Hesitant fuzzy Hamacher aggregating operators with immediate probabilities

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Abstract

In this paper, we investigate the probabilistic decision making problems with hesitant fuzzy information, some new probabilistic decision making analysis methods are developed. Then, we have developed some new probability aggregation operators with hesitant fuzzy information: probability hesitant fuzzy Hamacher weighted average(P-HFHWA) operator, immediate probability hesitant fuzzy Hamacher ordered weighted average (IP-HFOWA) operator and probability hesitant fuzzy Hamacher ordered weighted average (P-HFOWA) operator.

Keywords

decision making; hesitant fuzzy set; operational laws; probabilistic aggregating operators; Hamacher operations.

1. Introduction

Atanassov [1] introduced the concept of intuitionistic fuzzy set(IFS), which is a generalization of the concept of fuzzy set [2]. The intuitionistic fuzzy set has received more and more attention since its appearance[3-13]. Furthermore, Torra[14] proposed the hesitant fuzzy set which permits the membership having a set of possible values and discussed the relationship between hesitant fuzzy set and intuitionistic fuzzy set, and showed that the envelope of hesitant fuzzy set is an intuitionistic fuzzy set. Xia and Xu[15] gave an intensive study on hesitant fuzzy information aggregation techniques and their application in decision making. Xu et al. [16] developed several series of aggregation operators for hesitant fuzzy information with the aid of quasi-arithmetic means. Gu et al.[17] utilized the hesitant fuzzy weighted averaging (HFWA) operator to investigat the evaluation model for risk investment with hesitant fuzzy information. Motivated by the ideal of prioritized aggregation operators[18], Wei[19] developed some prioritized aggregation operators for aggregating hesitant fuzzy information. Wei et al.[20] proposed hesitant fuzzy choquet ordered averaging (HFCOA) operator and hesitant fuzzy choquet ordered geometric (HFCOG) operator and applied the HFCOA and HFCOG operators to multiple attribute decision making with hesitant fuzzy information. Zhu et al.[21] defined the hesitant fuzzy geometric Bonferroni mean (HFGBM) and the hesitant fuzzy Choquet geometric Bonferroni mean (HFCGBM). Then they gave the definition of hesitant fuzzy geometric Bonferroni element (HFGBE), which is considered as the basic calculational unit in the HFGBM and reflects the conjunction between two aggregated arguments. The properties and special cases of the HFGBM are studied in detail based on the discussion of the HFGBE. In addition, the weighted hesitant fuzzy geometric Bonferroni mean (WHFGBM) and the weighted hesitant fuzzy Choquet geometric Bonferroni mean (WHFCGBM) are proposed considering the importance of each argument and the correlations among them.

In this paper, we investigate the probabilistic decision making problems with hesitant fuzzy information, some new probabilistic decision making analysis methods are developed. Then, we have developed some new probability aggregation operators with hesitant fuzzy information: probability hesitant fuzzy Hamacher weighted average(P-HFHWA) operator, immediate probability hesitant fuzzy Hamacher ordered weighted average (IP-HFOWA) operator and probability hesitant fuzzy Hamacher ordered weighted average (P-HFOWA) operator.

2. Preliminaries

Atanassov [1] extended the fuzzy set to the intuitionistic fuzzy set(IFS). However, when giving the membership degree of an element, the difficulty of establishing the membership degree is not because we have a margin of error, or some possibility distribution on the possibility values, but because we have several possible values. For such cases, Torra [14] proposed another generation of FS.

Definition 1[14]. Given a fixed set X , then a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a sunset of $[0,1]$. The hesitant fuzzy set can be expressed by mathematical symbol:

$$E=\left(\left\langle x,h_E\left(x\right)\right\rangle \middle| x\in X\right) \tag{1}$$

where $h_E\left(x\right)$ is a set of some values in $[0,1]$, denoting the possible membership degree of the element $x\in X$ to the set E. For convenience, Xia and Xu[15] called $h=h_E\left(x\right)$ a hesitant fuzzy element(HFE) and H the set of all HFEs.

Definition 2[15]. For a HFE h, $s\left(h\right)=\frac{1}{\#h}\sum_{\gamma\in h}\gamma$ is called the score function of h, where $\#h$ is the number of the elements in h. For two HFEs h_1 and h_2, if $s\left(h_1\right)>s\left(h_2\right)$, then $h_1>h_2$; if $s\left(h_1\right)=s\left(h_2\right)$, then $h_1=h_2$.

