# Data Processing of Deformation Monitoring of Tall Buildings Based on Gray Model

Weidong Wu, Wenyan Pang

School of Civil Engineering Architecture, Southwest Petroleum University, Chengdu Sichuan,

610500, P. R. China

# Abstract

The main purpose of deformation monitoring is to analyze the information of the deformation body, and make accurate prediction of its future trends, so as to provide scientific basis and accurate guidance for prevention and control of geological disasters in engineering design and building, etc.,and also play an important role on the safe operation of existing buildings. This paper analyzes the high-rise buildings' deformation monitoring data,and sets up the traditional gray model based on gray theory on the , which is also called GM (1,1). Through early monitoring data we can predict post-settlement value, and analyze of the process error value.

# **Keywords**

# deformation monitoring gray theory GM (1,1) prediction settlement.

# **1.** Introduction

In the progress of human society and improved construction technology simultaneously, to solve the problem of land resource use, more and more high-rise and super high-rise buildings appear<sup>[2]</sup>. To ensure the normal life and security of buildings' structures, the necessity and importance of building deformation monitoring becomes more obvious. The deformation monitoring <sup>[1]</sup>, as a measure of a specialized subject areas, mainly uses a variety of instruments and tools for sustained observations of the deformable body to obtain data on the dynamic deformation of the deformable body change's mathematical prediction model, which can predict trends and size of the building's deformation and provide data and materials for construction's feasibility assessment, construction and post-operation. In China the research started from the early 1960s and in the 1980s with the development of modern science and technology, deformation prediction was promoted in our country, some new ideas, new theories, new technologies and methods have emerged .

At present, domestic and international scholars attach great importance to data processing and forecasting of sedimentation and deformation of based on the measured data.,andfocuse on the establishment of data processing and forecasting models using various forecasting methods. The representative model <sup>[4]</sup> includes regression model, time series analysis and forecasting models, Kalman filter model, gray prediction model, Markov prediction model, neural network forecasting model, wavelet analysis and forecasting models.

# 2. The gray system theory and establishment of model

Grey system theory first proposed in 1982 by the famous Chinese scholar Professor Deng Julong<sup>[3]</sup>. It is no special requirements and limitations of experimental observations and distribution of data. It is a new theories, which is very simple and easy to learn and easy to use .Gray theory studied a substantial lack of information or disordered uncertainty system. Once the gray system theory, produced, there will be great concern to the majority of domestic and foreign academic and practical workers.

### 2.1 The content of Grey System

Gray system mainly includes<sup>[4]</sup> theoretical system based ongray hazy, analysis system relying on space gray association, the gray process and generate space-based and content methodology, gray model (GM) as the core of the model system, analysis, assessment, modeling, forecasting, decision making, control, optimization technology as the main system, and launched a study gray mathematics.

In fact the nature of gray model <sup>[1]</sup> is the use of fewer or inaccurate columns to represent the behavior of the system features, and after the original series's generation and transformation, established approximate differential equations. General gray model can use h variables to reprent n-order model GM differential equation (n, h).Different n and h gray model have different meaning and purpose, gray model GM (1,1) gray system theory is a typical representative which is the most widely used in practice.

### 2.2 GM (1,1) model

Offers a non-negative discrete sequence

$$\mathbf{x}^{(0)} = \{\mathbf{x}^{(0)}(1), \mathbf{x}^{(0)}(2), \mathbf{x}^{(0)}(3) ..., \mathbf{x}^{(0)}(n)\}$$

Conduct a cumulative generation for  $\mathbf{x}^{(0)}$ 

$$\frac{\mathrm{d}\mathbf{x}^{(1)}}{\mathrm{d}\mathbf{t}} + \mathrm{a}\mathbf{x}^{(1)} = \mathbf{u}$$

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$$

Get a new sequence

Establishment a one-order differential equations for this new sequence

Denoted as GM (1, 1), a and u are gray parameters, sign  $\hat{a} = [a u]^T$ , It can be solved by least square principl

$$\hat{\mathbf{a}} = [\mathbf{a} \mathbf{u}]^{\mathrm{T}} = (\mathbf{B}^{\mathrm{T}} \mathbf{B})^{-1} \mathbf{B}^{\mathrm{T}} \mathbf{y}$$

Among:

$$\mathbf{B} = \begin{bmatrix} -\frac{1}{2} \mathbf{x}^{(1)}(2) + \mathbf{x}^{(1)}(1) & 1\\ -\frac{1}{2} \mathbf{x}^{(1)}(3) + \mathbf{x}^{(1)}(2) & 1\\ \vdots & \vdots\\ -\frac{1}{2} \mathbf{x}^{(1)}(n) + \mathbf{x}^{(1)}(n-1) & 1 \end{bmatrix}$$

