Two kinds of queuing management policy for improving customer satisfaction with waiting under the $M/M/C(C \geq 2)$ multi-server queuing system

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Abstract

the degree of customers' satisfaction depends largely on the length of the waiting time they perceive. The paper puts forward two strategies to shorter the waiting time they perceive: one is providing those in front of the queue with extra services, the other is offering extra services to those standing in the end of the queue. The waiting factor supposed in the paper helps to figure out the waiting time when the nth customer receive services in the queuing system $M/M/C(C \geq 2)$ .Firstly, the probability density function of customers' waiting time in the system is worked out with total probability formula, and then two kinds of optimization models of queuing managing strategies are worked out. As a result, the enterprises offering services such as banks and supermarkets can be provided with ways how to choose the queuing managing strategy scientifically by popularizing the two queuing managing strategies put forward to the queuing system $M/M/C(C \geq 2)$.

Keywords

queuing theory; customer' waiting; satisfaction; Optimization Model.

1. Introduction

Contemporary service industry in China is a basic one, developing modern service industry can be a weighty step for the development mode transformation of commercial trade system and development strategy adjustment. Waiting is common in services field, whereas waiting could have an impact oncustomer's satisfaction. Fan Chunmei [1]pointed out that waiting time could impact negatively customer's satisfaction, the level of satisfaction is spirit to services. Bielen and Demoulin[2] pointed out that customer's waiting satisfaction level is postively related to the satisfaction level of the whole services. Tom and Lucey[3], taking supermarket as an example, demonstrated that customer's waiting time in paying could significantly affect the satisfation level of the whole shopping experience, and the longer the waiting time, the lower the satisfaction level of the whole shopping experience. Moreover, not only is the waiting time associated with customer's waiting satisfaction level, bust also how much efforts customers could percieve from servicing parties to shorten waiting time. Namely enterprises should be commited to "ensuring the highest customer's satisfaction level", instead of simply pursuing " customer's satisfaction"[4]. Accordingly, enhancing customer's witing satisfaction seems to be Increasingly important, especiallly in service field, which have been a significant dynamic of economic growth, the measurement of regional comprehensive competitiveness and a vital symbol of modernization level.

Obviously, customer's satisfaction level in waiting is inversely proportional to the waiting time. However, there are two ways to shorten the waiting tiime for custmers: one is to enhance the efficiency of service; the other is to set priority service[5]. However, cooperation's service efficiency is diffcult to be enhanced effectively in a short time, while priority service could not cut the average waiting time, and it barely reduce the waiting time of priority customer's by increasing other's waiting time, hences it has to be explicit that customers could accept that consequence[6]. This way might work whe it comes to airline and railway services because paying more and enjoy priority service can be acceptable for customers, while in places, such as supermarket and shopping mall where customers can not be be distinguished, priority is diffcult to set, and it might bring about equality issue, which would seriously affcet customer's satisfaction and loyalty.

Psychology study found that customer's satisfaction level is determined by perceived waiting time rather than actual waiting time. Therefore the servicing parties should commit to decrease perceived waiting time by customers. .Norman [7] put forward "improving waiting time", filling waiting time and so on 8 principles with the purpose of reducing perceived waiting time. Zhou Wenhui[5] put forth two ways of improving customer's waiting satisfaction level regarding operability in the base of the above queuing management theory: the first one is additional service for the queue head, that is offering additional services to customers in the front of queue to get perceived waiting time decreased; the second is additional services for the queue tail, that is offering additional services to customers in the end of queue to decrease perceived waiting time. Furthermore, they eatsblished a complete mathematical mode beased on $M/M/1$ queuing system, with which the servicing parties choose properly services manners in accordance with the planned cost in order to achieve the highest satisfaction level in waiting, providing reliable and scientific basis for the servicing parties. Nevertheless, in real life situation, most of the service industries is multi-server, which means limitation in the above model.

This article from $M/M/2$ queuing systems presented waiting factor t_r consequently the customer waiting time probability density function in the system, and obtained optimization models of two-queue management strategy under this system, also consequently further expand to $M/M/C(C \ge 2)$ System,and studied two types of management strategies in a multi-server queueing system, then making the research have more widely actual significance.

2. Model description and proposed waiting factor

In systems $M/M/2$,as is show in figure 1 , and customers arrive to obey Poisson parameter λ .

