Some generalized Bonferroni mean operators with 2-tuple linguistic information and their application to multiple attribute decision making

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Abstract

In this paper, we investigate the multiple attribute decision making problems with 2-tuple linguistic information. Motivated by the ideal of generalized weighted Bonferroni mean and generalized weighted geometric Bonferroni mean, we develop the 2-tuple linguistic generalized Bonferroni mean (2TLGBM) operator for aggregating the 2-tuple linguistic information and 2-tuple linguistic generalized geometric Bonferroni mean (2TLGGBM) operator. For the situations where the input arguments have different importance, we then define the 2-tuple linguistic generalized weighted Bonferroni mean (2TLGWBM) operator and 2-tuple linguistic generalized weighted geometric Bonferroni mean (2TLGWGBM) operator, based on which we develop the procedure for multiple attribute decision making under the 2-tuple linguistic environments.

Keywords

multiple attribute decision making; 2-tuple linguistic variables; generalized Bonferroni mean; generalized geometric Bonferroni mean; generalized weighted Bonferroni mean; generalized weighted geometric Bonferroni mean.

1. Introduction

Multiple attribute decision making is a usual task in human activities. It consists of finding the most preferred alternative from a given alternative set. The increasing complexity of the socio-economic environment makes it less and less possible for a single decision maker to consider all relevant aspects of a problem. As a result, many decision making processes take place in group settings in the real life situation. However, under many conditions, for the real multiple attribute decision making problems, the decision information about alternatives is usually uncertain or fuzzy due to the increasing complexity of the socio-economic environment and the vagueness of inherent subjective nature of human think, thus, numerical values are inadequate or insufficient to model real-life decision problems. Indeed, human judgments including preference information may be stated in linguistic terms[1-30].

In this paper, we investigate the multiple attribute decision making problems with 2-tuple linguistic information. Motivated by the ideal of Bonferroni mean[31] and geometric Bonferroni mean[32], we develop the 2-tuple linguistic generalized Bonferroni mean (2TLGBM) operator for aggregating the 2-tuple linguistic information and 2-tuple linguistic generalized geometric Bonferroni mean (2TLGGBM) operator. For the situations where the input arguments have different importance, we then define the 2-tuple linguistic generalized weighted Bonferroni mean (2TLGWBM) operator and 2-tuple linguistic generalized weighted geometric Bonferroni mean (2TLGWGBM) operator, based on which we develop the procedure for multiple attribute decision making under the 2-tuple linguistic environments.

2. Preliminaries

Herrera[1-2] first introduced the 2-tuple fuzzy linguistic approach for overcoming the drawback of the classical computational models, which include the semantic model and symbolic model. The 2-tuple linguistic model is a kind of new information processing method. It takes 2-tuple to represent linguistic assessment information and carry out operation. The basic concept of linguistic 2-tuple is symbolic translation. The 2-tuple linguistic representation and computational model has received more and more attention since its appearance.

In the following, we shall introduce the definition of the 2-tuple linguistic representation and computational model.

Let $S=\{s_i \mid i=1,2,\cdots,t\}$ be a linguistic term set with odd cardinality. Any label, s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics [1-2]:

(1) The set is ordered: $s_i > s_j$, if $i > j$; (2) Max operator: $\max(s_i, s_j) = s_i$, if $s_i \geq s_j$; (3) Min operator: $\min(s_i, s_j) = s_i$, if $s_i \leq s_j$. For example, S can be defined as

$$S=\{s_1 = extremely\ poor(EP), s_2 = very\ poor(VP), s_3 = poor(P), s_4 = medium(M),\\ s_5 = good(G), s_6 = very\ good(VG), s_7 = extremely\ good(EG)\}$$

Herrera and Martinez[1-2] developed the 2-tuple fuzzy linguistic representation model based on the concept of symbolic translation. It is used for representing the linguistic assessment information by means of a 2-tuple (s_i, α_i), where s_i is a linguistic label from predefined linguistic term set S and α_i is the value of symbolic translation, and $\alpha_i \in [-0.5, 0.5)$.

