Geometric Bonferroni mean operators with fuzzy number intuitionistic fuzzy information and their applications to multiple attribute decision making

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Abstract

The aim of this paper is to investigate the fuzzy number intuitionistic fuzzy multiple attribute decision making problems based on the fuzzy number intuitionistic fuzzy sets and Geometric Bonferroni mean operator. Then, we have proposed two aggregation operators: fuzzy number intuitionistic fuzzy geometric Bonferroni mean (FNIFGBM) operator and fuzzy number intuitionistic fuzzy weighted geometric Bonferroni mean (FNIFWGBM) operator. Then, we have utilized FNIFWGBM operator to tackle the multiple attribute decision making problems with fuzzy number intuitionistic fuzzy information.

Keywords

Fuzzy number intuitionistic fuzzy numbers; geometric Bonferroni mean (GBM) operator; fuzzy number intuitionistic fuzzy geometric Bonferroni mean (FNIFGBM) operator; fuzzy number intuitionistic fuzzy weighted geometric Bonferroni mean (FNIFWGBM) operator

1. Introduction

Atanassov [1,2] introduced the concept of intuitionistic fuzzy set(IFS), which is a generalization of the concept of fuzzy set [3]. Later, Atanassov and Gargov [4-5] further introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers. Liu and Yuan[6] introduced the concept of fuzzy number intuitionistic fuzzy set(FNIFS) which fundamental characteristic of the FNIFS is that the values of its membership function and non-membership function are triangular fuzzy numbers. Wang[7] propose some aggregation operators, including fuzzy number intuitionistic fuzzy weighted geometric (FIFWG) operator, fuzzy number intuitionistic fuzzy ordered weighted geometric (FIFOWG) operator and fuzzy number intuitionistic fuzzy hybrid geometric (FIFHG) operator and develop an approach to multiple attribute group decision making (MAGDM) based on the FIFWG and the FIFHG operators with fuzzy number intuitionistic fuzzy information. Wang[8] propose the fuzzy number intuitionistic fuzzy weighted averaging (FIFWA) operator, fuzzy number intuitionistic fuzzy ordered weighted averaging (FIFOWA) operator and fuzzy number intuitionistic fuzzy hybrid aggregation (FIFHA) operator. Wei et al.[9] investigated some induced aggregating operators with fuzzy number intuitionistic fuzzy information. Wei et al.[10] investigated some aggregating operators based on the Choquet integral with fuzzy number intuitionistic fuzzy information. Wei et al.[11] proposed the induced Choquet integral with fuzzy number intuitionistic fuzzy information. Lin et al.[12] developed the fuzzy number intuitionistic fuzzy prioritized operators .

The aim of this paper is to investigate the fuzzy number intuitionistic fuzzy multiple attribute decision making problems based on the fuzzy number intuitionistic fuzzy sets and Geometric Bonferroni mean operator. Then, we have proposed two aggregation operators: fuzzy number intuitionistic fuzzy geometric Bonferroni mean (FNIFGBM) operator and fuzzy number intuitionistic fuzzy weighted geometric Bonferroni mean (FNIFWGBM) operator. Then, we have utilized FNIFWGBM operator to tackle the multiple attribute decision making problems with fuzzy number intuitionistic fuzzy information..

2. Preliminaries

Atanassov [1-2] extended the fuzzy set to the IFS, shown as follows:

Definition 1[1-2]. An IFS A in X is given by

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle | x\in X\} \tag{1}$$

where $\mu_A: X\to[0,1]$ and $\nu_A: X\to[0,1]$, with the condition $0\le\mu_A(x)+\nu_A(x)\le 1$, $\forall\, x\in X$. The numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to the set A. And let $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$, then $\pi_A(x)$ is called the degree of indeterminacy of x to A.

