An approach to hesitant fuzzy multiple attribute decision making and their applications to performance evaluation of emergency management

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Abstract

The Maclaurin symmetric mean (MSM) was originally introduced by Maclaurin. The prominent characteristic of the MSM is that it can capture the interrelationship among the multi-input arguments. However, the researches on MSM are very rare, especially in fuzzy decision making. In this paper, we investigate the MSM operator and extend the MSM operator to hesitant fuzzy environment and develop the hesitant fuzzy Maclaurin symmetric mean (HFMSM) operator and hesitant fuzzy weighted Maclaurin symmetric mean (HFWMSM) operator. Based on HFWMSM operator, an approach to multiple attribute decision making problems with hesitant fuzzy information is developed. Finally, an illustrative example for performance evaluation of emergency management is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords

hesitant fuzzy sets; operational laws; hesitant fuzzy Maclaurin symmetric mean (HFMSM) operator; hesitant fuzzy weighted Maclaurin symmetric mean (HFWMSM) operator; emergency management.

1. Introduction

Atanassov [1,2] introduced the concept of intuitionistic fuzzy set(IFS), which is a generalization of the concept of fuzzy set [3]. The intuitionistic fuzzy set has received more and more attention since its appearance [4-18]. Furthermore, Torra and Narukawa[19] and Torra[20] proposed the hesitant fuzzy set which permits the membership having a set of possible values and discussed the relationship between hesitant fuzzy set and intuitionistic fuzzy set, and showed that the envelope of hesitant fuzzy set is an intuitionistic fuzzy set. Xia and Xu[21] gave an intensive study on hesitant fuzzy information aggregation techniques and their application in decision making. Xu and Xia[22] defined the distance and correlation measures for hesitant fuzzy information and then discuss their properties in detail. Xu and Xia[23] proposed a variety of distance measures for hesitant fuzzy sets, based on which the corresponding similarity measures can be obtained and further developed a number of hesitant ordered weighted distance measures and hesitant ordered weighted similarity measures which can alleviate the influence of unduly large (or small) deviations on the aggregation results by assigning them low (or high) weights. Gu et al.[24] utilized the hesitant fuzzy weighted averaging (HFWA) operator to investigat the evaluation model for risk investment with hesitant fuzzy information. Xu et al. [25] developed several series of aggregation operators for hesitant fuzzy information with the aid of quasi-arithmetic means. Wang et al.[26] proposed the generalized hesitant fuzzy hybrid weighted distance (GHFHWD) measure, which is based on the generalized hesitant fuzzy weighted distance (GHFWD) measure and the generalized hesitant fuzzy ordered weighted distance (GHFOWD) measure[38] and studied some desirable properties of the GHFHWD measure. Zhu et al.[27] further defined the hesitant fuzzy geometric Bonferroni mean (HFGBM) and the hesitant fuzzy Choquet geometric Bonferroni mean (HFCGBM) and gave the definition of hesitant fuzzy geometric Bonferroni element (HFGBE), which is considered as the basic calculational unit in the HFGBM and reflects the conjunction between two aggregated arguments. In addition, the weighted hesitant fuzzy geometric Bonferroni mean (WHFGBM) and the weighted hesitant fuzzy Choquet geometric Bonferroni mean (WHFCGBM) are proposed considering the importance of each argument and the correlations among them. Wei et al.[28] proposed two hesitant fuzzy Choquet integral aggregation operators: hesitant fuzzy choquet ordered averaging (HFCOA) operator and hesitant fuzzy choquet ordered geometric (HFCOG) operator and applied the HFCOA and HFCOG operators to multiple attribute decision making with hesitant fuzzy information. Furthermore, they proposed the generalized hesitant fuzzy choquet ordered averaging (GHFCOA) operator and generalized hesitant fuzzy choquet ordered geometric (GHFCOG) operator. Wei[29] developed some prioritized aggregation operators for aggregating hesitant fuzzy information, and then apply them to develop some models for hesitant fuzzy multiple attribute decision making (MADM) problems in which the attributes are in different priority level.

