Robust positive real controller design for a class of two dimensional discrete systems

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Abstract

This paper considers the problem of positive real control for uncertain 2-D discrete systems in the General Model. The parameter uncertainty is assumed to be norm-bounded. The purpose is the design of state feedback controllers such that the closed-loop system is stable and the closed-loop transfer function is extend- ed strictly positive real. In terms of a linear matrix inequality, a condition for the solvability of the problem is obtained, and a desired state feedback controllers is given. Finally, we provide a numerical example to demo- nstrate the applicability of the proposed approach.

Keywords

2-D systems, the general model, linear matrix inequality(LMI), Positive real control.

1. Introduction

Two-dimensional (2-D) discrete systems have received much attention during the past decades since 2-D systems have extensive applications in image processing, seismographic data processing, thermal processes, water stream heating, and other areas [9]. Different kinds of 2-D models, such as 2-D Roesser models and 2-D Fornasini- Marchesini models [21] etc., have been proposed. A great number of fundamental notions and results of one dimensional (1-D) discrete systems were generalized to 2-D discrete systems [23]. Since the introduction of the general state space model of 2-D systems (2-D GM) in [23], a lot of research topics, such as controllability [10], minimum energy control [11], internal stability [12], computation of 2-D eigenvalues and the transfer function matrix [13] related to 2-D GM have been studied in the literature.

On the other hand, the concept of positive realness has played an important role in studying control and system theory [14]. The study of positive real control problem is motivated by robust and nonlinear control. If uncertainty or nonlinearity can be characterized by a positive real system, it is well known that the positive realness of a loop transfer function will guarantee the over all stability of a feedback system [15]. The problem of positive real control has been studied, which is concerned with the design of controllers such that the closed-loop system is stable and the closed-loop transfer function is positive real [16]. A solution to this problem for a known linear time-invariant system involves solving a pair of Riccati inequalities [6], while for uncertain systems, the solution can be characterized by solving certain linear matrix inequalities (LMIs) [17].Very recently, the problem of positive real control for 2-D discrete systems described by Roesser models, Fornasini-Marchesini model was considered in [23,24], where a state feedback controllers was designed such that the resulting closed-loop system is asymptotically stable and the closed-loop transfer function is positive real. However, the positive real control for 2-D GM systems has not been investigated.

In this paper, we study the problem of positive real control for 2-D discrete systems in the General model with parameter uncertainties. The parameter uncertainty is assumed to be unknown but norm bounded. The purpose is to design a state feedback controllers such that the closed-loop system is asymptotically stable while the associated closed-loop transfer function is extended strictly positive real (ESPR). A sufficient condition ensuring a 2-D discrete system to be ESPR property is proposed. Based on this, a condition for the solvability of the positive real control problem is obtained in terms of an LMI, and a desired state feedback controller is given when the proposed LMI is feasible. Finally, an illustrative example is provided to demonstrate the applicability of the proposed methods.

Notation. Throughout this paper, for Hermitian matrices X and Y , the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension. The superscript " T " " $-T$ " " $*$ "represents the transpose, inverse transpose, and the complex conjugate transpose. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem Statement and Preliminaries

Consider an uncertain 2-D discrete-time system described by the following general Model [22]:

$$\sum\nolimits_{\Delta}: \; x(i+1, j+1) = (A_1 + \Delta A_1)x(i+1, j) + (A_2 + \Delta A_2)x(i, j+1) + (A_0 + \Delta A_0)x(i, j)$$

$$+(B_1 + \Delta B_1)w(i+1, j) + (B_2 + \Delta B_2)w(i, j+1) + (B_0 + \Delta B_0)w(i, j) \tag{1a}$$

$$z(i, j) = Cx(i, j) + Dw(i, j) \tag{1b}$$

where $x(i, j) \in R^n$ is the local state vector, $w(i, j) \in R^q$ is the exogenous input, $z(i, j) \in R^q$ is the controlled output, where $A_1, A_2, A_0, B_1, B_2, B_0, C$ and D are known real constant matrices with appropriate dimensions. $\Delta A_1, \Delta A_2, \Delta A_0, \Delta B_1, \Delta B_2$ and ΔB_0 are time-invariant matrices representing nom-bounded parameter uncertainties, and are assumed to be of the form:

$$\begin{bmatrix} \Delta A_1 & \Delta A_2 & \Delta A_0 & \Delta B_1 & \Delta B_2 & \Delta B_0 \end{bmatrix} = MF \begin{bmatrix} N_{A_1} & N_{A_2} & N_{A_0} & N_{B_1} & N_{B_2} & N_{B_0} \end{bmatrix} \tag{2}$$

where $F \in R^{g \times l}$ is an unknown real matrix satisfying

$$F^T F \leq I \tag{3}$$

and $M, N_{A_1}, N_{A_2}, N_{A_0}, N_{B_1}, N_{B_2}$ and N_{B_0} are known real constant matrices with appropriate dimensions.