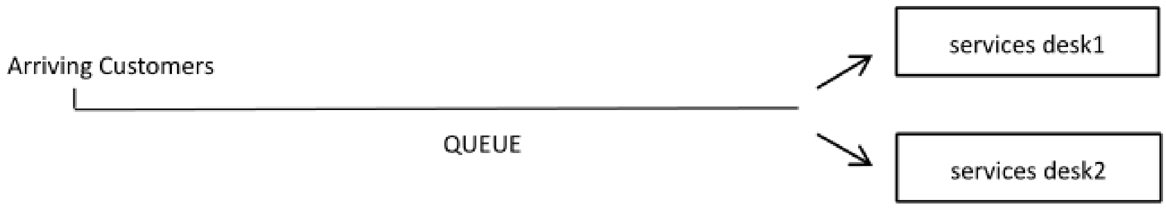

Figure 1

Reception Service subject to parameters μ exponential distribution. Set t_i as the wait service time of the i -th customer.However, the first two customers don't have to wait, so $t_1=t_2=0$, When the system the number of customers is more than 3,Then the rest customers will have to wait, And:

$$t_3=\min\{t_1',t_2'\} \tag{1}$$

$t_i', i=1,2,3,\ldots$ is the service time of the i -th customer,so the probability density function of t_1', t_2' is:

$$f_{t_1'}(t)=f_{t_2'}(t)=\mu e^{-\mu t}\ (t>0)$$

By (1) formula:

$$\begin{aligned} F_{t_3}(t) = p(t_3 \le t) = p\left(\min\left(t_1^{'},t_2^{'}\right) \le t\right) &= 1 - p\left(\min\left(t_1^{'},t_2^{'}\right) \ge t\right) \\ &= 1 - p\left(t_1^{'} \ge t\right) p\left(t_2^{'} \ge t\right) \\ &= 1 - \left(1 - p\left(t_1^{'} \le t\right)\right)\left(1 - p\left(t_2^{'} \le t\right)\right) \\ &= 1 - e^{-\mu t} e^{-\mu t} \\ &= 1 - e^{-2\mu t} \end{aligned}$$

$F_{t_3}(t)$ is the probability distribution function of the t_3,so the probability density function of the t_3 is:

$$f_{t_3}(t) = 2\mu e^{-2\mu t} \qquad (2)$$

At this time,wait time of the t_4 is:

$$t_4 = t_3 + \min\{\max\left(t_1^{'},t_2^{'}\right) - \min\left(t_1^{'},t_2^{'}\right), t_3^{'}\}$$

The above formula shows that $\max\left(t_1^{'},t_2^{'}\right) - \min\left(t_1^{'},t_2^{'}\right)$ mean the waiting time of t_4 after t_3 enter the service desk.Set:

$$t_r = \min\{\max\left(t_1^{'},t_2^{'}\right) - \min\left(t_1^{'},t_2^{'}\right), t_3^{'}\}$$

It is not difficult to get:

$$f(t_r) = f\left(\min\{\max\left(t_1^{'},t_2^{'}\right) - \min\left(t_1^{'},t_2^{'}\right), t_3^{'}\}\right) = 2\mu e^{-2\mu t}$$

In fact, when customer entered service, as memoryless exponential distribution, we found t_r is a factor, it represents wait time when it is in a wait queue first to enter service desk, so we define wait-factor. Can be obtain:

$$\begin{aligned} t_3 &= t_r \\ t_4 &= t_3 + t_r \\ t_5 &= t_4 + t_r \\ &\vdots \\ t_n &= t_{n-1} + t_r \end{aligned}$$

By mathematical induction:

$$t_n = (n-2)t_r, n \ge 3$$

Apparently $t_1 = t_2 = 0$, by above foemula can get $t_n, n \geq 2$ obey the Erlang distribution[9] with the parameter $n-2$.
The probability density function is,:

$$f_n(t) = \frac{\mu(\mu t)^{n-3}}{(n-3)!} e^{-\mu t} \tag{3}$$

For $n \geq 3$, distribution function of Customers wait time in the system is:

$$\begin{aligned} F(t) = p(T \leq t) &= \sum_{n=3}^{\infty} p_n p(t \leq t/n) = \sum_{n=3}^{\infty} p_n \int_0^t f_n(t) dt \\ &= \sum_{n=3}^{\infty} p_n \int_0^t \frac{\mu^{n-2} t^{n-3}}{(n-3)!} e^{-\mu t} dt \end{aligned} \tag{4}$$