Definition 2[4-5]. Let X be an universe of discourse, An IVIFS $\tilde{A}$ over X is an object having the form:

$$\tilde{A}=\{\langle x,\tilde{\mu}_A(x),\tilde{\nu}_A(x)\rangle | x\in X\} \tag{2}$$

Where $\tilde{\mu}_A(x)\subset[0,1]$ and $\tilde{\nu}_A(x)\subset[0,1]$ are interval numbers, and $0\le\sup(\tilde{\mu}_A(x))+\sup(\tilde{\nu}_A(x))\le 1$, $\forall\, x\in X$ .For convenience, let $\tilde{\mu}_A(x)=[a,b]$, $\tilde{\nu}_A(x)=[c,d]$, so $\tilde{A}=([a,b],[c,d])$.

Liu and Yuan [16] introduced the concept of fuzzy number intuitionistic fuzzy set(FNIFS) which fundamental characteristic of the FNIFS is that the values of its membership function and non-membership function are triangular fuzzy numbers.

Definition 3[16]. Let X be an universe of discourse, An FNIFS $\tilde{A}$ over X is an object having the form:

$$\tilde{A}=\{\langle x,\tilde{t}_A(x),\tilde{f}_A(x)\rangle | x\in X\} \tag{3}$$

Where $\tilde{t}_A(x)\subset[0,1]$ and $\tilde{f}_A(x)\subset[0,1]$ are triangular fuzzy numbers, and

$$\tilde{t}_A(x)=\left(\tilde{t}_A^1(x),\tilde{t}_A^2(x),\tilde{t}_A^3(x)\right): X\to[0,1],\qquad \tilde{f}_A(x)=\left(\tilde{f}_A^1(x),\tilde{f}_A^2(x),\tilde{f}_A^3(x)\right): X\to[0,1]$$

$$0\le\tilde{t}_A^3(x)+\tilde{f}_A^3(x)\le 1,\ \forall\, x\in X$$

For convenience, let $\tilde{t}_A(x)=(a,b,c)$, $\tilde{f}_A(x)=(l,m,p)$, so $\tilde{A}=\langle(a,b,c),(l,m,p)\rangle$.

Definition 4[7-8]. Let $\tilde{a}_1=\langle(a_1,b_1,c_1),(l_1,m_1,p_1)\rangle$ and $\tilde{a}_2=\langle(a_2,b_2,c_2),(l_2,m_2,p_2)\rangle$ be two fuzzy number intuitionistic fuzzy values, then

(1) $\tilde{a}_1+\tilde{a}_2=\langle(a_1+a_2-a_1a_2,b_1+b_2-b_1b_2,c_1+c_2-c_1c_2),(l_1l_2,m_1m_2,p_1p_2)\rangle$;

(2) $\tilde{a}_1\times\tilde{a}_2=\langle(a_1a_2,b_1b_2,c_1c_2),\ (l_1+l_2-l_1l_2,m_1+m_2-m_1m_2,p_1+p_2-p_1p_2)\rangle$;

(3) $\lambda\tilde{a}_1=\left\langle\left(1-(1-a_1)^\lambda,1-(1-b_1)^\lambda,1-(1-c_1)^\lambda\right),\left(l_1^\lambda,m_1^\lambda,p_1^\lambda\right)\right\rangle,\lambda>0$;

(4) $\tilde{a}_1^{\lambda}=\left\langle\left(a_1^\lambda,b_1^\lambda,c_1^\lambda\right),\ \left(1-(1-l_1)^\lambda,1-(1-m_1)^\lambda,1-(1-p_1)^\lambda\right)\right\rangle,\lambda\ge 0$;

Definition 5[7]. Let $\tilde{a}=\langle(a,b,c),(l,m,p)\rangle$ be a fuzzy number intuitionistic fuzzy value, a score function S of a fuzzy number intuitionistic fuzzy value can be represented as follows:

$$S(\tilde{a}) = \frac{a+2b+c}{4} - \frac{l+2m+p}{4}, \quad S(\tilde{a}) \in [-1,1]. \tag{4}$$

Definition 6[7]. Let $\tilde{a} = \langle (a,b,c),(l,m,p) \rangle$ be a fuzzy number intuitionistic fuzzy value, an accuracy function H of an fuzzy number intuitionistic fuzzy value can be represented as follows:

$$H(\tilde{a}) = \frac{(a+2b+c)+(l+2m+p)}{4}, \quad H(\tilde{a}) \in [0,1]. \tag{5}$$

to evaluate the degree of accuracy of the fuzzy number intuitionistic fuzzy value $\tilde{a} = \langle (a,b,c),(l,m,p) \rangle$, where $H(\tilde{a}) \in [0,1]$. The larger the value of $H(\tilde{a})$, the more the degree of accuracy of the fuzzy number intuitionistic fuzzy valu $\tilde{a}$.