The nominal 2-D discrete-time system of (1)can be written as

$$\sum: \; x(i+1, j+1) = A_1 x(i+1, j) + A_2 x(i, j+1) + A_0 x(i, j)$$

$$+B_1 w(i+1, j) + B_2 w(i, j+1) + B_0 w(i, j) \tag{4a}$$

$$z(i, j) = Cx(i, j) + Dw(i, j) \tag{4b}$$

Then, the transfer function of the 2-D discrete-time system (4)can be written as

$$G(z_1, z_2) = C(z_1 z_2 I - z_1 A_1 - z_2 A_2 - A_0)^{-1}(z_1 B_1 + z_2 B_2 + B_0) + D \tag{5}$$

We define the concept of positive realness for 2-D GM systems in the following.

Definition 2.1.

The 2-D discrete-time *system* (4)*is said to be positive real(PR) if its transfer function* $G(z_1, z_2)$ *is analytic in* $|z_1| > 1, |z_2| > 1$ *and satisfies* $G(z_1, z_2) + G^*(z_1, z_2) \geq 0$ *for* $|z_1| > 1, |z_2| > 1$.

The 2-D discrete-time *system* (4)*is said to be strictly positive real(SPR) if its transfer function* $G(z_1, z_2)$ *is analytic in* $|z_1| \geq 1, |z_2| \geq 1$ *and satisfies* $G(e^{j\theta_1}, e^{j\theta_2}) + G^*(e^{j\theta_1}, e^{j\theta_2}) > 0$ *for* $\theta_1, \theta_2 \in [0, 2\pi)$.

The 2-D discrete-time system (4)*is* said *to be extended strictly positive real(ESPR) if its SPR and* $G(\infty, \infty) + G(\infty, \infty)^T > 0$.

We end this section by presenting Lemmas that will be essential in the proof of our main results in the next section.

Lemma 2.1. [19]*Let* A, L, E, F *and* P *be real matrices of appropriate* dimensions *with* $P > 0$ *and* F *satisfying* $F^T F \leq I$ *.Then, for any scalar* $\varepsilon > 0$ *such that* $P - \varepsilon LL^T > 0$ *,we have*

$$(A+LFE)^T P^{-1}(A+LFE) \le A^T (P-\varepsilon LL^T)^{-1} A + \varepsilon^{-1} E^T E .$$

Lemma 2.2. [20]*Let* L, E, F *and* Q *be real* matrices *of appropriate dimensions with* Q *satisfying* $Q = Q^T$ *,then* $Q + LFE + (LFE)^T < 0$ *for all* F *satisfying* $F^T F \le I$ *,if and only if there exist a scalar* $\varepsilon > 0$ *such that*

$$Q + \varepsilon LL^T + \varepsilon^{-1} E^T E < 0$$

Lemma 2.3. [18] *The 2-D* linear *discrete-time system* (4)*is asymptotically stable if there exist matrices* $P_1 > 0, P_2 > 0$ *and* $P > 0$ *such that the following LMI holds:*

$$\begin{bmatrix} A_1^T PA_1 - P + P_0 + P_1 & A_1^T PA_2 & A_1^T PA_0 \\ A_2^T PA_1 & A_2^T PA_2 - P_1 & A_2^T PA_0 \\ A_0^T PA_1 & A_0^T PA_2 & A_0^T PA_0 \end{bmatrix} < 0 \tag{6}$$

3. Main Results

The following theorem provides the positive real lemma for the 2-D discrete systems in the General model.

Theorem 3.1.Th*e 2-D discrete-time system* (4) *is* asymptotically *stable with ESPR if there exist matrices* $P_0 > 0, P_1 > 0, P > 0, W > 0$ *and* $V > 0$ *such that the following LMI holds:*

$$\begin{bmatrix} \begin{Bmatrix} A_1^T PA_1 - P \\ +P_0 + P_1 \end{Bmatrix} & A_1^T PA_2 & A_1^T PA_0 & C^T - A_1^T PB_1 & -A_1^T PB_2 & -A_1^T PB_0 \\ A_2^T PA_1 & A_2^T PA_2 - P_1 & A_2^T PA_0 & -A_2^T PB_1 & -A_2^T PB_2 & -A_2^T PB_0 \\ A_0^T PA_1 & A_0^T PA_2 & A_0^T PA_0 - P_0 & -A_0^T PB_1 & -A_0^T PB_2 & -A_0^T PB_0 \\ C - B_1^T PA_1 & -B_1^T PA_2 & -B_1^T PA_0 & \begin{Bmatrix} B_1^T PB_1 - (D + \\ D^T) + W + V \end{Bmatrix} & B_1^T PB_2 & B_1^T PB_0 \\ -B_2^T PA_1 & -B_2^T PA_2 & -B_2^T PA_0 & B_2^T PB_1 & B_2^T PB_2 - W & B_2^T PB_0 \\ -B_0^T PA_1 & -B_0^T PA_2 & -B_0^T PA_0 & B_0^T PB_1 & B_0^T PB_2 & B_0^T PB_0 - V \end{bmatrix} < 0 \tag{7}$$

Proof: The proof is omitted here for brevity (see [25]).

