Combined method of multiple attribute decision making with intuitionistic fuzzy information

Xianping Jiang*, Linggang Ran

School of Economics and Management, Chongqing University of Arts and Sciences, Yongchuan, 402160, Chongqing, China

*Corresponding author:xianpingjiang20000@163.com

Abstract

With respect to multiple attribute decision making problems with intuitionistic fuzzy information, some operational laws of intuitionistic fuzzy numbers, score function and accuracy function of intuitionistic fuzzy numbers are introduced. An combined optimization model based on the deviation method and ideal solution, by which the attribute weights can be determined, is established. For the special situations where the information about attribute weights is completely unknown, we establish another combined optimization model. By solving this model, we get a simple and exact formula, which can be used to determine the attribute weights. We utilize the intuitionistic fuzzy weighted geometric (IFWG) operator to aggregate the intuitionistic fuzzy information corresponding to each alternative, and then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords

Multiple ttribute decision making; Intuitionistic fuzzy number; Intuitionistic fuzzy weighted geometric (IFWG) operator; Weight information

1. Introduction

Atanassov [1,2] introduced the concept of intuitionistic fuzzy set(IFS), which is a generalization of the concept of fuzzy set [3]. The intuitionistic fuzzy set has received more and more attention since its appearance[4-18]. In the process of MADM with intuitionistic fuzzy information, sometimes, the attribute values take the form of intuitionistic fuzzy numbers, and the information about attribute weights is incompletely known or completely unknown because of time pressure, lack of knowledge or data, and the expert's limited expertise about the problem domain. All of the above methods, however, will be unsuitable for dealing with such situations. Therefore, it is necessary to pay attention to this issue. In [8], Xu investigated the intuitionistic fuzzy MADM with the information about attribute weights is incompletely known or completely unknown, a method based on the ideal solution was proposed. The aim of this paper is to develop another combined method based on the deviation method and ideal solution, to overcome this limitation. The remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to intuitionistic fuzzy sets. In Section 3 we introduce the MADM problem with intuitionistic fuzzy information, in which the information about attribute weights is incompletely known, and the attribute values take the form of intuitionistic fuzzy numbers. To determine the attribute weights, an combined optimization model based on the deviation method and ideal solution, by which the attribute weights can be determined, is established. For the special situations where the information about attribute weights is completely unknown, we establish another combined optimization model. By solving this model, we get a simple and exact formula, which can be used to determine the attribute weights. We utilize the intuitionistic fuzzy weighted geometric (IFWG) operator to aggregate the intuitionistic fuzzy information corresponding to each alternative, and then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. In Section 4, an illustrative example is pointed out. In Section 5 we conclude the paper and give some remarks.

2. Preliminaries

In the following, we introduce some basic concepts related to intuitionistic fuzzy sets.

Definition 1. An IFS A in X is given by

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle | x\in X\} \tag{1}$$

Where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, with the condition

$$0 \le \mu_A(x)+\nu_A(x) \le 1,\ \forall\ x\in X$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to the set A [1,2].

Definition 2. Let $\tilde{a}=(\mu,\nu)$ be an intuitionistic fuzzy number, a score function S and accuracy function H of an intuitionistic fuzzy value can be represented as follows [9]:

$$S(\tilde{a})=\mu-\nu,\quad S(\tilde{a})\in[-1,1]. \tag{2}$$

$$H(\tilde{a})=\mu+\nu,\quad H(\tilde{a})\in[0,1]. \tag{3}$$

Definition 3. Let $\tilde{a}_1=(\mu_1,\nu_1)$ and $\tilde{a}_2=(\mu_2,\nu_2)$ be two intuitionistic fuzzy values, $s(\tilde{a}_1)=\mu_1-\nu_1$ and $s(\tilde{a}_2)=\mu_2-\nu_2$ be the scores of $\tilde{a}$ and $\tilde{b}$, respectively, and let $H(\tilde{a}_1)=\mu_1+\nu_1$ and $H(\tilde{a}_2)=\mu_2+\nu_2$ be the accuracy degrees of $\tilde{a}$ and $\tilde{b}$, respectively, then if $S(\tilde{a})<S(\tilde{b})$, then $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a}<\tilde{b}$; if $S(\tilde{a})=S(\tilde{b})$, then