3. Geometric Bonferroni means operator with fuzzy number intuitionistic fuzzy information

In the following, Zhu et al.[13] studied on the geometric Bonferroni mean (GBM) which utilizing both the BM and the geometric mean (GM).

Definition 7[13]. Let $p,q \ge 0$ and $a_i\ (i=1,2,\cdots,n)$ be a collection of non-negative real numbers. Then the aggregation functions:

$$GBM^{p,q}(a_1,a_2,\cdots,a_n) = \frac{1}{p+q}\left(\prod_{\substack{i,j=1 \\ i\neq j}}^{n}(pa_i+qa_j)\right)^{\frac{1}{n(n-1)}} \tag{6}$$

is named the geometric Bonferroni mean (GBM) operator.

But the geometric Bonferroni mean (BM) operator [13] have usually been utilized when inputting arguments are belonged to the non-negative real numbers. Hence, we should expand the GBM operators to make itself suitable to be used in the situations where the input arguments are belonged to fuzzy number intuitionistic fuzzy information. Therefore, we focus on the problem of using the GBM operator under fuzzy number intuitionistic fuzzy environments. Based on Definition 7, we put forward the definition of the fuzzy number intuitionistic fuzzy geometric Bonferroni mean (FNIFGBM) operator.

For convenience, let Q be the set of all fuzzy number intuitionistic fuzzy values.

Definition 8. Let $\tilde{a}_j = \langle (a_j,b_j,c_j),(l_j,m_j,p_j) \rangle$ $(j=1,2,\cdots,n)$ be a collection of fuzzy number intuitionistic fuzzy values, and let $\mathrm{FNIFGBM}: Q^n \to Q$, if

$$FNIFGBM(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n) = \frac{1}{p+q}\left(\bigotimes_{\substack{i,j=1 \\ i\neq j}}^{n}(p\tilde{a}_i+q\tilde{a}_j)\right)^{\frac{1}{n(n-1)}} \tag{7}$$

where $p,q \ge 0$, and p,q don't take the value 0 simultaneously, then the function FNIFGBM is called the fuzzy number intuitionistic fuzzy geometric Bonferroni mean (FNIFGBM) operator of dimension n.

Utilizing the operations of the fuzzy number intuitionistic fuzzy sets, the following equation can be defined.

Theorem 1. Let $\tilde{a}_j = \langle (a_j,b_j,c_j),(l_j,m_j,p_j) \rangle$ $(j=1,2,\cdots,n)$ be a collection of fuzzy number intuitionistic fuzzy values, where $p,q \ge 0$, and p,q don't take the value 0 simultaneously, and let $\mathrm{FNIFGBM}: Q^n \to Q$, if

$$
\begin{aligned}
& FNIFGBM^{p,q}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right) \\
& =\frac{1}{p+q}\left(\bigotimes_{\substack{i,j=1\\ i\neq j}}^{n}\left(p\tilde{a}_i+q\tilde{a}_j\right)\right)^{\frac{1}{n(n-1)}} \\
& =\left\langle\left(1-\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\left(1-a_i\right)^p\left(1-a_j\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}},\right.\right. \\
& \quad 1-\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\left(1-b_i\right)^p\left(1-b_j\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}, \\
& \quad \left.1-\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\left(1-c_i\right)^p\left(1-c_j\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}\right), \\
& \quad \left(\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\left(l_i\right)^p\left(l_j\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}},\right. \\
& \quad \left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\left(m_i\right)^p\left(m_j\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}, \\
& \quad \left.\left.\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\left(p_i\right)^p\left(p_j\right)^q\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}\right)\right\rangle
\end{aligned} \tag{8}
$$

In what follows, we investigate some desirable properties of the FNIFGBM operator.

