The Multivariable Linear Time-delay Systems and its Application in the Process of Oil-water Separation

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Abstract. Based on Lyapunov's theory of stability, analyze on a control process of oil-water separation with time-delay, coupling and uncertainty in the unite station. An equation of sufficient condition for multi-variable Linear Delayed Systems with delayed independent stabilization is derived and another forms are also given. Based on this, several simple criterions for judging independent stabilization from linear delayed system are presented. We also discuss the exponential stability for delayed system and also provide the sufficient condition of exponential stability with any appointed convergent rate and its corresponding deductions. Using these conditions, we can choose a set of suitable parameters to reduce the conservation. By calculating and being compared with methods of the literature, the results show that our methods have less conservation.

Keywords: Multi-variable linear systems, dependent stability, Supervision Oilfield.

1. Introduction

Disposal of crude oil in the unite-station is a synthesis productive process obtained oil-water separation, disposal of sewage, analyze of obtaining water in crude oil and disposal of gas from oil field. From degree of control, its controlled variation has the characteristic of time-delay, uncertainty and strong coupling. If only use routine PID method, it can not has the good effect. Thus, many scholars lead predictive control method to control the process of oil-water separation, but found characteristic is not good in some oil field [1]. In this paper, discuss the stability problem in the process of oil-water separation with time-lagged.

In fact, the independent stability for linear system with delayed has bring broad recognition for a long time. Many scholars have made deeply researches [2-4].The independent stability condition for delayed systems in the reference [2] is presented with norm and measurement. It is easy to use, but it is too conservative. In the reference [3], it obtained the stability condition of system with patulous Lyapunov's matrix equations, and presented corresponding stability criterions. But choosing the Lyapunov's function was limited some degree. In the reference [4], it provided bizarre value of matrix to judge system stability. In this paper, we provide a broader way to choosing Lyapunov's function, and apply Lyapunov's stability theory to obtain sufficient condition of delayed independent stability for judging linear system with delayed. Choosing a set of parameters may reduce conservation. We also provide the sufficient condition of the exponential stability for delayed system.

2. Independent stability for multi-variable linear systems with delayed state

A multi-variable linear system with delayed state as follows:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}\boldsymbol{x}(\boldsymbol{t}-\boldsymbol{h}) \tag{1}$$

Here: $\boldsymbol{x} \in \boldsymbol{R}^{\boldsymbol{n}}$ is state variable; $\boldsymbol{A},\boldsymbol{B} \in \boldsymbol{R}^{n\times n}$ is matrix of system; $\boldsymbol{h} > 0$ is delayed constant of system.

Define 1 To any $h > 0$ when system (1) is gradually close stability, we call it delayed independent gradually close stability.

Choosing two hypo-model Lyapunov's function:

$$V[x(t)] = x^T(t)P^* x(t) + \varepsilon\int_{-h}^{0} x^T(\theta)Dx(\theta)d\theta \quad (2)$$

Here: $\varepsilon > 0$ is a constant variable $D > 0$, $P^* > 0$ define as

$$P^* = P + \alpha I_n \quad (3)$$

Here: α is a real number; $p > 0$ is positive order symmetrical matrix.

Seeking differential coefficient of the equation (2) is obtained to.

$$\dot{V}[x(t)] = \dot{x}^T(t)P^* x(t) + x(t)P^* x^T(t) + \varepsilon x^T(t)Dx(t) - \varepsilon x^T(t-h)Dx(t-h)$$

$$= \begin{bmatrix} x^T(t) & x^T(t-h) \end{bmatrix} \begin{bmatrix} A^T P^* + P^* A + \varepsilon D & P^* B \\ B^T P^* & -\varepsilon D \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix} \quad (4)$$

By reference [4], we know that when matrix followed is negative order, $\dot{V}[x(t)]$ is also too, namely:

$$M = A^T P^* + P^* A + \varepsilon D + \tfrac{1}{\varepsilon} P^* B D^{-1} B^T P^* < 0 \quad (5)$$