By document 9 :

$$\begin{cases} p_0 = \left[\sum_{k=0}^{1} \frac{1}{k!} \left(\frac{\lambda}{\mu} \right)^k + \frac{1}{2} . \frac{1}{1-\rho} . \left(\frac{\lambda}{\mu} \right)^2 \right]^{-1} \\ p_n = \begin{cases} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n p_0 & (n \leq 2) \\ \frac{1}{2^{n-1}} \left(\frac{\lambda}{\mu} \right)^n p_0 & (n > 2) \end{cases} \end{cases}$$

Note:

$$w(t) = \sum_{n=3}^{\infty} p_n \frac{\mu^{n-2} t^{n-3}}{(n-3)!} \tag{5}$$

The probability density function of customers wait time in the system is obtained by the formula $(3),(4),(5)$.

$$f(t) = \begin{cases} p_0 + p_1 & t = 0 \\ w(t) e^{-\mu t} & t > 0 \end{cases} \tag{6}$$

By kallo N,koltaiT[10] Human studies, Waiting for the satisfaction of customers is the exponential function of the waiting time:

$$s = e^{-\beta t} \tag{7}$$

$\beta(\beta>0)$ is the sensitive degree of customer satisfaction to the waiting time, then the mathematical expectation of Customer satisfaction is:

$$E(s)=\int_0^{\infty} e^{-\beta t} f(t)dt = p_0 + p_1 + \int_0^{\infty} e^{-(\beta+\mu)t} w(t)dt \qquad (8)$$

In the enterprise perspective, On the one hand, the investment of additional service will improve customers waiting for satisfaction, on the one hand, to provide additional services can generate profits cannot directly bring cost directly, so companies will control the budget, this article assumes that the enterprise provides c_0 as the total cost of the additional services. Below will built two types optimization model of queue management strategy based on a given service cost

3. The extra Service Optimization Model of team head in the queue system $M/M/2$

H ,the extra Service strategy of team head, which provides additional service to these people in the top N of queue . However, Others outside this range should wait until they become top N . Specifically, as shown in Figure2

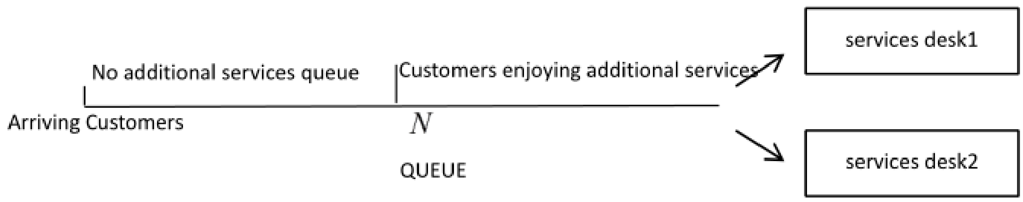

Figure.2

According to team head service strategy, the waiting time is divided in two types , existing additional service time or not , referred to as t_e^H, t_n^H Respectively.

waiting customer satisfaction in the head team additional services

Customers in the top N, $t_n^H=0, t_e^H \neq 0$,while others $t_n^H \neq 0$ (t_e^H =CON). Note $f_n^H(t), f_e^H(t)$ are Probability density functions of t_n^H, t_e^H . The following is obtained by the formula $(3),(4),(5),(6)$.

$$f_n^H(t)=\begin{cases} \sum_{n=0}^{N} p_n \\ \sum_{n=N+1}^{\infty} p_n f_{n-N}(t) = \sum_{n=N+1}^{\infty} p_n \dfrac{\mu^{n-N-2} t^{n-N-3}}{(n-N-3)!} e^{-\mu t} \end{cases} \qquad (9)$$

Similarly formula can be obtained:

$$f_e^H(t)=\begin{cases} p_0+p_1 \\ \sum_{n=2}^{N} p_n f_n(t) + \sum_{n=N+1}^{\infty} p_n f_N(t) = \sum_{n=2}^{N} p_n \dfrac{\mu^{n-2} t^{n-3}}{(n-3)!} e^{-\mu t} + \\ \qquad \sum_{n=N+1}^{\infty} p_n \dfrac{\mu^{N-2} t^{N-3}}{(N-3)!} e^{-\mu t} \end{cases} \qquad (10)$$

Formula(9) and (10), mean that a customer located in the top N, whose no extra service waiting time is zero, and the extra waiting time is determined by the position. If the customer is outside the range that they have to wait for a set of additional services ,waiting time is determined by N . and its extra services without waiting time decided by the location. From this we can understand the ultimate goal is to achievethe maximum satisfaction, while the value of N is certain and additional services costs c_0 is fixed.