if $H(\tilde{a})=H(\tilde{b})$, then $\tilde{a}$ and $\tilde{b}$ represent the same information, denoted by $\tilde{a}=\tilde{b}$; (2) if $H(\tilde{a})<H(\tilde{b})$, $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a}<\tilde{b}$ [6].

Definition 4 Let $a_j=(\mu_j,\nu_j)(j=1,2,\cdots,n)$ be a collection of intuitionistic fuzzy values, and let IFWG: $Q^n \to Q$, if

$$\mathrm{IFWG}_\omega(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\prod_{j=1}^{n}\tilde{a}_j^{\omega_j}=\left(\prod_{j=1}^{n}\mu_j^{\omega_j},1-\prod_{j=1}^{n}(1-_j)\nu_j^{\omega_j}\right) \tag{4}$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ be the weight vector of $\tilde{a}_j\,(j=1,2,\cdots,n)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$, then IFWA is called the intuitionistic fuzzy weightedgeometric (IFWG) operator [6].

Definition 5 Let $\tilde{a}_1=(\mu_1,\nu_1)$ and $\tilde{a}_2=(\mu_2,\nu_2)$ be two intuitionistic fuzzy numbers, then the normalized Hamming distance between $\tilde{a}_1=(\mu_1,\nu_1)$ and $\tilde{a}_2=(\mu_2,\nu_2)$ is defined as follows [8]:

$$d(\tilde{a}_1,\tilde{a}_2)=\frac{1}{2}(|\mu_1-\mu_2|+|\nu_1-\nu_2|) \tag{5}$$

3. Combined method for intuitionistic fuzzy decision making problems with incomplete weight information

Suppose that $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}=\left(\mu_{ij},\nu_{ij}\right)_{m\times n}$ is the intuitionistic fuzzy decision matrix, where μ_{ij} indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker, ν_{ij} indicates the degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker, $\mu_{ij}\subset[0,1]$, $\nu_{ij}\subset[0,1]$, $\mu_{ij}+\nu_{ij}\le 1$, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$.

Definition 6. Let $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}=\left(\mu_{ij},\nu_{ij}\right)_{m\times n}$ be an intuitionistic fuzzy decision matrix, $\tilde{r}_i=\left(\tilde{r}_{i1},\tilde{r}_{i2},\cdots,\tilde{r}_{in}\right)$ be the vector of attribute values corresponding to the alternative A_i, $i=1,2,\cdots,m$, then we call

$$\tilde{r}_i=\left(\mu_{ij},\nu_{ij}\right)=\mathrm{IFWG}_w\left(\tilde{r}_{i1},\tilde{r}_{i2},\cdots,\tilde{r}_{in}\right)=\left(\prod_{j=1}^{n}\mu_{ij}^{\ w_j},1-\prod_{j=1}^{n}\left(1-\nu_{ij}\right)^{w_j}\right),\quad i=1,2,\cdots,m. \quad (6)$$

the overall value of the alternative A_i, where $w=\left(w_1,w_2,\cdots,w_n\right)^T$ is the weight vector of attributes.

In the situation where the information about attribute weights is completely known, i.e., each attribute weight can be provided by the expert with crisp numerical value, we can weight each attribute value and aggregate all the weighted attribute values corresponding to each alternative into an overall one by using Eq. (6). Based on the overall attribute values $\tilde{r}_i$ of the alternatives $A_i\left(i=1,2,\cdots,m\right)$, we can rank all these alternatives and then select the most desirable one(s). The greater $\tilde{r}_i$, the better the alternative A_i will be.