In this paper, we investigate the MSM operator and extend the MSM operator to hesitant fuzzy environment and develop the hesitant fuzzy Maclaurin symmetric mean (HFMSM) operator and hesitant fuzzy weighted Maclaurin symmetric mean (HFWMSM) operator. Based on HFWMSM operator, an approach to multiple attribute decision making problems with hesitant fuzzy information is developed. In order to do so, the remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to interval-valued intuitionistic fuzzy sets. In Section 3 some new aggregation operator called the hesitant fuzzy Maclaurin symmetric mean (HFMSM) operator and hesitant fuzzy weighted Maclaurin symmetric mean (HFWMSM) operator are proposed. In Section 4, An HFWMSM operator-based approach is developed to solve the MADM under the hesitant fuzzy environment. In Section 5, an illustrative example for performance evaluation of emergency management is pointed out. In Section 6 we conclude the paper and give some remarks.

2. Preliminaries

Atanassov[1] extended the fuzzy set to the IFS. However, when giving the membership degree of an element, the difficulty of establishing the membership degree is not because we have a margin of error, or some possibility distribution on the possibility values, but because we have several possible values. For such cases, Torra[20] proposed another generation of FS.

Definition 1[20]. Given a fixed set X, then a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a sunset of $[0,1]$.

To be easily understood, Xia & Xu[21]express the HFS by mathematical symbol:

$$E=\left(\left\langle x,h_E\left(x\right)\right\rangle \middle| x\in X\right), \tag{1}$$

where $h_E\left(x\right)$ is a set of some values in $[0,1]$, denoting the possible membership degree of the element $x\in X$ to the set E. For convenience, Xu call $h=h_E\left(x\right)$ a hesitant fuzzy element(HFE) and H the set of all HFEs.

Definition 2[21-22]. For a HFE h, $s\left(h\right)=\frac{1}{\#h}\sum_{\gamma\in h}\gamma$ is called the score function of h, where $\#h$ is the number of the elements in h. For two HFEs h_1 and h_2, if $s\left(h_1\right)>s\left(h_2\right)$, then $h_1>h_2$; if $s\left(h_1\right)=s\left(h_2\right)$, then $h_1=h_2$.

Based on the relationship between the HFEs and IFVs, Xia and Xu[21] define some new operations on the HFEs h, h_1 and h_2:

$h^{\lambda}=\bigcup_{\gamma\in h}\left\{\gamma^{\lambda}\right\}$;

(2) $\lambda h=\bigcup_{\gamma\in h}\left\{1-\left(1-\gamma\right)^{\lambda}\right\}$;

(3) $h_1 \oplus h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$;

(4) $h_1 \otimes h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1\gamma_2\}$.

3. Hesitant fuzzy Maclaurin symmetric mean

The Maclaurin symmetric mean (MSM) was originally introduced by Maclaurin in [30], which is a useful technique characterized by the ability to capture the interrelationship among the multi-input arguments. The Definition of MSM is defined as follows.

Definition 3[30]. Let $a_j\,(j=1,2,\cdots,n)$ be a collection of nonnegative numbers, and $k=1,2,\cdots,n$. If

$$\mathrm{MSM}^{(k)}(a_1,a_2,\cdots,a_n)=\left(\frac{\sum_{1\le i_1\le\cdots\le i_k\le n}\prod_{j=1}^{k}a_{i_j}}{C_n^k}\right)^{1/k} \tag{2}$$

then $\mathrm{MSM}^{(k)}$ is called the Maclaurin symmetric mean (MSM), where $(i_1,i_2,\cdots,i_k)$ traversal all the k-tuple combination of $(1,2,\cdots,n)$, C_n^k is the binomial coefficient.

Obviously, the MSM have the following properties:

$\mathrm{MSM}^{(k)}(0,0,\cdots,0)=0$;

$\mathrm{MSM}^{(k)}(a,a,\cdots,a)=a$;

$\mathrm{MSM}^{(k)}(a_1,a_2,\cdots,a_n)\le\mathrm{MSM}^{(k)}(b_1,b_2,\cdots,b_n)$, if $a_i\le b_i$ for all i;

$\min_i\{a_i\}\le\mathrm{MSM}^{(k)}(a_1,a_2,\cdots,a_n)\le\max_i\{a_i\}$.