Similarly to Formula(8), waiting satisfaction degree in the two stages are:

$$E\left(S_n{}^H\right)=\sum_{n=0}^{N} p_n+\int_0^{\infty} e^{-\beta_n t} \sum_{n=N+1}^{\infty} p_n f_{n-N}(t)\,dt \tag{11}$$

$$E\left(S_e{}^H\right)=\sum_{n=0}^{1} p_n+\int_0^{\infty} e^{-\beta_n t}\left(\sum_{n=2}^{N} p_n f_n(t)+\sum_{n=N+1}^{\infty} p_n f_N(t)\right)dt \tag{12}$$

waiting satisfaction degree of customers under the strategy of team head extra satisfaction is as follows :

$$E\left(S^H\right)=\min\{E\left(S_n{}^H\right),E\left(S_e{}^H\right)\} \tag{13}$$

The Service cost in the head team additional services

c_e means the cost of additional services per unit time of everyone, Mathematical expectation of the $T_e{}^H$ in the head team additional services is:

$$E\left(T_e{}^H\right)=\sum_{n=1}^{N} p_n \frac{n}{2\mu}+\sum_{n=N+1}^{\infty} p_n \frac{N}{2\mu}$$

Obtained by the above formula:

$$c_H=\lambda c_e E\left(T_e{}^H\right)=\lambda c_e\left(\sum_{n=1}^{N} p_n \frac{n}{2\mu}+\sum_{n=N+1}^{\infty} p_n \frac{N}{2\mu}\right) \tag{14}$$

c_H , the average total cost of providing additional services.

The foundation of the additional service optimization model

Based on equation (13),(14),the additional service optimization model in $M/M/2$ is as follows:

$$\begin{aligned} &\max_{N} \quad E\left(S_H\right) \\ &s.t \quad c_H \le c_0 \end{aligned} \tag{p1}$$

Under the strategy of queue head additional service, N is determined by restriction of the service cost. And eventually the maximum satisfaction is acquired ,which is the most straightforward goal.

4. The additional service optimization model for queue tail under $M/M/2$ queuing system

The additional service strategy for queue tail, that is to probide additional service to N th customer and its subsequent ones, as figure 3 shows:

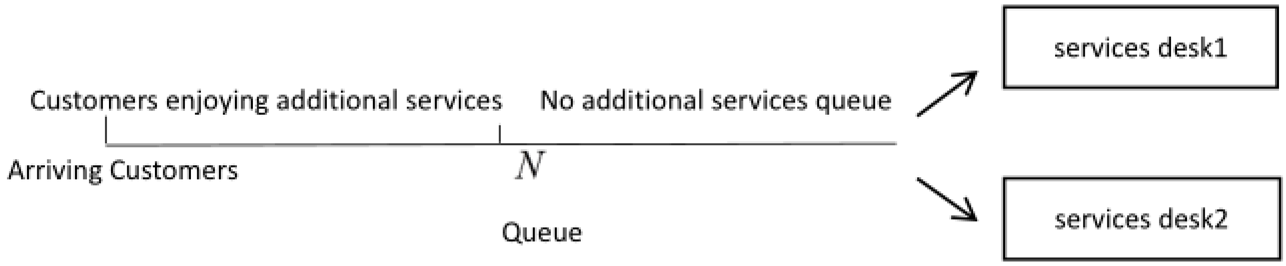

Figure 3

For queue head's additional service strategy,assuming t_e^T, t_n^T tand for additional time for customers or no additional time for them, if one lies in behind the Nth customer, then $t_e^T \neq 0$,while t_n^T is constant value;if one lies in with in the N th,then $t_e^T = 0, t_n^T \neq 0$;making $f_e^T(t), f_n^T(t)$ as probability density function for t_e^T, t_n^T separately,likewisely by using formula $(9),(10)$,it works out as follow;