Remark1:Theorem1 provides an LMI condition for the 2-D discrete system in the General Model to be asymptotically stable and ESPR.

Definition 3.1. The uncertain *2-D discrete-time system* (1)*is said to be strongly robustly stable with ESPR if there exist matrices* $P_0 > 0, P_1 > 0, P > 0, W > 0$ *and* $V > 0$ *such that following the LMI shown in (16), holds for all admissible uncertainties* ΔA_1, ΔA_2, $\Delta A_0, \Delta B_1, \Delta B_2$ *and* ΔB_0 *satisfying* (2).

$$\begin{bmatrix} \begin{Bmatrix} A_{1\Delta}^T PA_{1\Delta} - \\ P + P_0 + P_1 \end{Bmatrix} & A_{1\Delta}^T PA_{2\Delta} & A_{1\Delta}^T PA_{0\Delta} & C^T - A_{1\Delta}^T PB_{1\Delta} & -A_{1\Delta}^T PB_{2\Delta} & -A_{1\Delta}^T PB_{0\Delta} \\ A_{2\Delta}^T PA_{1\Delta} & A_{2\Delta}^T PA_{2\Delta} - P_1 & A_{2\Delta}^T PA_{0\Delta} & -A_{2\Delta}^T PB_{1\Delta} & -A_{2\Delta}^T PB_{2\Delta} & -A_{2\Delta}^T PB_{0\Delta} \\ A_{0\Delta}^T PA_{1\Delta} & A_{0\Delta}^T PA_{2\Delta} & A_{0\Delta}^T PA_{0\Delta} - P_0 & -A_{0\Delta}^T PB_{1\Delta} & -A_{0\Delta}^T PB_{2\Delta} & -A_{0\Delta}^T PB_{0\Delta} \\ C - B_{1\Delta}^T PA_{1\Delta} & -B_{1\Delta}^T PA_{2\Delta} & -B_{1\Delta}^T PA_{0\Delta} & \begin{Bmatrix} B_{1\Delta}^T PB_{1\Delta} - (D \\ +D^T) + W + V \end{Bmatrix} & B_{1\Delta}^T PB_{2\Delta} & B_{1\Delta}^T PB_{0\Delta} \\ -B_{2\Delta}^T PA_{1\Delta} & -B_{2\Delta}^T PA_{2\Delta} & -B_{2\Delta}^T PA_{0\Delta} & B_{2\Delta}^T PB_{1\Delta} & B_{2\Delta}^T PB_{2\Delta} - W & B_{2\Delta}^T PB_{0\Delta} \\ -B_{0\Delta}^T PA_{1\Delta} & -B_{0\Delta}^T PA_{2\Delta} & -B_{0\Delta}^T PA_{0\Delta} & B_{0\Delta}^T PB_{1\Delta} & B_{0\Delta}^T PB_{2\Delta} & B_{0\Delta}^T PB_{0\Delta} - V \end{bmatrix} < 0 \tag{8}$$

Where

$A_{1\Delta} = A_1 + \Delta A_1$, $A_{2\Delta} = A_2 + \Delta A_2$, $A_{0\Delta} = A_0 + \Delta A_0$, $B_{1\Delta} = B_1 + \Delta B_1$, $B_{2\Delta} = B_2 + \Delta B_2$, $B_{0\Delta} = B_0 + \Delta B_0$.

The following *theorem presents a necessary and sufficient condition for system* (1)*to be strongly robustly stable with ESPR.*

Theorem 3.2. *Consider the uncertain 2-D discrete-time system* (1).*This system is strongly robustly stable with ESPR for all admissible uncertainties if and only if there exists a scalar* $\varepsilon > 0$ *and matrices* $X > 0, Y > 0, Z > 0, W > 0$ *and* $V > 0$ *such that the following LMI holds.*

$$\begin{bmatrix} X+Y-Z & 0 & 0 & ZC^T & 0 & 0 & ZA_1^T & ZN_{A_1}^T \\ 0 & -Y & 0 & 0 & 0 & 0 & ZA_2^T & ZN_{A_2}^T \\ 0 & 0 & -X & 0 & 0 & 0 & ZA_0^T & ZN_{A_0}^T \\ CZ & 0 & 0 & \left\{\begin{array}{l} -(D+D^T) \\ +W+V \end{array}\right\} & 0 & 0 & -B_1^T & -N_{B1}^T \\ 0 & 0 & 0 & 0 & -W & 0 & -B_2^T & -N_{B2}^T \\ 0 & 0 & 0 & 0 & 0 & -V & -B_0^T & -N_{B0}^T \\ A_1Z & A_2Z & A_0Z & -B_1 & -B_2 & -B_0 & \varepsilon MM^T - Z & 0 \\ N_{A_1}Z & N_{A_2}Z & N_{A_0}Z & -N_{B_1} & -N_{B_2} & -N_{B_0} & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{9}$$

