The Research of Collision Detection Algorithm Based on Separating axis Theorem

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Abstract

Separating axis theorem is used in a large number of game development, it is mainly used to detect whether two convex polygon product a collision. When the number of sides of a convex polygon is less, separating axis theorem can quickly determine whether it products a collision. However, when the number of edges is increased, the polygon will become more complex and the greater costs will be. A improved algorithm is proposed for this shortcoming based on the closest point, where to find the parallel line of closest point as a potential separation axis, and then through potential separating axis to determine whether they product a collision.

Keywords

Separating axis theorem, convex polygon, closest point, collision detection.

1. Introduction

With the continuous development of computer technology, virtual reality technology is constantly improving, and collision detection technology is undoubtedly the virtual reality technology to best embody the "reality". Collision detection is to determine whether they product a collision between two or more virtual world objects in order to perform a series of actions after the collision, such as being blocked by an obstacle, rewind and so on. Today, a large number of collision detection algorithm is used in computer games, augmented reality, we have higher requirements for the real-time and accurate of the collision detection algorithms, so that it can give players a better sense of reality [1].

Separating axis theorem [2] (Separating Axis Theorem, Called the SAT) The main content is when two objects to detect collision, if we can find out a axis to show the two objects do not intersect which projection on this axis . then, it will prove two objects do not intersect. article[3] proposed that by using the normal of a edge of convex polygon as a potential separation axis, but this method has a drawback what is with growth of edges of a convex polygon, the number of potential separation of the respective axis will also increase. articles [4-8] summed up that by the bounding box for testing in advance, if two bounding boxes have product collision, then we will use separating axis theorem to accurate determination. otherwise it can be determined that it does not collide. This method does not solve with growth of edges of a convex polygon the potential separating axis will increase. Instead, the use of the bounding box to increase the amount of computation. In this paper, Improved algorithm is based on the closest point [9,10], try to find this potential separating axis that it must be parallel to the line of closest point, So as to solve this question.

2. Related knowledge

2.1 Euclidean distance

Euclidean distance (Euclidean distance) also referred to as the Euclid distance, it often used to define the Euclidean distance between the two-dimensional space.

\[ d(x, y) = \sqrt{\sum(x_i - y_i)^2} \quad i = 0,1,2,3,...,n \]  

(1)
The closest points can be determined by the Euclidean distance in this article, the real distance between two convex polygons obtained by Euclidean distance.

2.2 Convex polygon and properties

If all the edges of a polygon, there is an edge to the indefinite extension of the two parties to become a straight line, the other edges are in the same side of this line (or connection between any two points inside polygon), then the polygon is called convex polygon. The nature of convex polygon and Euclidean distance of points, we can draw the following one character.

Nature: the closest points of two convex polygons contains at least one vertex in the all vertexes of two convex polygons.

Center of gravity of convex polygon [11]: For center of gravity of a convex \( N \)-gon, it can be divided into a triangle and a convex \((N-1)\)-gon or quadrilateral and convex \((N-2)\)-gon, and then find out the center of gravity and connect into the line. As shown in Figure 1 Seeking the center of gravity a convex quadrilateral, which were taken diagonally into two triangles, respectively gain center of gravity and a line segment connecting the center of gravity as shown in Figure 1 (a), (b), the line segment \( AB, CD \) the intersection and \( E \) is the center of gravity of the quadrilateral.

![Fig. 1 Center of gravity of convex polygon](image)

2.3 Separating axis theorem

Separating axis theorem by S. Gottschalk [2] suggested that the main content is for two or more objects will collide, if I could find a axis so that these objects do not overlap on the projection of the axis, then we can believe that these objects do not intersect each other. Select theoretically axis is not fixed, as long as there is such an axis. However, due to the separation axis theorem applies to convex polygons, so for non-convex polygon can be decomposed into several all convex polygons. shown in Figure 2.

![Fig. 2 Separating axis theorem](image)

For 2D, the normal of each edge of convex polygon include the possibility of the axis in all directions, so when we select potential separating axis (Potential Separating Axis, PSA), generally we can choose normal of edges of the convex polygon. then the normal vector can be so defined:

\[
\begin{bmatrix}
-x \\
y
\end{bmatrix} = 0 \quad \text{or} \quad \begin{bmatrix}
x \\
y
\end{bmatrix} = 0
\]

(2)

For negative numbers, it just represents different directions of the two normal vectors, and it does not need to be considered when vertexes projected onto potential separating axis.

Separating axis theorem collision detection algorithm is an optimistic collision detection algorithm, once you have found such a disjoint axis that detection will no longer be implemented, otherwise would have been implemented. And when the convex polygon becomes more complex, the greater the corresponding overhead. So for these shortcomings, leading to the emergence of a large number
of calculations, there are many optimization schemes. The main idea is to reduce the number of potential separating axis, thereby reducing the amount of computation for collision detection. For example, two rectangles, because of its edges are parallel two by two, so we can merge potential separating axis into two separating axis, Under the best of circumstances the number of its maximum potential separating axis is two. The worst case is four. There is no need to test eight axes, We just need to detect two or four axes to determine whether the collision. Unfortunately, other irregularities can not use this method, only to detect potential separating axis one by one. There are other optimization programs, such as the object’s bounding box beforehand rough test, then more precise AABB bounding box test [12], after separating axis test, thereby reducing unnecessary calculations. Clearly these measures did not solve this question from within.