In the following, we shall extend MSM to hesitant fuzzy environment, and investigate some of their desirable properties.

Definition 4. Let $h_j\,(j=1,2,\cdots,n)$ be a collection of HFEs, and let HFMSM: $Q^n\to Q$, if

$$\mathrm{HFMSM}^{(k)}(h_1,h_2,\cdots,h_n)=\bigcup_{\gamma_{i_1}\in h_1,\gamma_{i_2}\in h_2,\cdots,\gamma_{i_n}\in h_n}\left\{\left(\frac{\bigoplus_{1\le i_1\le\cdots\le i_k\le n}\left(\bigotimes_{j=1}^{k}\gamma_{i_j}\right)}{C_n^k}\right)^{1/k}\right\} \tag{3}$$

then $\mathrm{HFMSM}^{(k)}$ is called the hesitant fuzzy Maclaurin symmetric mean (HFMSM), where $(i_1,i_2,\cdots,i_k)$ traversal all the k-tuple combination of $(1,2,\cdots,n)$, C_n^k is the binomial coefficient.

According to the operations of hesitant fuzzy numbers, we can derive the following Theorem 1.

Theorem 1. Let $h_j\,(j=1,2,\cdots,n)$ be a collection of HFEs, and let HFMSM: $Q^n\to Q$, if

$$\begin{aligned}&\mathrm{HFMSM}^{(k)}(h_1,h_2,\cdots,h_n)\\&=\bigcup_{\gamma_{i_1}\in h_1,\gamma_{i_2}\in h_2,\cdots,\gamma_{i_n}\in h_n}\left\{\left(\frac{\bigoplus_{1\le i_1\le\cdots\le i_k\le n}\left(\bigotimes_{j=1}^{k}\gamma_{i_j}\right)}{C_n^k}\right)^{1/k}\right\}\\&=\bigcup_{\gamma_{i_1}\in h_1,\gamma_{i_2}\in h_2,\cdots,\gamma_{i_n}\in h_n}\left\{\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\\le i_k\le n}}\left(1-\prod_{j=1}^{k}\gamma_{i_j}\right)\right)^{C_n^k}\right)^{1/k}\right\}\end{aligned} \tag{4}$$

then $\mathrm{HFMSM}^{(k)}$ is called the hesitant fuzzy Maclaurin symmetric mean (HFMSM), where $(i_1, i_2, \cdots, i_k)$ traversal all the k-tuple combination of $(1, 2, \cdots, n)$, C_n^k is the binomial coefficient.

The HFMSM operator has the following properties.

Theorem 2 (Commutativity). Let $h_j\ (j = 1, 2, \cdots, n)$ $(j = 1, 2, \cdots, n)$ be a collection of HFEs, if

$$\mathrm{HFMSM}^{(k)}(h_1, h_2, \cdots, h_n) = \mathrm{HFMSM}^{(k)}(h_1', h_2', \cdots, h_n')$$

where $(\tilde{a}_1', \tilde{a}_2', \cdots, \tilde{a}_n')$ is any permutation of $(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n)$.

Theorem 3 (Monotonicity). Let $h_j\ (j = 1, 2, \cdots, n)$ $(j = 1, 2, \cdots, n)$ be a collection of HFEs,

$$\mathrm{HFMSM}^{(k)}(h_1, h_2, \cdots, h_n) \le \mathrm{HFMSM}^{(k)}(h_1', h_2', \cdots, h_n')$$

if $h_j \le h_j'$.

In the following, we shall discuss some special cases of the HFMSM operator by taking different values of the parameter k.