Based on the relationship between the HFEs and IFVs, Xia and Xu[15] define some new operations on the HFEs h_1 and h_2:

$h_1^{\lambda}=\bigcup_{\gamma_1\in h_1}\left\{\gamma_1^{\lambda}\right\}$;

$\lambda h_1=\bigcup_{\gamma_1\in h_1}\left\{1-\left(1-\gamma_1\right)^{\lambda}\right\}$;

$h_1\oplus h_2=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\gamma_1+\gamma_2-\gamma_1\gamma_2\right\}$;

$h_1\otimes h_2=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\gamma_1\gamma_2\right\}$.

Merigó[22-23] developed the probabilistic weighted average (PWA) operator which is an aggregation operator that unifies the probability and the weighted average in the same formulation considering the degree of importance that each concept has in the aggregation.

Definition 3. An PWA operator of dimension n is a mapping PWA: $R^n\rightarrow R$, such that

$$\text{PWA}\left(a_1,a_2,\cdots,a_n\right)=\sum_{j=1}^{n}v_j a_j \tag{2}$$

where $\omega=\left(\omega_1,\omega_2,\cdots,\omega_n\right)^T$ be the weight vector of $a_j\left(j=1,2,\cdots,n\right)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$, and a probabilistic weight $p_j>0$, $\sum_{j=1}^{n}p_j=1$, $v_j=\beta p_j+\left(1-\beta\right)\omega_j$ with $\beta\in[0,1]$ and v_j is the weight that unifies probabilities and WAs in the same formulation,

In order to develop the analysis, Merigó[24] used in the same formulation the weights of the OWA operator and the probabilistic information and proposed the immediate probability OWA(IP-OWA) operator. It can be defined as follows.

Definition 4[24]. An IP-OWA operator of dimension n is a mapping IP-OWA: $R^n \rightarrow R$, that has an associated weight vector $w=(w_1,w_2,\cdots,w_n)^T$ such that $w_j>0$ and $\sum_{j=1}^{n} w_j=1$. Furthermore,

$$\text{IP-OWA}(a_1,a_2,\cdots,a_n)=\sum_{j=1}^{n}\hat{p}_j a_{\sigma(j)} \tag{3}$$

where $(\sigma(1),\sigma(2),\cdots,\sigma(n))$ is a permutation of $(1,2,\cdots,n)$, such that $\alpha_{\sigma(j-1)}\geq\alpha_{\sigma(j)}$ for all $j=2,\cdots,n$, each a_i has associated a probability p_i, p_j is the associated probability of $\alpha_{\sigma(j)}$, and $\hat{p}_j=\dfrac{w_j p_j}{\sum_{j=1}^{n} w_j p_j}$.

Definition 5[22-27]. An POWA operator of dimension n is a mapping PWA: $R^n \rightarrow R$ that has an associated weight vector $w=(w_1,w_2,\cdots,w_n)^T$ such that $w_j>0$ and $\sum_{j=1}^{n} w_j=1$, according to the following formula:

$$\text{POWA}(a_1,a_2,\cdots,a_n)=\sum_{j=1}^{n}\hat{p}_j a_{\sigma(j)} \tag{4}$$

where $(\sigma(1),\sigma(2),\cdots,\sigma(n))$ is a permutation of $(1,2,\cdots,n)$, such that $\alpha_{\sigma(j-1)}\geq\alpha_{\sigma(j)}$ for all $j=2,\cdots,n$, each a_i has associated a probability p_i, with $\sum_{i=1}^{n} p_i=1$ and $p_i\in[0,1]$, $\hat{p}_j=\beta p_j+(1-\beta)w_j$ with $\beta\in[0,1]$ and p_j is the associated probability of $\alpha_{\sigma(j)}$.

3. Some hesitant fuzzy Hamacher aggregating operator with immediate probabilities

Based on the aggregation principle for HFEs, Xia and Xu [15] developed the hesitant fuzzy weighted averaging(HFWA) operator and hesitant fuzzy ordered weighted averaging (HFOWA) operator.

Definition 6[15]. Let $h_j\,(j=1,2,\cdots,n)$ be a collection of HFEs. The hesitant fuzzy weighted averaging (HFWA) operator is a mapping $H^n \rightarrow H$ such that

$$HFWA(h_1,h_2,\cdots,h_n)=\overset{n}{\underset{j=1}{\oplus}}(\omega_j h_j)=\cup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\left\{1-\prod_{j=1}^{n}(1-\gamma_j)^{\omega_j}\right\} \tag{5}$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ be the weight vector of $h_j\,(j=1,2,\cdots,n)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$.