After calculated, differential equations can be solved

$$\hat{\mathbf{x}}^{(1)}(\mathbf{k}+1) = (\mathbf{x}^{0}(1) - \frac{\mathbf{u}}{a}) \ \mathbf{e}^{-\mathbf{a}\mathbf{k}} + \frac{\mathbf{u}}{a}$$

Then come to

$$\hat{\mathbf{x}}^{(0)}(\mathbf{k}+1) = (1 - \mathbf{e}^{a})(x^{0}(1) - \frac{u}{a})e^{-ak}$$

#### 2.3 residual test

The level of accuracy of the model directly determines the quality of Regression Level of data, gray model precision test mainly includes three ones: residual size test, association test and three posterior variance test<sup>[1]</sup>.Residual size test is to inspect calculated values and actual values point by point. By the GM (1,1) mode

$$\hat{\mathbf{x}^{(0)}} = \{ \hat{\mathbf{x}^{(0)}}(1), \hat{\mathbf{x}^{(0)}}(2), \dots, \hat{\mathbf{x}^{(0)}}(n) \}$$

Calculate residuals

$$e(k) = x^{(0)}(k) - x^{(0)}(k)$$
  
k = 1,2,..., n

Note the original series  $x^{(0)}$  and the residual number of columns e's variance, respectively as  $S_1^2$  and  $S_2^2$ , then

$$S_{1}^{2} = \frac{1}{n} \sum_{k=1}^{n} (x^{(0)}(k) - \overline{x^{(0)}})^{2}$$
$$S_{2}^{2} = \frac{1}{n} \sum_{k=1}^{n} (e^{-k}(k) - \overline{e})^{2}$$

The formula

$$\overline{\mathbf{x}^{(0)}} = \frac{1}{n} \sum_{k=1}^{n} \mathbf{x}^{(0)}(k)$$
$$\overline{\mathbf{e}} = \frac{1}{n} \sum_{k=1}^{n} \mathbf{e}(k)$$

Note posterior variance ratio  $C = \frac{S_2}{S_1}$ 

Small error probability  $P = \{|e(k)| \langle 0.6745S_1 \}$ 

#### 3. A numerical example

This paper selects the actual settlement monitoring data of a settlement point of a building in the construction process, a total of six<sup>[6]</sup>. Its monitoring results are [3.45 3.99 4.39 4.38 5.02 5.43 5.53 6.06].

#### 3.1 GM (1,1) model data processing

 $\mathbf{x}^{(0)} = \{3. 45, 3. 99, 4. 39, 4. 38, 5. 02, 5. 43, 5. 53, 6. 06\}$ 

conduct a cumulative of former 6 datas of raw sequence, generates

$$\mathbf{x}^{(1)} = \{3.45, 7.44, 11.83, 16.21, 21.23, 26.66\}$$

Calculate model's Data matrix B and  $y_6$ 

$$B = \begin{bmatrix} -5.44 & 1 \\ -9.64 & 1 \\ -14.02 & 1 \\ -18.72 & 1 \\ -23.95 & 1 \end{bmatrix} = \begin{bmatrix} 3.99 \\ 4.39 \\ 4.38 \\ 5.02 \\ 5.43 \end{bmatrix}$$

Demand model parameters  $\hat{a}$  ,  $\hat{u}$ 

$$\begin{bmatrix} \hat{a} \\ \hat{u} \end{bmatrix} = (\mathbf{B}^{\mathsf{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathsf{T}}\mathbf{y}_{6} = \begin{bmatrix} -0.0765 \\ 3.5433 \end{bmatrix}$$

Establish the Settlement gray prediction model

$$\hat{\mathbf{x}^{(1)}}(\mathbf{k}+1) = (\mathbf{x}^{0}(1) - \frac{\mathbf{u}}{a}) \ \mathbf{e}^{-\mathbf{a}\mathbf{k}} + \frac{\mathbf{u}}{a}$$
$$= 49.77 \mathbf{e}^{0.0765\mathbf{k}} - 46.32$$

Returns prediction results

$$\mathbf{x}^{(0)} = \{3.45, 3.96, 4.27, 4.61, 4.98, 5.37\}$$

#### 3.2 residual test

As shown below

k	Monitoring value	Predictive value	e(k)
1	3.45	3.45	0
2	3.99	3.96	0.03
3	4.39	4.27	0.12
4	4.38	4.61	-0.23
5	5.02	4.98	0.04
6	5.43	5.37	0.06
7	5.53	5.56	-0.03
8	6.06	6.08	-0.02

At the same time calculated:  $S_1^2 = 0.67 S_2^2 = 0.074$ 

$$C = 0.33 P = \{e(k) < 0.55\} = 100\%$$

According to the model accuracy class criteria, obtaining the GM (1,1) model prediction accuracy is grade level.