Definition 1[1-2]**.** Let β be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation, $\beta \in [1,t]$, being t the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [1,t]$ and $\alpha \in [-0.5, 0.5)$ then α is called a symbolic translation.

Definition 2[1-2]. Let $S=\{s_1, s_2, \cdots, s_t\}$ be a linguistic term set and $\beta \in [1,t]$ is a number value representing the aggregation result of linguistic symbolic. Then the function Δ used to obtain the 2-tuple linguistic information equivalent to β is defined as:

$$\Delta : [1,t] \to S \times [-0.5, 0.5) \tag{1}$$

$$\Delta(\beta) = \begin{cases} s_i, i = round(\beta) \\ \alpha = \beta - i, \alpha \in [-0.5, 0.5) \end{cases} \tag{2}$$

where round(.) is the usual round operation, s_i has the closest index label to β and α is the value of the symbolic translation.

Definition 3[1-2]. Let $S=\{s_1, s_2, \cdots, s_t\}$ be a linguistic term set and (s_i, α_i) be a 2-tuple. There is always a function Δ^{-1} can be defined, such that, from a 2-tuple (s_i, α_i) it return its equivalent numerical value $\beta \in [1,t] \subset R$, which is.

$$\Delta^{-1}: S \times [-0.5, 0.5) \to [1, t] \tag{3}$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \tag{4}$$

From Definitions 1 and 2, we can conclude that the conversion of a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation:

$$\Delta(s_i) = (s_i, 0) \tag{5}$$

Definition 4[1-2]. Let (s_k, a_k) and (s_l, a_l) be two 2-tuple, they should have the following properties:

(1)If $k < l$ then (s_k, a_k) is smaller than (s_l, a_l);

(2) If $k = l$ then

if $a_k = a_l$, then (s_k, a_k), (s_l, a_l) represents the same information;

if $a_k < a_l$ then (s_k, a_k) is smaller than (s_l, a_l);

if $a_k > a_l$ then (s_k, a_k) is bigger than (s_l, a_l).

3. Some generalized Bonferroni mean aggregating operators with 2-tuple linguistic information

Beliakov et al. [33] further extended the BM operator by considering the correlations of any three aggregated arguments instead of any two.

Definition 5. Let $p, q, r \geq 0$ and $a_i\ (i = 1, 2, \cdots, n)$ be a collection of nonnegative numbers. If

$$GBM^{p,q,r}(a_1, a_2, \cdots, a_n) = \left(\frac{1}{n(n-1)(n-2)} \sum_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} a_i^p a_j^q a_k^r \right)^{\frac{1}{p+q+r}} \tag{6}$$

then $GBM^{p,q,r}$ is called the generalized Bonferroni mean (GBM) operator.

In particular, if $r = 0$, then the GBM operator reduces to the BM operator. However, it is noted that both BM operator and the GBM operator do not consider the situation that $i = j$ or $j = k$ or $i = k$, and the weight vector of the aggregated arguments is not also considered. To overcome this drawback, Xia et al. [34] defined the weighted version of the GBM operator.

Definition 6. Let $p, q, r \geq 0$ and $a_i\ (i = 1, 2, \cdots, n)$ be a collection of nonnegative numbers with the weight vector $w = (w_1, w_2, \cdots, w_n)^T$ and $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$. If

$$GWBM^{p,q,r}(a_1, a_2, \cdots, a_n) = \left(\frac{1}{n(n-1)(n-2)} \sum_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} w_i w_j w_k a_i^p a_j^q a_k^r \right)^{\frac{1}{p+q+r}} \tag{7}$$

then $GWBM^{p,q,r}$ is called the generalized weighted Bonferroni mean (GWBM) operator.

In the following, we shall develop 2-tuple linguistic generalized Bonferroni mean (2TLGBM) operator and 2-tuple linguistic generalized weighted Bonferroni mean (2TLGWBM) operator.