$$f_n^T(t) = \begin{cases} p_0 + p_1 & t = 0 \\ \sum_{n=2}^{N} p_n f_n(t) + \sum_{n=N+1}^{\infty} p_n f_N(t) & t > 0 \end{cases} \tag{15}$$

$$f_e^T(t) = \begin{cases} \sum_{n=0}^{N} p_n & t = 0 \\ \sum_{n=N+1}^{\infty} p_n f_{n-N}(t) & t > 0 \end{cases} \tag{16}$$

waiting satisfaction level under the queue tail's additional services

Marking S_n^T, S_e^T as customer's waiting satisfaction level under additional services or no, Similar to $(11),(12)$ the expectation works out as follows:

$$\begin{aligned} E\left(S_n^T\right) &= p_0 + p_1 + \int_0^{\infty} e^{-\beta_n t} f_n^T(t)\,dt \\ &= p_0 + p_1 + \int_0^{\infty} e^{-\beta_n t} \left(\sum_{n=2}^{N} p_n f_n(t) + \sum_{n=N+1}^{\infty} p_n f_N(t) \right) dt \end{aligned} \tag{17}$$

$$E\left(S_n^T\right) = \sum_{n=0}^{N} p_n + \int_0^{\infty} e^{-\beta_e t} f_e^T(t)\,dt = \sum_{n=0}^{N} p_n + \int_0^{\infty} e^{-\beta_e t} \sum_{n=N+1}^{\infty} p_n f_{n-N}(t)\,dt \tag{18}$$

Customer's expected waiting satisfaction level under queue tail's additioanl service strategy:

$$E\left(S_T\right)=\min\{E\left(S_n^{\ T}\right),E\left(S_n^{\ T}\right)\} \tag{19}$$

the cost from queue tail's additional service

Marking c_T as the average total cost in per unit of time resulting from entriprise providing additional services under this strategy, similarly the folowing can be derived from formula (14):

$$\begin{aligned} c_T &= \lambda c_e E\left(T_e^{\ T}\right) \\ E\left(T_e^{\ T}\right) &= \sum_{n=N+1}^{\infty} p_n \frac{n-N}{2\mu} \end{aligned} \tag{20}$$

4.3 the establishment of queue tail's additional service optimization model

By using foumula $(19),(20)$, queue tail's additional services optimization model is as follows:

$$\begin{aligned} &\max_N \quad E\left(S_T\right) \\ &s.t \quad\ c_T \le c_0 \end{aligned} \qquad (p2)$$

The meaning of this model is to ascertain parameter N on the premise of additional service cost limitation under queue tail's additional services strategy, with intention of maxmizing customer's waiting satisfaction level.

5. Two additional services model of under the system $M/M/C\left(C\ge 2\right)$

mode description under the system of $M/M/C\left(C\ge 2\right)$

The waiting factor was put forth in the above discussion, two kinds of queuing management strategy regarding the optimization model of enhancing waiting satisfaction level,which is $(p1),(p2)$, are resulted based on that, providing basis on choosing additional service for service enterprises. Obviously this model could be extended to the systm of $M/M/C\left(C\ge 2\right)$,which endows this model with generalisation.

Provided that there are $c\left(c\ge 2\right)$ service desks in systems, as fig 4 innustrates:

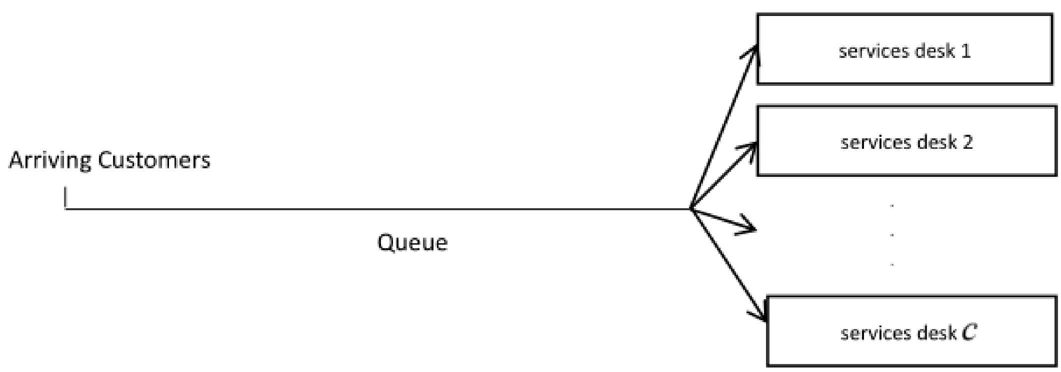

Figure 2

Assuming $t_{c,i}$ is the i th customer's waiting time under the system of $M/M/C(C \geq 2)$, and then the first c th customers have no need to wait meanwhile $t_{c,i} = 0, i \leq c$, while the rear need to wait meanwhile $t_{c,i} \neq 0, i > c$, obviously:

$$t_{c,c+1} = \min\left(t'_{c,1}, t'_{c,2}, ..., t'_{c,c}\right) \tag{21}$$

$t'_{c,i}$ is the Ith customer's served time under the system of $M/M/C(C \geq 2)$.

$$\begin{aligned}
F_{t_{c,c+1}}(t) &= p\left(t_{c,c+1} \leq t\right) = p\left(\min\left(t'_{c,1}, t'_{c,2}, ..., t'_{c,c}\right) \leq t\right) = 1 - p\left(\min\left(t'_{c,1}, t'_{c,2}, ..., t'_{c,c}\right) \geq t\right) \\
&= 1 - p\left(t'_{c,1} \geq t\right) ... p\left(t'_{c,c} \geq t\right) \\
&= 1 - \left(1 - p\left(t'_{c,1} \leq t\right)\right) ... \left(1 - p\left(t'_{c,c} \leq t\right)\right) \\
&= 1 - \overbrace{e^{-\mu t} ....e^{-\mu t}}^{c} \\
&= 1 - e^{-c\mu t}
\end{aligned}$$

$F_{t_{c,c+1}}(t)$ is a probability distribution function of $t_{c,c+1}$, the above formula can be justified because $t'_{c,1}, t'_{c,2}, ..., t'_{c,c}$ are independent respectively, thus the probability density function of $t_{c,c+1}$ is as follows:

$$f_{t_{c,c+1}}(t) = c\mu e^{-c\mu t} \tag{22}$$

By this time, the waiting time for $t_{c,c+2}$,

$$t_{c,c+2} = t_{c,c+1} + \min\{\max\left(t'_{c,1}, t'_{c,2}, ..., t'_{c,c}\right) - \min\left(t'_{c,1}, t'_{c,2}, ..., t'_{c,c}\right), t'_{c,c+1}\}$$

And $\min\{\max\left(t'_{c,1}, t'_{c,2}, ..., t'_{c,c}\right) - \min\left(t'_{c,1}, t'_{c,2}, ..., t'_{c,c}\right), t'_{c,c+1}\}$ is $t_{c,c+1}$ waiting time after $t_{c,c+2}$ entering into service desk, similar to discussion 2.1, define $t_{c,r}$ as the waiting factor:

$$t_{c,r} = \min\{\max\left(t'_{c,1}, t'_{c,2}, ..., t'_{c,c}\right) - \min\left(t'_{c,1}, t'_{c,2}, ..., t'_{c,c}\right), t'_{c,c+1}\}$$

And thus this can be derived easily:

$$f\left(t_{c,r}\right) = f\left(\min\{\max\left(t'_{c,1}, t'_{c,2}, ..., t'_{c,c}\right) - \min\left(t'_{c,1}, t'_{c,2}, ..., t'_{c,c}\right), t'_{c,c+1}\}\right) = c\mu e^{-c\mu t}$$

Similar to the disscusion in the first part of this paper, Owing to the memoryless of the exponential distribution , that means customers have received service for t time length, the probability distribution in the service time span is till the ones of parameter μ and have nothing to do with the served time span. Thus when customers queue up at first place, the waiting time until receiving service is a constant value $t_{c,r}$, thus the followings can be derived:

$$\begin{aligned} & t_{c,c+1} = t_{c,r} \\ & t_{c,c+2} = t_{c,c+1} + t_{c,r} \\ & \vdots \qquad \vdots \\ & t_{c,c+n} = t_{c,c+n-1} + t_{c,r} \end{aligned}$$