Definition 7[15]. Let $h_j\,(j=1,2,\cdots,n)$ be a collection of HFEs, the hesitant fuzzy ordered weighted averaging (HFOWA) operator of dimension n is a mapping HFOWA: $H^n \rightarrow H$, that has an associated weight vector $w=(w_1,w_2,\cdots,w_n)^T$ such that $w_j>0$ and $\sum_{j=1}^{n} w_j=1$. Furthermore,

$$\begin{aligned}HFOWA(h_1,h_2,\cdots,h_n)&=\overset{n}{\underset{j=1}{\oplus}}\left(w_j h_{\sigma(j)}\right)\\&=\cup_{\gamma_{\sigma(1)}\in h_{\sigma(1)},\gamma_{\sigma(2)}\in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)}\in h_{\sigma(n)}}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{w_j}\right\}\end{aligned} \tag{6}$$

where $\left(\sigma(1),\sigma(2),\cdots,\sigma(n)\right)$ is a permutation of $\left(1,2,\cdots,n\right)$, such that $h_{\sigma(j-1)} \geq h_{\sigma(j)}$ for all $j=2,\cdots,n$.

Motivated by the arithmetic aggregation operators[28], the Hamacher product $\otimes$ and the Hamacher sum $\oplus$, then the generalized intersection and union on two HFEs h_1 and h_2 become the Hamacher product(denoted by $h_1 \otimes h_2$) and Hamacher sum(denoted by $h_1 \oplus h_2$) of two HFEs h_1 and h_2, respectively, as follows:

(1) $h_1 \oplus h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \dfrac{\gamma_1 + \gamma_2 - \gamma_1\gamma_2 - (1-\gamma)\gamma_1\gamma_2}{1-(1-\gamma)\gamma_1\gamma_2} \right\}$;

(2) $h_1 \otimes h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \dfrac{\gamma_1\gamma_2}{\gamma + (1-\gamma)\left(\gamma_1 + \gamma_2 - \gamma_1\gamma_2\right)} \right\}$.

(3) $\lambda h_1 = \bigcup_{\gamma_1 \in h_1} \left\{ \dfrac{\left(1+(\gamma-1)\gamma_1\right)^{\lambda} - \left(1-\gamma_1\right)^{\lambda}}{\left(1+(\gamma-1)\gamma_1\right)^{\lambda} + (\gamma-1)\left(1-\gamma_1\right)^{\lambda}} \right\}, \lambda > 0$;

(4) $\left(h_1\right)^{\lambda} = \bigcup_{\gamma_1 \in h_1} \left\{ \dfrac{\gamma\left(\gamma_1\right)^{\lambda}}{\left(1+(\gamma-1)(1-\gamma_1)\right)^{\lambda} + (\gamma-1)\left(\gamma_1\right)^{\lambda}} \right\}, \lambda > 0$.

In the following, we shall develop some hesitant fuzzy Hamacher aggregation operator with immediate probabilities. These operators include: probability hesitant fuzzy Hamacher weighted average (P-HFHWA) operator, immediate probability hesitant fuzzy Hamacher ordered weighted average (IP-HFOWA) operator and probability hesitant fuzzy Hamacher ordered weighted average (P-HFOWA) operator.

Definition 8. Let $h_j \left(j=1,2,\cdots,n \right)$ be a collection of HFEs, an P-HFHWA operator of dimension n is a mapping P-HFHWA: $Q^n \to Q$, such that

$$\text{P-HFHWA}_{\hat{v},\lambda}\left(h_1,h_2,\cdots,h_n\right) = \mathop{\oplus}_{j=1}^{n}\left(\hat{v}_j h_j\right) = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \cdots, \gamma_n \in h_n} \left\{ \frac{\prod_{j=1}^{n}\left(1+(\lambda-1)\gamma_j\right)^{\hat{v}_j} - \prod_{j=1}^{n}\left(1-\gamma_j\right)^{\hat{v}_j}}{\prod_{j=1}^{n}\left(1+(\lambda-1)\gamma_j\right)^{\hat{v}_j} + (\lambda-1)\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\hat{v}_j}} \right\}, \lambda > 0. \tag{7}$$

where $\omega = \left(\omega_1,\omega_2,\cdots,\omega_n\right)^T$ be the weight vector of $a_j \left(j=1,2,\cdots,n \right)$, and $\omega_j > 0$, $\sum_{j=1}^{n}\omega_j = 1$, and a probabilistic weight $p_j > 0$, $\sum_{j=1}^{n} p_j = 1$, $\hat{v}_j = \beta p_j + \left(1-\beta\right)\omega_j$ with $\beta \in \left[0,1\right]$ and $\hat{v}_j$ is the weight that unifies probabilities and HFHWAs in the same formulation.