Definition 7. Let $p,q,r \geq 0$ and $\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple linguistic variables, If

$$2TLGBM^{p,q,r}\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\} = \Delta\left(\frac{1}{n(n-1)(n-2)}\sum_{\substack{i,j,k=1 \\ i\neq j\neq k}}^{n}\left(\Delta^{-1}(r_i,a_i)\right)^p\left(\Delta^{-1}(r_j,a_j)\right)^q\left(\Delta^{-1}(r_k,a_k)\right)^r\right)^{\frac{1}{p+q+r}} \tag{8}$$

then $2TLGBM^{p,q,r}$ is called the 2-tuple linguistic generalized Bonferroni mean (2TLGBM) operator.

If $r=0$, then the 2TLGBM operator reduces to the 2TLBM operator.

$$\begin{aligned} &2TLGBM^{p,q,0}\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\} \\ &= 2TLBM^{p,q}\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\} \\ &= \Delta\left(\frac{1}{n(n-1)}\sum_{\substack{i,j \\ i\neq j}}^{n}\left(\Delta^{-1}(r_i,a_i)\right)^p\left(\Delta^{-1}(r_j,a_j)\right)^q\right)^{\frac{1}{p+q}} \end{aligned} \tag{9}$$

However, it is noted that both 2TLBM operator and the 2TLGBM operator do not consider the situation that $i=j$ or $j=k$ or $i=k$, and the weight vector of the aggregated arguments is not also considered. To overcome this drawback, we shall propos the weighted version of the 2TLGBM operator.

Definition 8. Let $\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple linguistic variables, and let $p,q,r \geq 0$. $w=(w_1,w_2,\cdots,w_n)^T$ is the weight vector of $\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}(i=1,2,\cdots,n)$, where w_i indicates the importance degree of (r_i,a_i), satisfying $w_i>0(i=1,2,\cdots,n)$, and $\sum_{i=1}^{n} w_i = 1$. If

$$2TLGWBM_w^{p,q,r}\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\} = \Delta\left(\sum_{i,j,k=1}^{n}\left(w_iw_jw_k\Delta^{-1}(r_i,a_i)\right)^p\left(\Delta^{-1}(r_j,a_j)\right)^q\left(\Delta^{-1}(r_k,a_k)\right)^r\right)^{\frac{1}{p+q+r}} \tag{10}$$

then $2TLGWBM_w^{p,q,r}$ is called the 2-tuple linguistic generalized weighted Bonferroni mean (2TLGWBM) operator.

It can be easily proved that the 2TLGWBM operator has the following properties.

Theorem 1. (Idempotency)

Let $x=\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple linguistic variables. If all $(r_j,a_j)(j=1,2,\cdots,n)$ are equal, i.e. $(r_j,a_j)=(r,a)$ for all j, then

$$2TLGWBM_{w}^{p,q,r}\left\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right\}=(r,a) \tag{11}$$

Theorem 2. (Boundedness)

Let $x=\left\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right\}$ be a set of 2-tuple linguistic variables, and let

$$\left(r^-,a^-\right)=\min_j\left(r_j,a_j\right),\ \left(r^+,a^+\right)=\max_j\left(r_j,a_j\right)$$

Then

$$\left(r^-,a^-\right)\le 2TLGWBM_{w}^{p,q,r}\left\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right\}\le\left(r^+,a^+\right) \tag{12}$$

Theorem 3. (Monotonicity)

Let $x=\left\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right\}$ and $x'=\left((r_1',a_1'),(r_2',a_2'),\ldots,(r_n',a_n')\right)$ be two set of 2-tuple linguistic variables, if $\left(r_j,a_j\right)\le\left(r_j',a_j'\right)$, for all j, then

$$\begin{aligned}&2TLGWBM_{w}^{p,q,r}\left((r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right)\le\\&2TLGWBM_{w}^{p,q,r}\left((r_1',a_1'),(r_2',a_2'),\ldots,(r_n',a_n')\right)\end{aligned} \tag{13}$$

Theorem 4. (Commutativity)

Let $x=\left\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right\}$ and $x'=\left((r_1',a_1'),(r_2',a_2'),\ldots,(r_n',a_n')\right)$be two set of 2-tuple, where $\left((r_1',a_1'),(r_2',a_2'),\ldots,(r_n',a_n')\right)$is any permutation of $\left\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right\}$, then

$$\begin{aligned}&2TLGWBM_{w}^{p,q,r}\left((r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right)=\\&2TLGWBM_{w}^{p,q,r}\left((r_1',a_1'),(r_2',a_2'),\ldots,(r_n',a_n')\right)\end{aligned} \tag{14}$$

Some special cases can be obtained as the change of the parameters as follows.