Proof : (*Necessity) Suppose the uncertain 2-D system* (1)*is strongly robustly stable with ESPR, that is, there exist* matrices $P_0 > 0, P_1 > 0, P > 0, W > 0$ *and* $V > 0$ *such that above the LMI (8) holds. By Schur complements, it follows from (8) that*

$$\begin{bmatrix} P_0+P-P_1 & 0 & 0 & C^T & 0 & 0 & A_{1\Delta}^T \\ 0 & -P_1 & 0 & 0 & 0 & 0 & A_{2\Delta}^T \\ 0 & 0 & -P_0 & 0 & 0 & 0 & A_{0\Delta}^T \\ C & 0 & 0 & \left\{\begin{array}{l} -(D+D^T) \\ +W+V \end{array}\right\} & 0 & 0 & -B_{1\Delta}^T \\ 0 & 0 & 0 & 0 & -W & 0 & -B_{2\Delta}^T \\ 0 & 0 & 0 & 0 & 0 & -V & -B_{0\Delta}^T \\ A_{1\Delta} & A_{2\Delta} & A_{0\Delta} & -B_{1\Delta} & -B_{2\Delta} & -B_{0\Delta} & -P^{-1} \end{bmatrix} < 0$$

That is

$$\begin{bmatrix} P_0+P-P_1 & & & C^T & & & A_1^T \\ & -P_1 & & & & & A_2^T \\ & & -P_0 & & & & A_0^T \\ C & & & \left\{\begin{array}{l} -(D+D^T) \\ +W+V \end{array}\right\} & & & -B_1^T \\ & & & & -W & & -B_2^T \\ & & & & & -V & -B_0^T \\ A_1 & A_2 & A_0 & -B_1 & -B_2 & -B_0 & -P^{-1} \end{bmatrix} + \begin{bmatrix} N_{A_1}^T \\ N_{A_2}^T \\ N_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \\ 0 \end{bmatrix} F^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ M \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ M \end{bmatrix} F \begin{bmatrix} N_{A_1}^T \\ N_{A_2}^T \\ N_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \\ 0 \end{bmatrix}^T < 0$$

Therefore, using Lemma 2.2, we have that there exist a scalar $\varepsilon > 0$ *, such that*

$$\begin{bmatrix} P_0+P-P_1 & 0 & 0 & C^T & 0 & 0 & A_1^T \\ 0 & -P_1 & 0 & 0 & 0 & 0 & A_2^T \\ 0 & 0 & -P_0 & 0 & 0 & 0 & A_0^T \\ C & 0 & 0 & \left\{\begin{array}{l} -(D+D^T) \\ +W+V \end{array}\right\} & 0 & 0 & -B_1^T \\ 0 & 0 & 0 & 0 & -W & 0 & -B_2^T \\ 0 & 0 & 0 & 0 & 0 & -V & -B_0^T \\ A_1 & A_2 & A_0 & -B_1 & -B_2 & -B_0 & -P^{-1} \end{bmatrix} + \varepsilon \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ M \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ M \end{bmatrix}^T + \varepsilon^{-1} \begin{bmatrix} N_{A_1}^T \\ N_{A_2}^T \\ N_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \\ 0 \end{bmatrix} \begin{bmatrix} N_{A_1}^T \\ N_{A_2}^T \\ N_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \\ 0 \end{bmatrix}^T < 0 \tag{10}$$

That is

$$\begin{bmatrix} P_0+P_1-P & 0 & 0 & C^T & 0 & 0 & A_1^T & N_{A_1}^T \\ 0 & -P_1 & 0 & 0 & 0 & 0 & A_2^T & N_{A_2}^T \\ 0 & 0 & -P_0 & 0 & 0 & 0 & A_0^T & N_{A_0}^T \\ C & 0 & 0 & \begin{Bmatrix} -(D+D^T) \\ +W+V \end{Bmatrix} & 0 & 0 & -B_1^T & -N_{B1}^T \\ 0 & 0 & 0 & 0 & -W & 0 & -B_2^T & -N_{B2}^T \\ 0 & 0 & 0 & 0 & 0 & -V & -B_0^T & -N_{B0}^T \\ A_1 & A_2 & A_0 & -B_1 & -B_2 & -B_0 & \varepsilon MM^T-P^{-1} & 0 \\ N_{A_1} & N_{A_2} & N_{A_0} & -N_{B_1} & -N_{B_2} & -N_{B_0} & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{11}$$