By mathematical induction

$$t_{c,n} = (n-c)t_{c,r}, n \geq c$$

Obviously, in the case of $n < c$, customers have no need to wait, that is $t_{c,1} = t_{c,2} = ... = t_{c,c} = 0$, and then $t_{c,n}, n \geq c$, which comply with $n < c$ jieaierlang distribution, that is :

$$f_{c,n}(t) = \frac{\mu(\mu t)^{n-c-1}}{(n-c-1)!}e^{-\mu t} \tag{23}$$

In the case of $n \geq c$, the distribution function of cutomers' waiting time under the system is

$$\begin{aligned} F_c(t) = p(t \leq t) &= \sum_{n=c+1}^{\infty} p_{c,n} p(t_{c,n} \leq t/n) = \sum_{n=3}^{\infty} p_{c,n} \int_0^t f_{c,n}(t)dt \\ &= \sum_{n=3}^{\infty} p_{c,n} \int_0^t \frac{\mu^{n-c}t^{n-c-1}}{(n-c-1)!}e^{-\mu t}dt \end{aligned} \tag{24}$$

Derived form article 9,

$$\begin{cases} p_0 = \left[\sum_{k=0}^{c-1} \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k + \frac{1}{c}.\frac{1}{1-\rho}.\left(\frac{\lambda}{\mu}\right)^c \right]^{-1} \\ p_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n p_0 & (n \leq c) \\ \frac{1}{c!c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n p_0 & (n > c) \end{cases} \end{cases}$$

Marking

$$w(t) = \sum_{n=c+1}^{\infty} p_{c,n} \frac{\mu^{n-c}t^{n-c-1}}{(n-c-1)!} \tag{25}$$

Probability density function of customer's waiting time under the system can be derived from(23) (24) (25)

$$f_c(t)=\begin{cases} p_0+p_1 & t=0 \\ w(t)e^{-\mu t} & t>0 \end{cases} \tag{26}$$

$F_c(t)$ is a distribution function of customer's waiting time in the system of $M/M/C(C\geq 2)$, $f_c(t)$ stand for the probability density function of customers' waiting time in the system of $M/M/C(C\geq 2)$. For the convinence of understanding and application, this paper will not differentiate particular system marking $F_c(t)$, $f_c(t)$ as $F(t)$, $f(t)$,both the two signify the probability distribution function and density function of customers' waiting time in that system.

the optimization model extenstion of two additional service strategies to ehancing satisfaction level

1) The optimization model of queue head additional service

Adopting queue head additional service strategy in the system of $M/M/C(C\geq 2)$ to enhance satisfaction level is similar to formula $(9),(10)$, assumed that $t_{c,n}{}^{H}$, $t_{c,e}{}^{H}$ is the waiting time without receiving addition services and that time with receiving additional services respectively, and $f_n{}^{H}(t), f_e{}^{H}(t)$ is the probability density function of $t_n{}^{H}$, $t_e{}^{H}$ respectively, the followings could be derived from formula $(3),(4),(5),(6)$

$$f_{c,n}{}^{H}(t)=\begin{cases} \sum_{n=0}^{N} p_{c,n} \\ \sum_{n=N+1}^{\infty} p_{c,n} f_{n-N}(t) \end{cases} \tag{27}$$

$$f_{c,e}{}^{H}(t)=\begin{cases} \sum_{n=0}^{c-1} p_n \\ \sum_{n=c}^{N} p_{c,n} f_{c,n}(t)+\sum_{n=N+1}^{\infty} p_{c,n} f_{c,N}(t) \end{cases} \tag{28}$$

Of which, $f_{c,n}{}^{H}(t), f_{c,e}{}^{H}(t)$ are the probability density function of $t_{c,n}{}^{H}, t_{c,e}{}^{H}$

Similar to $(11),(12),(13)$:

$$E\left(S_{c,n}{}^{H}\right)=\sum_{n=0}^{N} p_{c,n}+\int_0^{\infty} e^{-\beta_n t} \sum_{n=N+1}^{\infty} p_{c,n} f_{c,n-N}(t)\,dt \tag{29}$$

$$E\left(S_{c,e}{}^{H}\right)=\sum_{n=0}^{c-1} p_{c,n}+\int_0^{\infty} e^{-\beta_n t}\left(\sum_{n=c}^{N} p_{c,n} f_{c,n}(t)+\sum_{n=N+1}^{\infty} p_{c,n} f_{c,N}(t)\right)dt \tag{30}$$

Of which, $E\left(S_{c,n}{}^{H}\right)$, $E\left(S_{c,e}{}^{H}\right)$ 为 stand for customer's expectation value concerning waiting satisfaction level in the case of additional services and no additional services under that system, and then customer's satisfaction level under the strategy of queue head additional service

$$E\left(S_c{}^H\right)=\min\{E\left(S_{c,n}{}^H\right),E\left(S_{c,e}{}^H\right)\} \tag{31}$$