The IP-HFHOWA operator is an aggregation operator that uses probabilities and HOWAs in the same formulation and information represented with hesitant fuzzy information.

Definition 9. Let $h_j \left(j=1,2,\cdots,n \right)$ be a collection of HFEs, an IP-HFHOWA operator of dimension n is a mapping IP-HFHOWA: $Q^n \to Q$, that has an associated weight vector $w = \left(w_1,w_2,\cdots,w_n\right)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$\text{IP-HFHOWA}_{\hat{p},\lambda}\left(h_1,h_2,\cdots,h_n\right)=\overset{n}{\underset{j=1}{\oplus}}\left(\hat{p}_j h_{\sigma(j)}\right)$$
$$=\bigcup_{\gamma_{\sigma(1)}\in h_{\sigma(1)},\gamma_{\sigma(2)}\in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)}\in h_{\sigma(n)}}\left\{\frac{\prod_{j=1}^{n}\left(1+(\lambda-1)\gamma_{\sigma(j)}\right)^{\hat{p}_j}-\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{\hat{p}_j}}{\prod_{j=1}^{n}\left(1+(\lambda-1)\gamma_{\sigma(j)}\right)^{\hat{p}_j}+(\lambda-1)\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{\hat{p}_j}}\right\} \tag{8}$$

where $\left(\sigma(1),\sigma(2),\cdots,\sigma(n)\right)$ is a permutation of $\left(1,2,\cdots,n\right)$, such that $h_{\sigma(j-1)}\geq h_{\sigma(j)}$ for all $j=2,\cdots,n$, each a_i has associated a probability p_i, p_j is the associated probability of $h_{\sigma(j)}$, and $\hat{p}_j=\frac{w_j p_j}{\sum_{j=1}^{n} w_j p_j}$.

The P-HFHOWA operator unifies the probability and the HOWA operator in the same formulation considering the degree of importance of each concept in the aggregation. It also uses information represented in the form of hesitant fuzzy information.

Definition 10. Let $h_j\left(j=1,2,\cdots,n\right)$ be a collection of HFEs, an P-HFHOWA operator of dimension n is a mapping P-HFHOWA: $Q^n\to Q$, that has an associated weight vector $w=\left(w_1,w_2,\cdots,w_n\right)^T$ such that $w_j>0$ and $\sum_{j=1}^{n} w_j=1$. Furthermore,

$$\text{P-HFHOWA}_{\tilde{p},\lambda}\left(h_1,h_2,\cdots,h_n\right)=\overset{n}{\underset{j=1}{\oplus}}\left(\tilde{p}_j h_{\sigma(j)}\right)$$
$$=\bigcup_{\gamma_{\sigma(1)}\in h_{\sigma(1)},\gamma_{\sigma(2)}\in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)}\in h_{\sigma(n)}}\left\{\frac{\prod_{j=1}^{n}\left(1+(\lambda-1)\gamma_{\sigma(j)}\right)^{\hat{p}_j}-\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{\hat{p}_j}}{\prod_{j=1}^{n}\left(1+(\lambda-1)\gamma_{\sigma(j)}\right)^{\hat{p}_j}+(\lambda-1)\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{\hat{p}_j}}\right\},\lambda>0 \tag{9}$$

where $\left(\sigma(1),\sigma(2),\cdots,\sigma(n)\right)$ is a permutation of $\left(1,2,\cdots,n\right)$, such that $h_{\sigma(j-1)}\geq h_{\sigma(j)}$ for all $j=2,\cdots,n$, each $\tilde{a}_i$ has associated a probability p_i, with $\sum_{i=1}^{n} p_i=1$ and $p_i\in\left[0,1\right]$, $\tilde{p}_j=\beta p_j+\left(1-\beta\right)w_j$ with $\beta\in\left[0,1\right]$ and p_j is the associated probability of $\tilde{\alpha}_{\sigma(j)}$.

4. Conclusion

In this paper, we investigate the probabilistic decision making problems with hesitant fuzzy information, some new probabilistic decision making analysis methods are developed. Then, we have developed some new probability aggregation operators with hesitant fuzzy information: probability hesitant fuzzy Hamacher weighted average(P-HFHWA) operator, immediate probability hesitant fuzzy Hamacher ordered weighted average (IP-HFOWA) operator and probability hesitant fuzzy Hamacher ordered weighted average (P-HFOWA) operator.

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