Pre-multiplying and Post-multiplying (11) by $diag\left\{P^{-1},P^{-1},P^{-1},I,I,I,I,I\right\}$ *and setting* $X=P^{-1}P_0P^{-1}, Y=P^{-1}P_1P^{-1}, Z=P^{-1}$*, the* desired *result follows immediately.*

(Sufficiency):Suppose that exist a scalar $\varepsilon>0$ *and matrices* $X>0,Y>0,Z>0,W>0$ *and* $V>0$ *such that (9)is satisfied .Then ,from(9)it is easy to see that*

$$Z-\varepsilon MM^T>0 \tag{12}$$

By Lemma 2.1, it can *be show that*

$$\left(\begin{bmatrix} A_1^T \\ A_2^T \\ A_0^T \\ -B_1^T \\ -B_2^T \\ -B_0^T \end{bmatrix}+\begin{bmatrix} N_{A_1}^T \\ N_{A_2}^T \\ N_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \end{bmatrix}F^TM^T\right)Z^{-1}\left(\begin{bmatrix} A_1^T \\ A_2^T \\ A_0^T \\ -B_1^T \\ -B_2^T \\ -B_0^T \end{bmatrix}+\begin{bmatrix} N_{A_1}^T \\ N_{A_2}^T \\ N_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \end{bmatrix}F^TM^T\right)^T$$

$$\leq\begin{bmatrix} A_1^T \\ A_2^T \\ A_0^T \\ -B_1^T \\ -B_2^T \\ -B_0^T \end{bmatrix}(Z-\varepsilon MM^T)^{-1}\begin{bmatrix} A_1^T \\ A_2^T \\ A_0^T \\ -B_1^T \\ -B_2^T \\ -B_0^T \end{bmatrix}^T+\varepsilon^{-1}\begin{bmatrix} N_{A_1}^T \\ N_{A_2}^T \\ N_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \end{bmatrix}\begin{bmatrix} N_{A_1}^T \\ N_{A_2}^T \\ N_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \end{bmatrix}^T \tag{13}$$

Let $\tilde{X}=Z^{-1}XZ^{-1}, \tilde{Y}=Z^{-1}YZ^{-1}, J=J^T=diag\left\{Z,Z,Z,I,I,I\right\}$

Then, by considering (9) and *using Schur complements, we get the following equation.*

$$\begin{bmatrix} ZA_1^T \\ ZA_2^T \\ ZA_0^T \\ -B_1^T \\ -B_2^T \\ -B_0^T \end{bmatrix}(Z-\varepsilon MM^T)^{-1}\begin{bmatrix} ZA_1^T \\ ZA_2^T \\ ZA_0^T \\ -B_1^T \\ -B_2^T \\ -B_0^T \end{bmatrix}^T+\varepsilon^{-1}\begin{bmatrix} ZN_{A_1}^T \\ ZN_{A_2}^T \\ ZN_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \end{bmatrix}\begin{bmatrix} ZN_{A_1}^T \\ ZN_{A_2}^T \\ ZN_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \end{bmatrix}^T+\begin{bmatrix} X+Y-Z & 0 & 0 & ZC^T & 0 & 0 \\ 0 & -Y & 0 & 0 & 0 & 0 \\ 0 & 0 & -X & 0 & 0 & 0 \\ CZ & 0 & 0 & -(D+D^T)+W+V & 0 & 0 \\ 0 & 0 & 0 & 0 & -W & 0 \\ 0 & 0 & 0 & 0 & 0 & -V \end{bmatrix}$$

$$
= J\left(\begin{bmatrix} A_1^T \\ A_2^T \\ A_0^T \\ -B_1^T \\ -B_2^T \\ -B_0^T \end{bmatrix} (Z-\varepsilon MM^T)^{-1} \begin{bmatrix} A_1^T \\ A_2^T \\ A_0^T \\ -B_1^T \\ -B_2^T \\ -B_0^T \end{bmatrix}^T + \varepsilon^{-1} \begin{bmatrix} N_{A_1}^T \\ N_{A_2}^T \\ N_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \end{bmatrix} \begin{bmatrix} N_{A_1}^T \\ N_{A_2}^T \\ N_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \end{bmatrix}^T \right) J^T
$$

$$
+J\begin{bmatrix} \tilde{X}+\tilde{Y}-Z^{-1} & 0 & 0 & C^T & 0 & 0 \\ 0 & -\tilde{Y} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\tilde{X} & 0 & 0 & 0 \\ C & 0 & 0 & -(D+D^T)+W+V & 0 & 0 \\ 0 & 0 & 0 & 0 & -W & 0 \\ 0 & 0 & 0 & 0 & 0 & -V \end{bmatrix} J^T
$$