The deviation method [15] is selected here to compute the differences of the performance values of each alternative. For the attribute $G_j\in G$, the deviation of alternative A_i to all the other alternatives can be defined as follows:

$$D_{ij}\left(w\right)=\sum_{k=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_{kj}\right)w_j, i=1,2,\cdots,m, j=1,2,\cdots,n.$$

Let $D_j\left(w\right)=\sum_{i=1}^{m}D_{ij}\left(w\right)=\sum_{i=1}^{m}\sum_{k=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_{kj}\right)w_j, j=1,2,\cdots,n$

Then $D_j\left(w\right)$ represent the deviation value of all alternatives to other alternatives for the attribute $G_j\in G$.

Based on the above analysis, we have to choose the weight vector w to maximize all deviation values for all the attributes. To do so, we can construct a non-linear programming model as follows:

$$(\mathrm{M.1})\quad\begin{cases}\max\ D\left(w\right)=\sum_{j=1}^{n}\sum_{i=1}^{m}D_{ij}\left(w\right)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_{kj}\right)w_j\\ \text{Subject to}\quad w\in H,\sum_{j=1}^{n}w_j=1, w_j\ge 0, j=1,2,\cdots,n\end{cases}$$

Definition 7. Let $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}=\left(\mu_{ij},\nu_{ij}\right)_{m\times n}$ be an interval-valued intuitionistic fuzzy decision matrix, $\tilde{r}^+=\left(\left(\mu_1^+,\nu_1^+\right),\left(\mu_2^+,\nu_2^+\right),\cdots,\left(\mu_n^+,\nu_n^+\right)\right)$ be the ideal point of attribute values, defined as follows

$$\left(\mu_j^+,\nu_j^+\right)=\left(\max_i\mu_{ij},\min_i\nu_{ij}\right), j\in 1,2,\cdots,n.$$

In the real life, there always exist some differences between the vector of attribute values corresponding to ideal point and the vector of attribute values corresponding to the alternative $A_i\left(i=1,2,\cdots,m\right)$. By Definitions 8, in what follows we define the weighted hamming distance $d\left(\tilde{r}_i,\tilde{r}^+\right)$ between the vector of attribute values $\tilde{r}^+$ of ideal point and the vector of attribute values $\tilde{r}_i$ corresponding to the alternative $A_i\left(i=1,2,\cdots,m\right)$:

$$N_i\left(w\right)=d\left(\tilde{r}_i,\tilde{r}^+\right)=\sum_{j=1}^{n}d\left(\tilde{r}_{ij},\tilde{r}^+\right)w_j \tag{7}$$

Obviously, the smaller $N_i\left(w\right)$, the better the alternative A_i will be. Since each alternative is noninferior, so there exists no preference relation on the all the alternatives. So, we can establish the following single objective optimization model to calculate the weight information:

$$\left(\text{M.2}\right)\begin{cases}\min N\left(w\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}d\left(\tilde{r}_{ij},\tilde{r}^+\right)w_j\\ \text{Subject to}\quad w\in H,\sum_{j=1}^{n}w_j=1,w_j\geq 0,\,j=1,2,\cdots,n\end{cases}$$

Based the model $\left(\text{M.1}\right)$ and $\left(\text{M.2}\right)$, An combined optimization model $\left(\text{M.3}\right)$ is established as follows:

$$\begin{cases}\max C\left(w\right)=\left[\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_{kj}\right)-\sum_{i=1}^{m}\sum_{j=1}^{n}d\left(\tilde{r}_{ij},\tilde{r}^+\right)\right]w_j\\ \text{Subject to}\quad w\in H,\sum_{j=1}^{n}w_j=1,w_j\geq 0,\,j=1,2,\cdots,n\end{cases}$$

By solving the model $\left(\text{M.3}\right)$, we get the optimal solution $w=\left(w_1,w_2,\cdots,w_n\right)$, which can be used as the weight vector of attributes.