3. Improved algorithm

3.1 Closest points

Closest points are two convex polygon the nearest two points, namely European nearest point. If \( s_1, s_2 \) are the closest points, you must satisfy the following condition:

\[
D(s_1, s_2) = \{(s_1, s_2) | \text{dis}(s_1, s_2) \leq \text{dis}(a_i, b_j), a_i \in P_1, b_j \in P_2, \quad i = 0, 1, 2...n, j = 0, 1, 2...N\} \tag{3}
\]

Which \( s_1, s_2 \) represent points on the convex polygon \( P_1, P_2 \) point set, \( \text{dis}(a, b) \) represents the Euclidean distance between two points.

These two points can determine whether a collision between each other very accurately, this approach which is also the best. By the nature of a convex polygon can be found the nearest point or a group contains at least a vertex of convex polygon. If \( A \) is one of the nearest points, then by the following formula, you can easily draw another point. Figure 3 shows a schematic diagram.

![Fig. 3 Closest points](image)

\[ \text{So } n = (n_1, n_2), A(x_A, y_A), \text{ the point where the straight line } A: \]

\[
\frac{x - x_A}{y - y_A} = \frac{n_1}{n_2} \tag{4}
\]

(4)and edge \( CD \) form a set of equations to solve another point, the use of the Euclidean distance we can determine which is another closest point. However, we do not know which one the vertex of convex polygon is closest point from the beginning, but we can achieve by computer programming, this would need to traverse all points of the two convex polygons. Obviously this method seek out axis as a potential separating axis is undesirable. As shown in Figure 4 and Formula (5), we find the longer the \( OE \), the more close to the vertex and the other convex polygons. So we will get two potential nearest points \( B \) and \( C \). The normal vectors of the edges of containing point \( B \) and point \( C \) as potential separating axis, so there are only four potential separating axis.

\[
OE = \frac{OB}{\cos \angle BOE} \tag{5}
\]

3.2 Potential separating axis

From the separating axis definition, which it is a kind of "optimists" algorithm. The literature [2]approach to take potential separating axis, as shown in Figure 5 with the edges of convex polygon increases, the potential separating axis also will increase.
Improved algorithm starts to find an effective center of two convex polygons - Center of gravity of convex polygon, the connection of the centers as a judging condition of the beginning, thus $AABB$ is replaced by this method as a rough pre-judgment conditions [6]. The specific approach is the all vertexes are projected onto the connection of the centers. if there is no overlap phenomenon, it can be judged without a collision, otherwise go to the next stage of testing.

Fig.6 (b) is known, when the rough detection stage detects the two objects are overlap, but this phenomenon does not explain its has intersect, So it will enter accurate testing phase. The normal of the straight line of two potential closest points is used as potential separating axis. Fig. 6 (b) as shown, if $B$ is one of the nearest points, then the another is the point $B$ to the vertical line of edge $CD$, so one of the four separating axes is a line that this parallel to the line of the nearest points. As long as there is no overlap, you can determine that it does not collide. If overlap occurs then the two objects intersect, it is a good solution to the problems separating axis theorem.

4. **Improved algorithm overall description**

The following specific description of the improved algorithm:

Step 1  Find out effective centers of two convex polygons.

Step 2  Get in touch with the center wire four vertices intersect segment.

Step 3  Enter the rough testing phase, the four vertices are projected onto the effective center connection, if there is no overlap, the two objects do not collide. Otherwise it proceeds to step Step4.

Step 4  Step 4 will connect the vertical edge intersects with an effective separation of the central axis as a potential crash test accurate, if not overlap, you can determine that it does not collide, otherwise collision.
We can know from the improved algorithm, one can improve the accuracy rate of collision detection, on the other hand can provide efficiency of collision detection, reduce the average time of detection.

5. Efficiency

Based on separating axis collision detection traditional algorithm, the entire algorithm takes place in two main stages: First, to establish the potential separating axis of each edge of the convex polygon, at this stage we need to traverse the vertex of each edge, and then find out the normal of edge as a potential separating axis (PSA). Second, traverse all vertexes projected onto the potential separating axis, if you can find a this potential separating axis (PSA) so that the two objects do not overlap on this projection, then the two objects will without collision. However, in two phases with the growth of the number of vertexes while becoming more complex.

Compared with the separation axis collision detection traditional algorithm, this paper mainly takes the time to find the center of gravity.

6. Conclusion

In this paper, with SAT algorithm is proposed based on a vector to the closest point where the potential separating axis to determine whether two convex polygon collision. this paper improved collision detection algorithm to shorten the average time, improve the accuracy of collision detection, center of gravity of the convex polygon can get it in the pre-treatment stage. Most of the game engine have a fairly complete physical systems, such as Unity3D and Cocos2d-x are relatively well-known domestic, which contains its collision detection algorithms, but there are still some problems to be solved. In this paper, as well as combining popular game, we can try to extend our algorithm to the three-dimensional

References