0.2mm, 0.5mm and 1.0mm, the frequency of the power spectral density of the slider with joint clearance 1.0 centralizes in the lower frequency range than the others. However, the frequency of the power spectral density of the slider with joint clearance 0.2 centralizes in the highest frequency range of all. Thus the journal colliding with the sleeve is the main reason that influences the slider velocity. As a result, in mechanism motion process, when the joint clearance is increasing, the frequency that the journal collides with the sleeve is lower, which indicates that the time they are in contact increases.

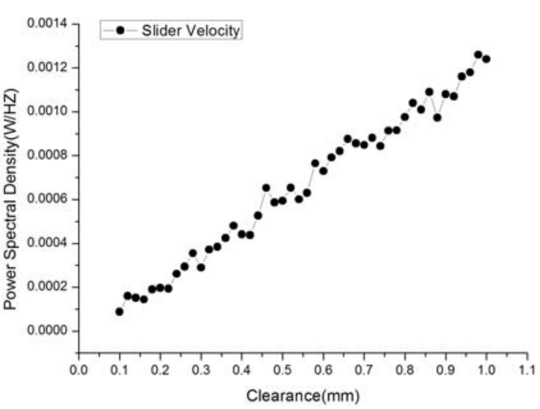

Fig. 6 PSD statistic analysis of slider velocity

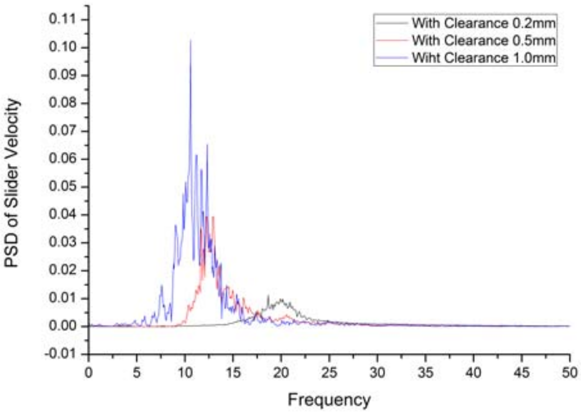

Fig. 7 PSD of slider velocity

6. Summary

In mechanism motion process, joint clearance is one of the most important factors which affects dynamic characteristics. With the increase of joint clearance, most of the dynamic characteristics of the mechanism are influenced greatly. The main conclusions are as follows:

When the joint clearance is increasing, its influence on the slider velocity and slider displacement and link displacement are increasing.

The smaller the joint clearance is, the more frequently the journal collides with the sleeve.

With the increase of the joint clearance, the time that the journal contacting with the sleeve is increasing and the collisions between them are decreasing.

Acknowledgements

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