If the information about attribute weights is completely unknown, we can establish another combined programming model:

$$\left(\text{M.4}\right)\begin{cases}\max C\left(w\right)=\left[\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_{kj}\right)-\sum_{i=1}^{m}\sum_{j=1}^{n}d\left(\tilde{r}_{ij},\tilde{r}^+\right)\right]w_j\\ s.t.\ \sum_{j=1}^{n}w_j^2=1,w_j\geq 0,j=1,2,\cdots,n\end{cases}$$

By solving the model $\left(\text{M.4}\right)$, we get a simple and exact formula for determining the attribute weights as follows:

$$w_j^*=\frac{\sum_{i=1}^{m}\sum_{k=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_{kj}\right)-\sum_{i=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_j^-\right)}{\sqrt{\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{k=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_{kj}\right)-\sum_{i=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_j^-\right)\right]^2}} \tag{8}$$

By normalizing $w_j^*\left(j=1,2,\cdots,n\right)$ be a unit, we have

$$w_j=\frac{\sum_{i=1}^{m}\sum_{k=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_{kj}\right)-\sum_{i=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_j^-\right)}{\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{k=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_{kj}\right)-\sum_{i=1}^{m}d\left(\tilde{r}_{ij},\tilde{r}_j^-\right)\right]} \tag{9}$$

Based on the above models, we develop a practical method for solving the MADM problems, in which the information about attribute weights is incompletely known or completely unknown, and the attribute values take the form of intuitionistic fuzzy information. The method involves the following steps:

Step 1. Let $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}$ be an intuitionistic fuzzy decision matrix, where $\tilde{r}_{ij}=\left(\mu_{ij},\nu_{ij}\right)$, which is an attribute value, given by an expert, for the alternative $A_i\in A$ with respect to the attribute $G_j\in G$, $w=\left(w_1,w_2,\cdots,w_n\right)$ be the weight vector of attributes, where $w_j\in[0,1]$, $j=1,2,\cdots,n$, H is a set of the known weight information.

Step 2. If the information about the attribute weights is partly known, then we solve the model (M.3) to obtain the attribute weights. If the information about the attribute weights is completely unknown, then we can obtain the attribute weights by using Eq. (9).

Step 3. Utilize the weight vector $w=\left(w_1,w_2,\cdots,w_n\right)$ and by Eq. (6), we obtain the overall values $\tilde{r}_i$ of the alternative $A_i\left(i=1,2,\cdots,m\right)$.

Step 4. calculate the scores $S\left(\tilde{r}_i\right)$ of the overall intuitionistic fuzzy preference value $\tilde{r}_i\left(i=1,2,\cdots,m\right)$ to rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ and then to select the best one(s) (if there is no difference between two scores $S\left(\tilde{r}_i\right)$ and $S\left(\tilde{r}_j\right)$, then we need to calculate the accuracy degrees $H\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_j\right)$ of the overall intuitionistic fuzzy preference value $\tilde{r}_i$ and $\tilde{r}_j$, respectively, and then rank the alternatives A_i and A_j in accordance with the accuracy degrees $H\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_j\right)$.

Step 5. Rank all the alternatives A_i and select the best one(s) in accordance with $S\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_i\right)\left(i=1,2,\cdots,m\right)$.

Step 6. End.

4. Numerical example

Let us suppose there is an investment company, which wants to invest a sum of money in the best option (adapted from [16]). There is a panel with five possible alternatives to invest the money: ① A_1 is a car company; ② A_2 is a food company; ③ A_3 is a computer company; ④ A_4 is an arms company; ⑤ A_5 is a TV company. The investment company must take a decision according to the following four attributes: ① G_1 is the risk analysis; ② G_2 is the growth analysis; ③ G_3 is the social-political impact analysis; ④ G_4 is the environmental impact analysis. The five possible alternatives $A_i\left(i=1,2,3,4,5\right)$ are to be evaluated using the intuitionistic fuzzy information by the decision maker under the above four attributes, as listed in the following matrix.