If $k = 1$, then based on the definition of HFMSM operator, we have

$$\begin{aligned}
&\mathrm{HFMSM}^{(1)}(h_1, h_2, \cdots, h_n) \\
&= \bigcup_{\gamma_{i_1} \in h_1, \gamma_{i_2} \in h_2, \cdots, \gamma_{i_n} \in h_n} \left\{ \left(\frac{\bigoplus_{1 \le i_1 \le \cdots \le i_k \le n} \left(\bigotimes_{j=1}^{1} \gamma_{i_j} \right)}{C_n^k} \right)^{1/1} \right\} \\
&= \bigcup_{\gamma_{i_1} \in h_1, \gamma_{i_2} \in h_2, \cdots, \gamma_{i_n} \in h_n} \left\{ \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ \le i_k \le n}} \left(1 - \prod_{j=1}^{1} \gamma_{i_j} \right) \right)^{C_n^1} \right)^{1/1} \right\} \\
&= \bigcup_{\gamma_{i_1} \in h_1, \gamma_{i_2} \in h_2, \cdots, \gamma_{i_n} \in h_n} \left\{ 1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ \le i_k \le n}} \left(1 - \gamma_{i_j} \right) \right)^{C_n^1} \right\}
\end{aligned}$$

If $k = 2$, then based on the definition of HFMSM operator, we have

$$\begin{aligned}
&\mathrm{HFMSM}^{(2)}(h_1, h_2, \cdots, h_n) \\
&= \bigcup_{\gamma_{i_1} \in h_1, \gamma_{i_2} \in h_2, \cdots, \gamma_{i_n} \in h_n} \left\{ \left(\frac{\bigoplus_{1 \le i_1 \le \cdots \le i_k \le n} \left(\bigotimes_{j=1}^{2} \gamma_{i_j} \right)}{C_n^2} \right)^{1/2} \right\} \\
&= \bigcup_{\gamma_{i_1} \in h_1, \gamma_{i_2} \in h_2, \cdots, \gamma_{i_n} \in h_n} \left\{ \left(1 - \left(\prod_{\substack{1 \le i_1 \le \cdots \\ \le i_k \le n}} \left(1 - \prod_{j=1}^{2} \gamma_{i_j} \right) \right)^{C_n^2} \right)^{1/2} \right\} \\
&= \bigcup_{\gamma_{i_1} \in h_1, \gamma_{i_2} \in h_2, \cdots, \gamma_{i_n} \in h_n} \left\{ \left(1 - \prod_{\substack{i_1 = i_2 = 1 \\ i_1 \ne i_2}}^{n} \left(1 - \gamma_{i_1} \gamma_{i_2} \right)^{1/n(n-1)} \right)^{1/2} \right\}
\end{aligned}$$

It can be seen that HFMSM operator does not consider the importance of the aggregated arguments. Thus, in the following, we shall develop the hesitant fuzzy weighted Maclaurin symmetric mean (HFWMSM) operator.

Definition 5. Let $h_j\left(j=1,2,\cdots,n\right)$ be a collection of HFEs, and $w=\left(w_1,w_2,\cdots,w_n\right)^T$ is the weight vector of h_j, such that $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$, and let HFWMSM: $Q^n\rightarrow Q$, if

$$\mathrm{HFWMSM}^{(k)}\left(h_1,h_2,\cdots,h_n\right)=\bigcup_{\gamma_{i_1}\in h_1,\gamma_{i_2}\in h_2,\cdots,\gamma_{i_n}\in h_n}\left\{\left(\frac{\bigoplus\limits_{1\le i_1\le\cdots\le i_k\le n}\left(\bigotimes\limits_{j=1}^{k}w_{i_j}\gamma_{i_j}\right)}{C_n^k}\right)^{1/k}\right\} \tag{5}$$

then $\mathrm{HFWMSM}^{(k)}$ is called the hesitant fuzzy weighted Maclaurin symmetric mean (HFWMSM), where $\left(i_1,i_2,\cdots,i_k\right)$ traversal all the k-tuple combination of $\left(1,2,\cdots,n\right)$, C_n^k is the binomial coefficient and $w_{i_j}=\frac{s\left(\gamma_{i_j}\right)}{\sum_{j=1}^{n}s\left(\gamma_{i_j}\right)}$, $j=1,2,\cdots,n$, $\sum_{j=1}^{n}w_{i_j}=1$.

According to the operations of hesitant fuzzy numbers, we can derive the following Theorem 1.