If $r=0$, then the 2-tuple linguistic generalized weighted Bonferroni mean (2TLGWBM) operator reduces to the 2-tuple linguistic weighted Bonferroni mean (2TLGWBM) operator.

$$\begin{aligned}&\mathrm{2TLGWBM}_{w}^{p,q,0}\left\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right\}\\&=\mathrm{2TLWBM}_{w}^{p,q,0}\left\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right\}\\&=\Delta\left(\sum_{i,j=1}^{n}\left(w_iw_j\Delta^{-1}(r_i,a_i)\right)^p\left(\Delta^{-1}(r_j,a_j)\right)^q\right)^{\frac{1}{p+q}}\end{aligned} \tag{15}$$

If $r=0,q=0$, the 2-tuple linguistic generalized weighted Bonferroni mean (2TLGWBM) operator reduces to the following:

$$\begin{aligned} & 2TLGWBM_{w}^{p,0,0}\left\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right\} \\ & =\Delta\left(\sum_{i,j,k=1}^{n}\left(w_i w_j w_k \Delta^{-1}(r_i,a_i)\right)^{p}\left(\Delta^{-1}(r_j,a_j)\right)^{0}\left(\Delta^{-1}(r_k,a_k)\right)^{0}\right)^{\frac{1}{p}} \\ & =\Delta\left(\sum_{i=1}^{n}\left(w_i\Delta^{-1}(r_i,a_i)\right)^{p}\right)^{\frac{1}{p}} \end{aligned} \tag{16}$$

Which is the 2-tuple linguistic generalized weighted averaging (2TLGWA) operator. Furthermore, in this case, let us look at the 2TLGWBM operator for some special cases of p.

If $p=1$, the 2-tuple linguistic generalized weighted Bonferroni mean (2TLGWBM) operator reduces to 2-tuple linguistic weighted averaging (2TLWA) operator.

If $p\to 0$, the 2-tuple linguistic generalized weighted Bonferroni mean (2TLGWBM) operator reduces to 2-tuple linguistic weighted geometric (2TLWG) operator.

If $p\to\infty$, the 2-tuple linguistic generalized weighted Bonferroni mean (2TLGWBM) operator reduces to 2-tuple linguistic max operator.

In the following, Zhu et al.[35] explored the geometric Bonferroni mean (GBM) considering both the BM and the geometric mean (GM).

Definition 9[35]. Let $p,q\ge 0$ and $a_i\left(i=1,2,\cdots,n\right)$ be a collection of non-negative real numbers. Then the aggregaton functions:

$$GBM^{p,q}\left(a_1,a_2,\cdots,a_n\right)=\frac{1}{p+q}\left(\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(pa_i+qa_j\right)\right)^{\frac{1}{n(n-1)}} \tag{17}$$

is called the geometric Bonferroni mean (GBM) operator.

Then, we shall give the definition of the generalized geometric Bonferroni mean (GGBM) operator and the generalized weighted geometric Bonferroni mean (GWGBM) operator.

Definition 10. Let $p,q,r\ge 0$ and $a_i\left(i=1,2,\cdots,n\right)$ be a collection of nonnegative numbers. If

$$GGBM^{p,q,r}\left(a_1,a_2,\cdots,a_n\right)=\frac{1}{p+q+r}\prod_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(pa_i+qa_j+ra_k\right)^{\frac{1}{n(n-1)(n-2)}} \tag{18}$$

then $GGBM^{p,q,r}$ is called the generalized geometric Bonferroni mean (GGBM) operator.