$$\tilde{R}=\begin{bmatrix}(0.4,0.5) & (0.5,0.4) & (0.2,0.7) & (0.1,0.8)\\ (0.6,0.4) & (0.6,0.3) & (0.6,0.3) & (0.3,0.6)\\ (0.5,0.5) & (0.4,0.5) & (0.4,0.4) & (0.5,0.4)\\ (0.7,0.2) & (0.5,0.4) & (0.2,0.5) & (0.1,0.7)\\ (0.5,0.3) & (0.3,0.4) & (0.6,0.2) & (0.4,0.4)\end{bmatrix}$$

Then, we utilize the approach developed to get the most desirable alternative(s).

Case 1: The information about the attribute weights is partly known and the known weight information is given as follows:

$$H=\{0.15\le w_1\le 0.2, 0.16\le w_2\le 0.18, 0.30\le w_3\le 0.31,$$
$$0.35\le w_4\le 0.45, w_j\ge 0, j=1,2,3,4, \sum_{j=1}^{4}w_j=1\}$$

Step 1 Utilize the model (M.3) to establish the following single-objective programming model:

$$\begin{cases}\max C(w) = 0.85w_1 + 0.80w_2 + 1.65w_3 + 2.10w_4 \\ s.t.\ \ w \in H\end{cases}$$

Solving this model, we get the weight vector of attributes: $w = \begin{pmatrix}0.17 & 0.18 & 0.30 & 0.35\end{pmatrix}^T$

Step 2 Utilize the weight vector w and by Eq. (6), we obtain the overall values $\tilde{r}_i$ of the alternatives $A_i\ (i = 1,2,3,4,5)$.

$$\tilde{r}_1 = (0.2082, 0.6783), \tilde{r}_2 = (0.4708, 0.4394), \tilde{r}_3 = (0.4492, 0.4371)$$
$$\tilde{r}_4 = (0.2290, 0.5320), \tilde{r}_5 = (0.4455, 0.3285)$$

Step 3 Calculate the scores $S(\tilde{r}_i)$ of the overall intuitionistic fuzzy preference values $\tilde{r}_i\ (i = 1,2,3,4,5)$

$$S(\tilde{r}_1) = -0.4701, S(\tilde{r}_2) = 0.0314, S(\tilde{r}_3) = 0.0121$$
$$S(\tilde{r}_4) = 0.3030, S(\tilde{r}_5) = 0.1170$$

Step 4 Rank all the alternatives $A_i\ (i = 1,2,3,4,5)$ in accordance with the scores $S(\tilde{r}_i)$ of the overall intuitionistic fuzzy preference values $\tilde{r}_i$: $A_5 \succ A_3 \succ A_2 \succ A_4 \succ A_1$, and thus the most desirable alternative is A_5.

Case 2: If the information about the attribute weights is completely unknown, we utilize another approach developed to get the most desirable alternative(s).

Step 1 Utilize the Eq. (9) to get the weight vector of attributes:

$$w = \begin{pmatrix}0.1574 & 0.1481 & 0.3056 & 0.3889\end{pmatrix}^T$$

Step 2 Utilize the weight vector w and by Eq. (6), we obtain the overall values $\tilde{r}_i$ of the alternative $A_i\ (i = 1,2,3,4,5)$.

$$\tilde{r}_1 = (0.1951, 0.6923), \tilde{r}_2 = (0.4582, 0.4504), \tilde{r}_3 = (0.4519, 0.4325)$$
$$\tilde{r}_4 = (0.2131, 0.5465), \tilde{r}_5 = (0.4494, 0.3288)$$

Step 3 Calculate the scores $S(\tilde{r}_i)$ of the overall intuitionistic fuzzy preference values $\tilde{r}_i\ (i = 1,2,3,4,5)$

$$S(\tilde{r}_1) = -0.4972, S(\tilde{r}_2) = 0.0078, S(\tilde{r}_3) = 0.0194$$
$$S(\tilde{r}_4) = -0.3335, S(\tilde{r}_5) = 0.1206$$

Step 4 Rank all the alternatives $A_i(i=1,2,3,4,5)$ in accordance with the scores $S(\tilde{r}_i)$: $A_5 \succ A_3 \succ A_2 \succ A_4 \succ A_1$, and thus the most desirable alternative is A_5.