Theorem 4. Let $h_j\left(j=1,2,\cdots,n\right)$ be a collection of HFEs, and let HFMSM: $Q^n\rightarrow Q$, if

$$\begin{aligned}&\mathrm{HFWMSM}^{(k)}\left(h_1,h_2,\cdots,h_n\right)\\&=\bigcup_{\gamma_{i_1}\in h_1,\gamma_{i_2}\in h_2,\cdots,\gamma_{i_n}\in h_n}\left\{\left(\frac{\bigoplus\limits_{1\le i_1\le\cdots\le i_k\le n}\left(\bigotimes\limits_{j=1}^{k}w_{i_j}\gamma_{i_j}\right)}{C_n^k}\right)^{1/k}\right\}\\&=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\left\{\left(1-\left(\prod_{\substack{1\le i_1\le\cdots\\\le i_k\le n}}\left(1-\prod_{j=1}^{k}\left(1-\left(1-\gamma_{i_j}\right)^{w_{i_j}}\right)\right)\right)^{C_n^k}\right)^{1/k}\right\}\end{aligned} \tag{6}$$

then $\mathrm{HFWMSM}^{(k)}$ is called the hesitant fuzzy weighted Maclaurin symmetric mean (HFWMSM), where $\left(i_1,i_2,\cdots,i_k\right)$ traversal all the k-tuple combination of $\left(1,2,\cdots,n\right)$, C_n^k is the binomial coefficient.

4. An approach to hesitant fuzzy multiple attribute decision making based on HFWMSM operator

The following assumptions or notations are used to represent the MADM problems for performance evaluation of emergency management with hesitant fuzzy information. Let $A=\left\{A_1,A_2,\cdots,A_m\right\}$ be a discrete set of alternatives, and $G=\left\{G_1,G_2,\cdots,G_n\right\}$ be the state of nature. If the decision makers provide several values for the alternative A_i under the state of nature G_j with anonymity, these values can be considered as a hesitant fuzzy element h_{ij}. In the case where two decision makers provide the same value, then the value emerges only once in h_{ij}. Suppose that the decision matrix $H=\left(h_{ij}\right)_{m\times n}$ is the hesitant fuzzy decision matrix, where $h_{ij}\left(i=1,2,\cdots,m,\ j=1,2,\cdots,n\right)$ are in the form of HFEs.

In the following, we apply the HFWMSM operator to multiple attribute decision making for performance evaluation of emergency management based on hesitant fuzzy information. The method involves the following steps:

Step 1. We utilize the decision information given in matrix H , and the HFWMSM operator

$$h_i = \mathrm{HFWMSM}^{(k)}\left(h_{i1}, h_{i2}, \cdots, h_{in}\right), \qquad i = 1, 2, \cdots, m. \tag{7}$$

to derive the overall preference values $h_i\left(i=1,2,\cdots,m\right)$ of the alternative A_i .

Step 2. Calculate the scores $S\left(h_i\right)\left(i=1,2,\cdots,m\right)$ of the overall hesitant fuzzy preference values $h_i\left(i=1,2,\cdots,m\right)$ to rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ and then to select the best one(s).

Step 3. Rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ and select the best one(s) in accordance with $S\left(h_i\right)\left(i=1,2,\cdots,m\right)$.

Step 4. End.