## 4. Model Improvement

### 4.1 Advantages and disadvantages of Grey Model

Gray system forecasting model requires less sample data, simple principle, easy operation, high short-term forecasting accuracy, inspection, etc., but it also has some limitations <sup>[1]</sup>.It embodied in:

① it is mainly applied to a single model of exponential growth, the situation on the abnormal sequence data is difficult to be considered;

②it mainly rely on sequence data itself to find the law, the model cannot reflect both qualitative factors and quantitative factors to reflect lost application flexibility;

③ it cannot predicted multiple sequences simultaneously, only can model separately for each sequence, which cannot take advantage of the relationship between the sequence;

Because of these limitations, many scholars in various fields began research on improved gray model extensively, generally used to improve the idea is:

(1) through some transformations, improved raw data smooth characteristics, and thus get some deformation of gray model and transform raw data;

(2) modifing traditional GM (1,1) model, it means changes GM (1,1) model or the form of differential equations to adapt the characteristics of the original data.

③ combining gray models and other data processing model to form a variety of gray combined model.

### **4.2 Model Improvement Method**

Reference 5 proposed a combination of gray linear regression model, that means gray model is combined with the upcoming linear regression model. It is on the basis of gray model and it takes full advantage of the characteristics of the linear regression model, so that the result is very good fit to the data, and also proved that Combined Model in precision deformation monitoring data processing to achieve a higher level.

Reference 6 considered the advantages of Kalman filter, and proposed the establishment of GM model-based Kalman filter and the corresponding precision evaluation method. After combining engineering instances calculations and tests found, GM model based on Kalman filter has a strong practical application. This method can simulate the dynamic changes of the target system, in real time, quickly process a large number of dynamic monitoring data, andthe accuracy of dynamic deformation monitoring data is corrected effectively and in real-time, the deformation monitoring of buildings has its unique advantages.

Reference 7 presented the one based on wavelet analysis and gray model, and on the basis of two different methods of applying wavelet analysis, wavelet gray gray model series and parallel combinations of models were applied.

Wherein the series combination model is the use of wavelet Mallat algorithm raw data translation invariant wavelet thresholding hard and improve, new data was subjected to GM (1,1) model model, to obtain the predicted value. Parallel combination model uses a  $^$  trous algorithm wavelet to decompose the original data, and the lays the gray modelingafter coefficients of decomposition. and finally to get the final predicted value by remodeling. This model is applied in deformation monitoring data processing, and with higher accuracy, it can meet the practical engineering.

## 5. Conclusion

As used herein, the monitoring of engineering data is little, it only based on settlement data, and with the gray model GM (1,1) prediction model, to predict the amount of the sedimentation, it can be seen high precision of conventional gray short-term forecasting models. After building structure subsidence monitoring data processing, it needs for more increasion and improvement in gray model, to make monitoring and prediction in line with the actual situation, timely and effectively predict that

may occur in the project building deformation, and act accordingly measures to guide the next step of construction, provide great help to the realization of information of construction management.

### Acknowledgement

Fund Project: Supported by Graduate Innovation Foundation of Southwest Petroleum University, SGIFSWPU, number CXJJ2015048

### References

- [1]Zhu Jian.Study on the Application of Modified Gray Model in Data Analyzing of Deformation Monitoring[D].A Disseration Submitted for the Degree of Master of Chan'an University,2007
- [2]Xiong Junnan, Wang Zegeng, Xu Zhongyin. Method of Tall Buildings Deformation Monitoring Data Processing Method and Engineering Applications [J]. Geomatics Scienc, 2010, 35 Volume (S1):69-71.
- [3]Chen Gang.Subsidence Deformation Monitoring Data Processing and Forecasting Method of Buildings [D].A Disseration Submitted for the Degree of Master of Jiang'xi University, 2011.
- [4]Du Kun.The Study of Deformation Monitoring Data Processing.A Disseration Submitted for the Degree of Master of Central South University, 2013.
- [5]Chen Huaizheng, Wang Yong. Application of Grey Linear Regression Combination Model in Processing and Predicting Deformation Observation Data[J]. Resoursec & Industries, 2011, 13Volume(S1):165-167.
- [6]Chen Mengwei,Zhou Zili, Yu Dongxue.Application of Grey System Theory in Deformation Monitoring Data Processing [J].Jiang'xi Mapping, 2013, Total No. 95(1):15-17.
- [7]Xu Xiujie,Huang Zhangyu,Ling Chenyang,Application of Grey Theory and Wavelet in Deformation Monitoring Data Processing[J].Geotechnil Investigation & Surveying, 2014. 4:80-83.