This together with (13) implies *that*

$$
\left(\begin{bmatrix} A_1^T \\ A_2^T \\ A_0^T \\ -B_1^T \\ -B_2^T \\ -B_0^T \end{bmatrix} + \begin{bmatrix} N_{A_1}^T \\ N_{A_2}^T \\ N_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \end{bmatrix} F^T M^T \right) Z^{-1} \left(\begin{bmatrix} A_1^T \\ A_2^T \\ A_0^T \\ -B_1^T \\ -B_2^T \\ -B_0^T \end{bmatrix} + \begin{bmatrix} N_{A_1}^T \\ N_{A_2}^T \\ N_{A_0}^T \\ -N_{B_1}^T \\ -N_{B_2}^T \\ -N_{B_0}^T \end{bmatrix} F^T M^T \right)^T + \begin{bmatrix} \tilde{X}+\tilde{Y}-Z^{-1} & 0 & 0 & C^T & 0 & 0 \\ 0 & -\tilde{Y} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\tilde{X} & 0 & 0 & 0 \\ C & 0 & 0 & -(D+D^T)+W+V & 0 & 0 \\ 0 & 0 & 0 & 0 & -W & 0 \\ 0 & 0 & 0 & 0 & 0 & -V \end{bmatrix} < 0
$$

This leads to the following *equation.*

$$
\begin{bmatrix}
\begin{Bmatrix} A_{1\Delta}^T Z^{-1} A_{1\Delta} - \\ Z^{-1}+\tilde{X}+\tilde{Y} \end{Bmatrix} & A_{1\Delta}^T Z^{-1} A_{2\Delta} & A_{1\Delta}^T Z^{-1} A_{0\Delta} & C^T - A_{1\Delta}^T Z^{-1} B_{1\Delta} & -A_{1\Delta}^T Z^{-1} B_{2\Delta} & -A_{1\Delta}^T Z^{-1} B_{0\Delta} \\
A_{2\Delta}^T Z^{-1} A_{1\Delta} & A_{2\Delta}^T Z^{-1} A_{2\Delta} - \tilde{Y} & A_{2\Delta}^T Z^{-1} A_{0\Delta} & -A_{2\Delta}^T Z^{-1} B_{1\Delta} & -A_{2\Delta}^T Z^{-1} B_{2\Delta} & -A_{2\Delta}^T Z^{-1} B_{0\Delta} \\
A_{0\Delta}^T Z^{-1} A_{1\Delta} & A_{0\Delta}^T Z^{-1} A_{2\Delta} & A_{0\Delta}^T Z^{-1} A_{0\Delta} - \tilde{X} & -A_{0\Delta}^T Z^{-1} B_{1\Delta} & -A_{0\Delta}^T Z^{-1} B_{2\Delta} & -A_{0\Delta}^T Z^{-1} B_{0\Delta} \\
C - B_{1\Delta}^T Z^{-1} A_{1\Delta} & -B_{1\Delta}^T Z^{-1} A_{2\Delta} & -B_{1\Delta}^T Z^{-1} A_{0\Delta} & \begin{Bmatrix} B_{1\Delta}^T Z^{-1} B_{1\Delta} - (D \\ +D^T) + W + V \end{Bmatrix} & B_{1\Delta}^T Z^{-1} B_{2\Delta} & B_{1\Delta}^T Z^{-1} B_{0\Delta} \\
-B_{2\Delta}^T Z^{-1} A_{1\Delta} & -B_{2\Delta}^T Z^{-1} A_{2\Delta} & -B_{2\Delta}^T Z^{-1} A_{0\Delta} & B_{2\Delta}^T Z^{-1} B_{1\Delta} & B_{2\Delta}^T Z^{-1} B_{2\Delta} - W & B_{2\Delta}^T Z^{-1} B_{0\Delta} \\
-B_{0\Delta}^T Z^{-1} A_{1\Delta} & -B_{0\Delta}^T Z^{-1} A_{2\Delta} & -B_{0\Delta}^T Z^{-1} A_{0\Delta} & B_{0\Delta}^T Z^{-1} B_{1\Delta} & B_{0\Delta}^T Z^{-1} B_{2\Delta} & B_{0\Delta}^T Z^{-1} B_{0\Delta} - V
\end{bmatrix} < 0
$$

By definition 3.1, it follows *that uncertain 2-D system* (1)*is strongly robustly stable with ESPR for all admissible uncertainties.*