5. Numerical example

The process of human development is always accompanied by a number of risks, all types of emergencies occur frequently, and the destructive power of emergencies is increasing. Also, due to the correlation, dependency, and coupling between different social subsystems, a partial or routine emergency more likely to become a highly destructive unconventional emergency. In recent years, China suffered a series of large-scale emergencies. It not only caused heavy casualties, but also leaded to serious economic losses which are about 3.5% of GDP. Over the same time, those unconventional emergencies posed some serious threat to social stability and sustainable development of economic. Improving the government’s response ability to unconventional emergency contributes to increase the governance ability, and helps to enhance government’s social administration capacity, and it is also the requirements of building a service-oriented government. Strengthen the research on management theory of unconventional emergencies, is not only China’s needs, but also the whole world. The correlation and dependence are enhanced constantly among various social function systems. All kinds of unexpected events are more likely to turn into unconventional emergencies with large scale and serious consequences. Unconventional emergency management research has become a pivotal frontier and multidisciplinary field. One of major research goals in the field is how to obtain some valuable information rapidly and accurately from data, information and knowledge included in unconventional emergencies, and acquire multi-dimensional multi-variable information visualization expression through data processing and information fusion to support the intelligent decision-making process in emergency response. Complex system science, emergency management and information visualization were made as the theoretical basis. The dissertation integrated multi-disciplinary theories and adopted the methods of system science, management science, information science, mathematics, deductive induction and empirical analysis. The research studied some key issues like information system, information flow, data characteristics, etc in unconventional emergency management, and built unconventional emergency management visual information system, and proposed some models and methods about visual information fusion. It has important theoretical and practical significance. Thus, in this section we shall present a numerical example for performance evaluation of emergency management with hesitant fuzzy information in order to illustrate the method proposed in this paper. There are five prospect cities $A_i\left(i=1,2,3,4,5\right)$ for performance evaluation of emergency management according to four attributes $G_j\left(j=1,2,3,4\right)$. The four attributes include the social public satisfaction $\left(G_1\right)$, resources and environment protection $\left(G_2\right)$, internal organization and management $\left(G_3\right)$ and the ability to learn and grow $\left(G_4\right)$, respectively. In order to avoid influence each other, the decision makers are required to evaluate the five possible cities $A_i\left(i=1,2,\cdots,5\right)$ under the above four attributes in anonymity and the decision matrix $H=\left(h_{ij}\right)_{m\times n}$ is presented in Table 1, where $h_{ij}\left(i=1,2,3,4,5,\ j=1,2,3,4\right)$ are in the form of HFEs.

Table 1 Hesitant fuzzy decision matrix

	G_1	G_2	G_3	G_4
A_1	(0.2,0.3)	(0.2,0.5,0.6)	(0.7,0.8)	(0.1,0.3,0.4)
A_2	(0.2,0.4,0.5)	(0.3,0.6)	(0.2,0.4,0.6,0.7)	(0.5,0.8)
A_3	(0.5,0.6,0.7)	(0.2,0.4)	(0.8,0.9)	(0.4,0.5,0.8)
A_4	(0.3,0.7)	(0.2,0.3,0.6)	(0.6,0.8)	(0.2,0.4,0.7)
A_5	(0.3,0.7,0.9)	(0.4,0.5,0.6)	(0.2,0.6)	(0.6,0.7)

Suppose that the weight vector of the attribute is: $w=\left(0.3,0.1,0.2,0.4\right)$

Then, we utilize the approach developed to get the most desirable alternative(s).

Step 1. Utilize the weight vector $w=\left(0.3,0.1,0.2,0.4\right)$ and HFWMSM operator, we obtain the overall values h_i of the cities $A_i\left(i=1,2,\cdots,m\right)$.

Step 2. calculate the scores $S\left(h_i\right)\left(i=1,2,\cdots,m\right)$ of the overall hesitant fuzzy values $h_i\ \left(i=1,2,\cdots,m\right)$

$$S\left(h_1\right)=0.7103, S\left(h_2\right)=0.5802, S\left(h_3\right)=0.6365$$

$$S\left(h_4\right)=0.5987, S\left(h_5\right)=0.6022$$

Step 3. Rank all the cities $A_i\left(i=1,2,3,4,5\right)$ in accordance with the scores $S\left(\tilde{h}_i\right)$: $A_1\succ A_3\succ A_5\succ A_4\succ A_2$, and thus the most desirable city is A_1.

6. Conclusion

The Maclaurin symmetric mean (MSM) was originally introduced by Maclaurin. The prominent characteristic of the MSM is that it can capture the interrelationship among the multi-input arguments. However, the researches on MSM are very rare, especially in fuzzy decision making. In this paper, we investigate the MSM operator and extend the MSM operator to hesitant fuzzy environment and develop the hesitant fuzzy Maclaurin symmetric mean (HFMSM) operator and hesitant fuzzy weighted Maclaurin symmetric mean (HFWMSM) operator. Based on HFWMSM operator, an approach to multiple attribute decision making problems with hesitant fuzzy information is developed. Finally, an illustrative example for performance evaluation of emergency management is given to verify the developed approach and to demonstrate its practicality and effectiveness. In the future, we shall extend the proposed approached to other domain[31-45].

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