Definition 11. Let $p,q,r\ge 0$ and $a_i\left(i=1,2,\cdots,n\right)$ be a collection of nonnegative numbers with the weight vector $w=\left(w_1,w_2,\cdots,w_n\right)^{T}$ and $w_j>0$, $\sum_{j=1}^{n}w_j=1$. If

$$GWGBM_{w}^{p,q,r}\left(a_1,a_2,\cdots,a_n\right)=\frac{1}{p+q+r}\prod_{i,j,k=1}^{n}\left(pa_i+qa_j+ra_k\right)^{w_i w_j w_k} \tag{19}$$

then $GWGBM_w^{p,q,r}$ is called the generalized weighted Bonferroni mean (GWBM) operator.

In the following, we shall develop 2-tuple linguistic generalized geometric Bonferroni mean (2TLGWGBM) operator and the 2-tuple linguistic generalized weighted geometric Bonferroni mean (2TLGWGBM) operator.

Definition 12. Let $p, q, r \geq 0$ and $\{(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\}$ be a set of 2-tuple linguistic variables, If

$$2TLGGBM^{p,q,r}\{(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\} = \Delta\left(\frac{1}{p+q+r}\prod_{i,j,k=1}^{n}\left(p\Delta^{-1}(r_i, a_i) + q\Delta^{-1}(r_j, a_j) + r\Delta^{-1}(r_k, a_k)\right)^{\frac{1}{n(n-1)(n-2)}}\right) \tag{20}$$

then $2TLGGBM^{p,q,r}$ is called the 2-tuple linguistic generalized geometric Bonferroni mean (2TLGGBM) operator.

Definition 13. Let $\{(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\}$ be a set of 2-tuple linguistic variables, and let $p, q, r \geq 0$. $w = (w_1, w_2, \cdots, w_n)^T$ is the weight vector of $\{(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\}(i = 1, 2, \cdots, n)$, where w_i indicates the importance degree of (r_i, a_i), satisfying $w_i > 0 (i = 1, 2, \cdots, n)$, and $\sum_{i=1}^{n} w_i = 1$. If

$$\text{2TLGWGBM}_w^{p,q,r}\{(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\} = \Delta\left(\frac{1}{p+q+r}\prod_{i,j,k=1}^{n}\left(p\Delta^{-1}(r_i, a_i) + q\Delta^{-1}(r_j, a_j) + r\Delta^{-1}(r_k, a_k)\right)^{w_i w_j w_k}\right) \tag{21}$$

then $\text{2TLGWGBM}_w^{p,q,r}$ is called the 2-tuple linguistic generalized weighted geometric Bonferroni mean (2TLGWGBM) operator.

It can be easily proved that the 2TLGWGBM operator has the following properties.

Theorem 5. (Idempotency)

Let $x = \{(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\}$ be a set of 2-tuple linguistic variables. If all $(r_j, a_j)(j = 1, 2, \cdots, n)$ are equal, i.e. $(r_j, a_j) = (r, a)$ for all j, then

$$2TLG\text{WG}BM_w^{p,q,r}\{(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\} = (r, a) \tag{22}$$

Theorem 6. (Boundedness)

Let $x = \{(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\}$ be a set of 2-tuple linguistic variables, and let

$$(r^-, a^-) = \min_j (r_j, a_j),\ (r^+, a^+) = \max_j (r_j, a_j)$$

Then

$$(r^-, a^-) \leq 2TLG\text{WG}BM_w^{p,q,r}\{(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\} \leq (r^+, a^+) \tag{23}$$

Theorem 7. (Monotonicity)

Let $x=\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ and $x'=((r_1',a_1'),(r_2',a_2'),\ldots,(r_n',a_n'))$ be two set of 2-tuple linguistic variables, if $(r_j,a_j)\le(r_j',a_j')$, for all j, then

$$\begin{aligned}&2TLGWGBM_w^{p,q,r}\left((r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right)\le\\&2TLGWGBM_w^{p,q,r}\left((r_1',a_1'),(r_2',a_2'),\ldots,(r_n',a_n')\right)\end{aligned}\tag{24}$$

Theorem 8. (Commutativity)