Theorem 2 (Commutativity).

$$FNIFGBM^{p,q}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)=FNIFGBM^{p,q}\left(\tilde{a}_1',\tilde{a}_2',\cdots,\tilde{a}_n'\right)$$

where $\left(\tilde{a}_1',\tilde{a}_2',\cdots,\tilde{a}_n'\right)$ is any permutation of $\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)$.

Theorem 3 . (Idempotency) If $\tilde{a}_j=\tilde{a}$ for all j , then

$$FNIFGBM^{p,q}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)=\tilde{a}$$

Theorem 4. (Monotonicity) Let $\tilde{a}_j=\left\langle\left(a_j,b_j,c_j\right),\left(l_j,m_j,p_j\right)\right\rangle$, $\tilde{a}_j'=\left\langle\left(a_j',b_j',c_j'\right),\left(l_j',m_j',p_j'\right)\right\rangle$ $\left(j=1,2,\cdots,n\right)$ be two collection of fuzzy number intuitionistic fuzzy values, If $a_j\le a_j',b_j\le b_j',c_j\le c_j'$, and $l_j\ge l_j',m_j\ge m_j',p_j\ge p_j'$ for all j, then

$$FNIFGBM^{p,q}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)\le FNIFGBM^{p,q}\left(\tilde{a}_1',\tilde{a}_2',\cdots,\tilde{a}_n'\right)$$

Theorem 5 . (Boundedness): Let $\tilde{a}_j=\left\langle\left(a_j,b_j,c_j\right),\left(l_j,m_j,p_j\right)\right\rangle$ $\left(j=1,2,\cdots,n\right)$ be a collection of fuzzy number intuitionistic fuzzy values,

$$\min_j\left\{\tilde{a}_j\right\}\le FNIFGBM^{p,q}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)\le\max_j\left\{\tilde{a}_j\right\}$$

It should be noted that the FNIFGBM operator does not consider the importance of the aggregated arguments, but in many practical problem, especially in some group decision making, the attributes have different weights, to overcome this drawback, we define the fuzzy number intuitionistic fuzzy weighted geometric Bonferroni mean (FNIFWGBM) operator.

Definition 9. Let $\tilde{a}_j = \langle (a_j, b_j, c_j), (l_j, m_j, p_j) \rangle$ $(j = 1, 2, \cdots, n)$ be a collection of fuzzy number intuitionistic fuzzy values, and let $\mathrm{FNIFWGBM} : Q^n \to Q$, if

$$
\begin{aligned}
& FNIFWGBM_w^{p,q}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) \\
&= \frac{1}{p+q}\left(\bigotimes_{\substack{i,j=1 \\ i \neq j}}^{n} \left(p(\tilde{a}_i)^{w_i} + q(\tilde{a}_j)^{w_j} \right) \right)^{\frac{1}{n(n-1)}} \\
&= \Bigg\langle \Bigg(1 - \left(1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - \left(1 - (a_i)^{w_i}\right)^p \left(1 - (a_j)^{w_j}\right)^q \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}}, \\
& \quad 1 - \left(1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - \left(1 - (b_i)^{w_i}\right)^p \left(1 - (b_j)^{w_j}\right)^q \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}}, \\
& \quad 1 - \left(1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - \left(1 - (c_i)^{w_i}\right)^p \left(1 - (c_j)^{w_j}\right)^q \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \Bigg), \\
& \quad \Bigg(\left(1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - \left(1 - (1 - l_i)^{w_i}\right)^p \left(1 - (1 - l_j)^{w_j}\right)^q \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}}, \\
& \quad \left(1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - \left(1 - (1 - m_i)^{w_i}\right)^p \left(1 - (1 - m_j)^{w_j}\right)^q \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}}, \\
& \quad \left(1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - \left(1 - (1 - p_i)^{w_i}\right)^p \left(1 - (1 - p_j)^{w_j}\right)^q \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \Bigg) \Bigg\rangle
\end{aligned}
\tag{9}
$$

where $p, q \geq 0$, and p, q don't take the value 0 simultaneously, $w = (w_1, w_2, \cdots, w_n)^T$ denotes the weight vector of $\tilde{a}_j = \langle (a_j, b_j, c_j), (l_j, m_j, p_j) \rangle (j = 1, 2, \cdots, n)$, in which w_j indicates the importance degree of $\tilde{a}_j$, satisfying $w_j > 0 (j = 1, 2, \cdots, n)$, and $\sum_{j=1}^{n} w_j = 1$.then the function FNIFGBM is called the fuzzy number intuitionistic fuzzy weighted geometric Bonferroni mean (FNIFWGBM) operator of dimension n.