4. Robust Positive Real Control

In this section, we consider the problem of positive real control for uncertain 2-D discrete-time system. An LMI design approach will be developed. The uncertain 2-D discrete-time system $\sum_{\Delta u}$ to be considered in this section is described by the following 2-D general model:

$$
\begin{aligned}
\sum_{\Delta u}:\ x(i+1,j+1) &= (A_1+\Delta A_1)x(i+1,j)+(A_2+\Delta A_2)x(i,j+1)+(A_0+\Delta A_0)x(i,j) \\
&+(B_1+\Delta B_1)w(i+1,j)+(B_2+\Delta B_2)w(i,j+1)+(B_0+\Delta B_0)w(i,j) \\
&+(L_1+\Delta L_1)u(i+1,j)+(L_2+\Delta L_2)u(i,j+1)+(L_0+\Delta L_0)u(i,j)
\end{aligned} \tag{14a}
$$

$$
z(i,j) = Cx(i,j)+Dw(i,j) \tag{14b}
$$

where $x(i,j)\in R^n$ is the local state vector, $u(i,j)\in R^m$ is the control input, $w(i,j)\in R^q$ is the exogenous input, $z(i,j)\in R^q$ is the controlled output, where L_1, L_2 and L_0 are known real constant matrices with appropriate dimensions. $\Delta L_1, \Delta L_2$ and ΔL_0 are time-invariant matrices representing nom-bounded parameter uncertainties, and are assumed to be of the form

$$[\Delta L_1 \quad \Delta L_2 \quad \Delta L_0] = MF[N_{L_1} \quad N_{L_2} \quad N_{L_0}] \tag{15}$$

where $F\in R^{g\times l}$ is an unknown real matrix satisfying (3)and N_{L_1}, N_{L_2} and N_{L_0} are known real constant matrices with appropriate dimensions. The remaining matrices are the same as in system (1).It is assumed that all the state variables are available for feedback.

The objective of the robust positive real control is the design of feedback controllers for system (22)such that the resulting closed-loop system is strongly robustly stable with ESPR for all admissible uncertainties.

Now, applying the state feedback controller

$$u(i,j) = Kx(i,j) \tag{16}$$

to the system (22),we obtain the closed-loop systems

$$\begin{aligned}\textstyle\sum_c: x(i+1,j+1) &= A_{1c}x(i+1,j) + A_{2c}x(i,j+1) + A_{0c}x(i,j)\\ &\quad +(B_1+\Delta B_1)w(i+1,j) + (B_2+\Delta B_2)w(i,j+1) + (B_0+\Delta B_0)w(i,j)\\ z(i,j) &= Cx(i,j) + Dw(i,j)\end{aligned} \tag{17}$$

Here, $A_{1c} = (A_1+\Delta A_1) + (L_1+\Delta L_1)K$, $A_{2c} = (A_2+\Delta A_2)+(L_2+\Delta L_2)K$, $A_{0c} = (A_0+\Delta A_0)+(L_0+\Delta L_0)K$

The main result of this section is given in the following theorem.

Theorem 4.1. Co*nsider the uncertain 2-D discrete-time system* (22).*There exist a static state feedback controller for* system (22)*such that the resulting closed-loop system is strongly robustly stable with ESPR for all admissible uncertains if and only if there exit a scalar* $\varepsilon > 0$ *and matrices* $X>0, Y>0, Z>0, W>0, V>0$ *and* K *such that the following LMI holds.*

$$\begin{bmatrix} \Xi & \Omega^T & \Upsilon^T \\ \Omega & \varepsilon MM^T - Z & 0 \\ \Upsilon & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{18}$$

Where

$$\Xi = \begin{bmatrix} X+Y-Z & 0 & 0 & ZC^T & 0 & 0 \\ 0 & -Y & 0 & 0 & 0 & 0 \\ 0 & 0 & -X & 0 & 0 & 0 \\ CZ & 0 & 0 & \begin{Bmatrix} -(D+D^T) \\ +W+V \end{Bmatrix} & 0 & 0 \\ 0 & 0 & 0 & 0 & -W & 0 \\ 0 & 0 & 0 & 0 & 0 & -V \end{bmatrix}$$

$$\Omega = \begin{bmatrix} A_1Z+L_1K & A_2Z+L_2K & A_0Z+L_0K & -B_1 & -B_2 & -B_0 \end{bmatrix}$$

$$\Upsilon = [N_{A_1}Z + N_{L_1}K \quad N_{A_2}Z+N_{L_2}K \quad N_{A_0}Z+N_{L_0}K \quad -N_{B_1} \quad -N_{B_2} \quad -N_{B_0}]$$

Furthermore,in this case, a suitable state feedback controller can be chosen as

$u(i,j) = KZ^{-1}x(i,j)$

Proof: *the theorem can be carried out by using a similar approach as in the proof of theorem 2.*

Remark2:Therorem4.1 provides a sufficient condition for the designing a state feedback controller which stabilizes the uncertain 2-D discrete system described by the General Model and achieves the *extended strictly positive realness property of the closed-loop system. It is worth pointing out that the LMI(26) in the Therorem4.1 can be solved efficiently, and no tuning of parameters is required.*

5. Numerical Example.

In this section, we give an example to illustrate the effectiveness of the proposed method.