Also this can be derived easily:

$$c_{c,H}=\lambda c_{c,e}E\left(T_{c,e}{}^H\right)=\lambda c_{c,e}\left(\sum_{n=1}^{N}p_{c,n}\frac{n}{c\mu}+\sum_{n=N+1}^{\infty}p_{c,n}\frac{N}{c\mu}\right) \tag{32}$$

$c_{c,H}$ is the average total cost caused by additional services, $c_{c,e}$ is per unit time and per labor force cost from additional services under that system, and then the optimization model of queue head additional service can be derived from formula $(31),(32)$:

$$\begin{aligned}&\max \quad E\left(S_c{}^H\right)\\&s.t \quad c_{c,H}\le c_o\end{aligned} \qquad (p3)$$

2) the optimization of queue tail additional service

similar to the queue head additional service strategy, assumed that $t_{c,e}{}^T,t_{c,n}{}^T$ signify the waiting without receiving additional services and that time with receiving additional services, if a customer lie behind N th, $t_{c,e}{}^T\neq 0$ and $t_{c,n}{}^T$ is a constant value; if a customer lie within Nth, $t_{c,e}{}^T=0,t_{c,n}{}^T\neq 0$. Marking $f_{c,e}{}^T(t),f_{c,n}{}^T(t)$ as thye probability density function of $t_{c,e}{}^T,t_{c,n}{}^T$, likewaise the followings can be derived:

$$f_{c,n}{}^T(t)=\begin{cases}\sum_{n=0}^{c-1}p_n & t=0\\ \sum_{n=c}^{N}p_{c,n}f_{c,n}(t)+\sum_{n=N+1}^{\infty}p_{c,n}f_{c,N}(t) & t>0\end{cases} \tag{33}$$

$$f_{c,e}{}^T(t)=\begin{cases}\sum_{n=0}^{N}p_{c,n} & t=0\\ \sum_{n=N+1}^{\infty}p_{c,n}f_{c,n-N}(t) & t>0\end{cases} \tag{34}$$

The satisfaction level under queue tailo additional services is:

Marking $S_{c,n}{}^T,S_{c,e}{}^T$ as the customer's waiting satisfaction level in the case of additional services and no additional services respectively, similar to (17)(18), the expectation value can be derived as follows:

$$E\left(S_{c,n}{}^T\right)=\sum_{n=0}^{c-1}p_n+\int_0^{\infty}e^{-\beta_n t}f_{c,n}{}^T(t)\,dt \tag{35}$$

$$E\left(S_{c,n}{}^{T}\right)=\sum_{n=0}^{N}p_{c,n}+\int_{0}^{\infty}e^{-\beta_{e}t}f_{c,e}{}^{T}\left(t\right)dt \tag{36}$$

Customers' expecting waiting satisfaction level under the strategy of queue tail additional services is :

$$E\left(S_{c,T}\right)=\min\{E\left(S_{c,n}{}^{T}\right),E\left(S_{c,n}{}^{T}\right)\} \tag{37}$$

Marking $c_{c,T}$ is the average total cost by per nuit time from offering additional services under this stratey, the followings can be derived from foumula (20) likewisely:

$$\begin{aligned} c_{c,T} &= \lambda c_{c,e}E\left(T_{c,e}{}^{T}\right) \\ E\left(T_{c,e}{}^{T}\right) &= \sum_{n=N+1}^{\infty}p_{c,n}\frac{n-N}{c\mu} \end{aligned} \tag{38}$$

By formula (37), (38), the most optimized model of waiting satisfaction level is:

$$\begin{aligned} &\max_{N} \quad E\left(S_{c,T}\right) \\ &s.t \quad c_{c,T}\le c_{0} \end{aligned} \tag{p4}$$

6. Conclusion

On the basis of clarifing detailedly $M/M/2$ system, this paper put forth the waiting factor tr, establishs the optimization model of two queuing management strategy under that system, and extends this model into the system of $M/M/C\left(C\ge 2\right)$ endowing this model with universal guidance signifance. In practice, most service enterprises are equipped with multiple service desk, this model, therefore, would be a scientific basis for enterprises, like bank, supermarket, on how too provide additional service to maximiza customer's satisfaction level

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