5. Conclusion

In this paper, we have investigated the problem of MADM with incompletely known information on attribute weights to which the attribute values are given in terms of intuitionistic fuzzy numbers. To determine the attribute weights, a combined optimization model based on the deviation method and ideal solution, by which the attribute weights can be determined, is established. For the special situations where the information about attribute weights is completely unknown, we establish another combined optimization model. By solving this model, we get a simple and exact formula, which can be used to determine the attribute weights. We utilize the intuitionistic fuzzy weighted geometric (IFWG) operator to aggregate the intuitionistic fuzzy information corresponding to each alternative, and then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. Finally, an illustrative example is given. In the future, we shall continue working in the application of the intuitionistic fuzzy multiple attribute decision-making to other domains.

Acknowledgements

The work was supported by the Humanities and Social Sciences Foundation of Ministry of Education of the People's Republic of China (No. 14YJA630017 and No. 14XJCZH002).

References

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87-96.
[2] K. Atanassov, More on intuitionistic fuzzy sets, Fuzzy Sets and Systems 33 (1989) 37- 46.
[3] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965)338-356.
[4] W. L. Gau and D. J. Buehrer, Vague sets, IEEE Transactions on Systems, Man and Cybernetics 23 (2) (1993) 610-614.
[5] H. Bustine and P. Burillo, Vague sets are intuitionistic fuzzy sets, Fuzzy Sets and Systems 79 (1996) 403-405.
[6] Z. S. Xu and R. R. Yager, Some geometric aggregation operators based on intuitionistic fuzzy sets, International Journal of General System 35 (2006) 417-433.
[7] Z. S. Xu, Intuitionistic fuzzy aggregation operators, IEEE Transations on Fuzzy Systems 2007.(in press)
[8] Z. S. Xu, Models for multiple attribute decision-making with intuitionistic fuzzy information, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 15(3) (2007) 285-297.
[9] S. M. Chen and J. M. Tan, Handling multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets and Systems 67 (1994) 163-172.
[10] D. H. Hong and C. H. Choi, Multicriteria fuzzy problems based on vague set theory, Fuzzy Sets and Systems 114 (2000) 103-113.
[11] K.S. Park and S.H. Kim, Tools for interactive multi-attribute decision making with incompletely identified information, European Journal of Operational Research 98 (1997) 111-123.
[12] S.H. Kim, S.H. Choi and J.K. Kim, An interactive procedure for multiple attribute group decision making with incomplete information: range-based approach, European Journal of Operational Research 118 (1999) 139-152.
[13] S.H. Kim and B.S. Ahn, Interactive group decision making procedure under incomplete information, European Journal of Operational Research 116 (1999) 498-507.
[14] K.S. Park, Mathematical programming models for charactering dominance and potential optimality when multicriteria alternative values and weights are simultaneously incomplete. IEEE transactions on systems, man, and cybernetics-part A, Systems and Humans, 34 (2004) 601-614.
[15]Y.M.Wang, Using the method of maximizing deviations to make decision for multi-indices. System Engineering and Electronics 7 (1998) 24-26, 31.
[16] F. Herrera and E. Herrera-Viedma, Linguistic decision analysis: steps for solving decision problems under linguistic information. Fuzzy Sets and Systems 115 (2000) 67-82.
[17] G. W. Wei. Some geometric aggregation functions and their application to dynamic multiple attribute decision making in intuitionistic fuzzy setting. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 17(2) (2009) 179-196.
[18] G.W. Wei. Maximizing deviation method for multiple attribute decision making in intuitionistic fuzzy setting. Knowledge-Based Systems 21(8) (2008) 833-836.