Let $x=\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ and $x'=((r_1',a_1'),(r_2',a_2'),\ldots,(r_n',a_n'))$ be two set of 2-tuple, where $((r_1',a_1'),(r_2',a_2'),\ldots,(r_n',a_n'))$ is any permutation of $\{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$, then

$$\begin{aligned}&2TLGWGBM_w^{p,q,r}\left((r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\right)=\\&2TLGWGBM_w^{p,q,r}\left((r_1',a_1'),(r_2',a_2'),\ldots,(r_n',a_n')\right)\end{aligned}\tag{10}$$

4. An approach to multiple attribute decision making with linguistic information

In this section, we shall utilize the developed operators to multiple attribute decision making. For a multiple attribute decision making problems with linguistic information, let $A=\{A_1,A_2,\cdots,A_m\}$ be a discrete set of alternatives, $G=\{G_1,G_2,\cdots,G_n\}$ be the set of attributes, whose weight vector is $\omega=(\omega_1,\omega_2,\cdots,\omega_n)$,with $\omega_j\ge 0$, $j=1,2,\cdots,n$, $\sum_{j=1}^{n}\omega_j=1$. Suppose that $R=(r_{ij})_{m\times n}$ is the multiple attribute decision making matrix, where $r_{ij}\in S$ is an attribute values, which take the form of linguistic variable, given by the decision maker for the alternative $A_i\in A$ with respect to the attribute $G_j\in G$.

In what follows, we shall apply the 2TLGWBM operator or 2TLGWGBM to solve the MADM problems with linguistic variables.

Step 1. Transforming linguistic decision matrix $R=(r_{ij})_{m\times n}$ into 2-tuple linguistic decision matrix $\tilde{R}=(r_{ij},0)_{m\times n}$.

Step 2. We utilize the decision information given in matrix $\tilde{R}$, and the 2TLGWBM operator

$$\begin{aligned}\tilde{r}_i&=2TLGWBM_w^{p,q,r}\{(r_{i1},0),(r_{i2},0),\ldots,(r_{in},0)\}\\&=\Delta\left(\sum_{m,s,t=1}^{n}\left(w_m w_s w_t\Delta^{-1}(r_{im},0)\right)^p\left(\Delta^{-1}(r_{is},0)\right)^q\left(\Delta^{-1}(r_{it},0)\right)^r\right)^{\frac{1}{p+q+r}}\end{aligned}$$

Or the 2TLGWGBM operator

$$\begin{aligned}\tilde{r}_i&=2TLGWGBM_w^{p,q,r}\{(r_{i1},0),(r_{i2},0),\ldots,(r_{in},0)\}\\&=\Delta\left(\frac{1}{p+q+r}\prod_{m,s,t=1}^{n}\left(p\Delta^{-1}(r_{im},0)+q\Delta^{-1}(r_{is},0)+r\Delta^{-1}(r_{it},0)\right)^{w_m w_s w_t}\right)\end{aligned}$$

to derive the overall preference values $\tilde{r}_i\left(i=1,2,\cdots,m\right)$ of the alternative A_i.

Step 3. Rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ and select the best one(s) in accordance with $\tilde{r}_i\left(i=1,2,\cdots,m\right)$. If any alternative has the highest $\tilde{r}_i$ value, then, it is the most important alternative.

Step 4. End.

5. Conclusion

In this paper, we investigate the multiple attribute decision making problems with 2-tuple linguistic information. Motivated by the ideal of generalized weighted Bonferroni mean and generalized weighted geometric Bonferroni mean, we develop the 2-tuple linguistic generalized Bonferroni mean (2TLGBM) operator for aggregating the 2-tuple linguistic information and 2-tuple linguistic generalized geometric Bonferroni mean (2TLGGBM) operator. For the situations where the input arguments have different importance, we then define the 2-tuple linguistic generalized weighted Bonferroni mean (2TLGWBM) operator and 2-tuple linguistic generalized weighted geometric Bonferroni mean (2TLGWGBM) operator, based on which we develop the procedure for multiple attribute decision making under the 2-tuple linguistic environments. In the future, we shall continue working in the extension and application of the developed operators to other domains.

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