Consider the 2-D discrete-time system (14) with parameters given by

$$A_1=\begin{bmatrix}0.1&0.1&0.1\\-0.1&-0.5&0.3\\-0.2&-0.1&0.3\end{bmatrix},A_2=\begin{bmatrix}0&-0.1&0.5\\-0.5&-0.5&0.2\\0.2&0.1&0.5\end{bmatrix},A_0=\begin{bmatrix}0.1&-0.5&0.5\\-0.4&-0.5&0.2\\0&0.2&0.5\end{bmatrix},B_1=\begin{bmatrix}0.5&0.3&0.1\\0&0.2&0.5\\-1&0.1&0.4\end{bmatrix}$$

$$B_2=\begin{bmatrix}-0.2&0&0.1\\0.3&0.1&-0.5\\0.6&-0.1&0.3\end{bmatrix},B_0=\begin{bmatrix}0.5&0.3&0.1\\0&0.2&0.5\\-1&0.1&0\end{bmatrix},L_1=\begin{bmatrix}0.1&0\\1&-1\\1&1\end{bmatrix},L_2=\begin{bmatrix}1&1\\1&0\\1&-0.6\end{bmatrix},L_0=\begin{bmatrix}1&1\\1&0\\1&0.1\end{bmatrix},$$

$$C=\begin{bmatrix}0.1&0.3&0.5\\0.1&0&0.3\\0.2&0.2&0\end{bmatrix},D=\begin{bmatrix}1.5&0.5&0\\0.1&0.8&0.2\\0.3&0.1&1.6\end{bmatrix},M=\begin{bmatrix}-0.1\\-0.1\\0.1\end{bmatrix},N_{A_1}=\begin{bmatrix}0.1\\-0.1\\0.1\end{bmatrix}^T,N_{A_2}=\begin{bmatrix}0.2\\-0.2\\0.2\end{bmatrix}^T,N_{A_0}=\begin{bmatrix}0.2\\0.1\\0.1\end{bmatrix}^T,N_{B_1}=\begin{bmatrix}0.1\\0.2\\0\end{bmatrix}^T$$

$$N_{B_2}=\begin{bmatrix}0.1&0.1&0\end{bmatrix},N_{B_0}=\begin{bmatrix}-0.1&0.1&0.1\end{bmatrix},N_{L_1}=\begin{bmatrix}0.1&0.2\end{bmatrix},N_{L_2}=\begin{bmatrix}0.1&0.3\end{bmatrix},N_{L_0}=\begin{bmatrix}0.1&0.2\end{bmatrix}.$$

It is required to construct a static feedback controller that stabilizes the given 2-D system while ensuring that the resulting closed-loop system ESPR .Now using matlab LMI control toolbox and solving the LMI (18), we obtain

$$X=\begin{bmatrix}1.9709&-0.9571&0.1511\\-0.9571&1.8199&-0.4671\\0.1511&-0.4671&0.3171\end{bmatrix},Y=\begin{bmatrix}2.1493&0.0726&-0.2920\\0.0726&0.6972&-0.3003\\-0.2920&-0.3003&0.3106\end{bmatrix},Z=\begin{bmatrix}6.8599&-1.5726&-2.7144\\-1.5726&4.5488&-2.3991\\-2.7144&-2.3991&7.9801\end{bmatrix},$$

$$W=\begin{bmatrix}0.4823&-0.0240&0.1130\\-0.0240&0.1078&-0.0519\\0.1130&-0.0519&0.6133\end{bmatrix},V=\begin{bmatrix}0.5711&-0.0205&0.1106\\-0.0205&0.3346&0.3257\\0.1106&0.3257&0.7544\end{bmatrix},K=\begin{bmatrix}1.7139&1.6944&-3.8699\\0.2008&-0.2171&-0.4113\end{bmatrix},$$

$$\varepsilon=16.9973$$

Therefore, from Theorem 3, there exist a solution to the positive real control problem. Furthermore, a desired state feedback controller can be chosen as

$$u(i,j)=\begin{bmatrix}0.1640&0.2411&-0.3567\\-0.0341&-0.1103&-0.0963\end{bmatrix}x(i,j)$$

6. Conclusions.

This paper has studied the problem of positive real control for 2-D discrete systems in the general model. A Necessary and Sufficient Conditions for the solvability of this problem has been proposed. A desired dynamic output feedback controller can be constructed by solving a given LMI.

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