4. An Approach to multiple attribute Decision Making under Fuzzy Number Intuitionistic Fuzzy Environment

In this section, we shall develop an approach based on the FNIFWGBM operator to multiple attribute decision making under fuzzy number intuitionistic fuzzy environment as follows.

Let $A = \{A_1, A_2, \cdots, A_m\}$ be a discrete set of alternatives, and $G = \{G_1, G_2, \cdots, G_n\}$ be the set of attributes, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$ is the weighting vector of the attribute $G_j (j = 1, 2, \cdots, n)$, where $\omega_j \in [0,1]$, $\sum_{j=1}^{n} \omega_j = 1$. Suppose that $\tilde{R} = (\tilde{r}_{ij})_{m\times n} = \langle (a_{ij}, b_{ij}, c_{ij}), (l_{ij}, m_{ij}, p_{ij}) \rangle_{m\times n}$ is the fuzzy number intuitionistic fuzzy decision matrix, where (a_{ij}, b_{ij}, c_{ij}) indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker, (l_{ij}, m_{ij}, p_{ij}) indicates the degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker, $(a_{ij}, b_{ij}, c_{ij}) \subset [0,1]$, $(l_{ij}, m_{ij}, p_{ij}) \subset [0,1]$, $c_{ij} + p_{ij} \le 1$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$. In the following, we apply the FNIFWGBM operator to multiple attribute decision making based on fuzzy number intuitionistic fuzzy information.

Step 1. Utilize the decision information given in the fuzzy number intuitionistic fuzzy decision matrix $\tilde{R}$, and the FIFWA operator

$$\tilde{r}_i = FNIFWGBM_w^{p,q}(\tilde{r}_{i1}, \tilde{r}_{i2}, \cdots, \tilde{r}_{in}), i = 1, 2, \cdots, m. \tag{10}$$

to derive the overall preference fuzzy number intuitionistic fuzzy value $\tilde{r}_i (i = 1, 2, \cdots, m)$ of the alternative A_i.

Step 2. calculate the scores $S(\tilde{r}_i)(i = 1, 2, \cdots, m)$ of the collective overall fuzzy number intuitionistic fuzzy preference values $\tilde{r}_i (i = 1, 2, \cdots, m)$ to rank all the alternatives $A_i (i = 1, 2, \cdots, m)$ and then to select the best one(s) (if there is no difference between two scores $S(\tilde{r}_i)$ and $S(\tilde{r}_j)$, then we need to calculate the accuracy degrees $H(\tilde{r}_i)$ and $H(\tilde{r}_j)$ of the collective overall fuzzy number intuitionistic fuzzy preference values $\tilde{r}_i$ and $\tilde{r}_j$, respectively, and then rank the alternatives A_i and A_j in accordance with the accuracy degrees $H(\tilde{r}_i)$ and $H(\tilde{r}_j)$).

Step 3. End.

5. Conclusion

The aim of this paper is to investigate the fuzzy number intuitionistic fuzzy multiple attribute decision making problems based on the fuzzy number intuitionistic fuzzy sets and Geometric Bonferroni mean operator. Then, we have proposed two aggregation operators: fuzzy number intuitionistic fuzzy geometric Bonferroni mean (FNIFGBM) operator and fuzzy number intuitionistic fuzzy weighted geometric Bonferroni mean (FNIFWGBM) operator. Then, we have utilized FNIFWGBM operator to tackle the multiple attribute decision making problems with fuzzy number intuitionistic fuzzy information.

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