Choosing positive order symmetrical matrix Q, ordering

$$A^T P^* + P^* A + \varepsilon D + \tfrac{1}{\varepsilon} P^* B D^{-1} B^T P^* + Q = 0 \quad (6)$$

Have

$$\dot{V}[x(t)] \le -x^T(t)Qx(t) < 0 \quad (7)$$

Theorem 1 Considering system (1) of possessing constant coefficient matrix A and B, Given α, ε, D and positive order symmetrical matrix Q, if equation (6) exists the answer P of positive order symmetrical, system (1) is delayed independent stability. If $\dot{x} = Ax$ is gradually close stable, Lyapunov's equation as follows come into existence.

$$A^T P + PA = -Q \quad (8)$$

Here: P is the answer of positive order symmetrical for positive order symmetrical matrix Q. Theorem 1 can present as follows.

Theorem 2 If given α, ε, D and positive order symmetrical matrix Q is satisfied with

$$\alpha A^T + \alpha A + \varepsilon D + \tfrac{1}{\varepsilon} P^* B D^{-1} B^T P^* + Q < 0 \quad (9)$$

System (1) is delayed independent gradually close stable, here P is the answer of positive order symmetrical of Lyapunov's equation (8).

Using $P = I$ and $Q = I$ to fill in respectively theorem 1 and theorem 1', we may derive deduction as follows.

Deduction 1 If equation as follows comes into existence, namely,

$$\lambda_{\max}[(1+\alpha)A^T + (1+\alpha)A + \varepsilon D + \tfrac{(1+\alpha)^2}{\varepsilon} BD^{-1}B^T] < 0 \quad (10)$$

System (1) is delayed independent gradually close stability.

Deduction 2 If equation as follows comes into exits, namely,

$$\lambda_{\max}[\alpha A^T + \alpha A + \varepsilon D + \tfrac{1}{\varepsilon} P^* B D^{-1} B^T P^*] < 1 \quad (11)$$

It provided that seeking suitable non- bizarre matrix by resemble transform try to reduce conservation. In this paper, we adapt the method of reference [4] to reduce conservation.

3. Exponential stability for multi-variable linear systems with delated state

Define 2 when the answer of system (1) is satisfied as follows

$$\|x(t)\| < Ke^{-\mu t} \quad (12)$$

When K is a positive constant, we call this system is exponential stable and its convergent rate is μ [1].The relation between exponential stability and gradually close stability of delayed system is provided below.

Lemma 1 If system below is gradually close,

$$\dot{\boldsymbol{x}}'(\boldsymbol{t}) = \boldsymbol{A}'\boldsymbol{x}'(\boldsymbol{t}) + \boldsymbol{B}'\boldsymbol{x}'(\boldsymbol{t}-\boldsymbol{h}) \tag{13}$$

Here

$$\boldsymbol{A}' = \boldsymbol{A} + \mu\boldsymbol{I}\ ,\ \ \boldsymbol{B}' = \boldsymbol{B}\boldsymbol{e}^{\mu\boldsymbol{h}} \tag{14}$$

Theorem 2 Given $\alpha, \varepsilon, \boldsymbol{D}$ and positive order matrix Q, if equation below exists the answer P of positive order symmetrical,

$$A^T P^* + P^* A + \varepsilon D + 2\mu P^* + \frac{1}{\varepsilon} P^* B D^{-1} B^T P^* + Q = 0 \tag{15}$$

We call system (1) is exponential stability and its convergent rate is μ.

Deduction 3 If formula below comes into existence, namely

$$\lambda_{\max}[(1+\alpha)A^T + (1+\alpha)A + \varepsilon D + \frac{(1+\alpha)^2}{\varepsilon} B D^{-1} B^T e^{2\mu h}] < -2\mu(1+\alpha) \tag{16}$$

We call system (1) is exponential stability and its convergent rate is μ.

Deduction 4 If formula below comes into existence,

$$\alpha A^T + \alpha A + \varepsilon D + 2\mu P^* + \frac{1}{\varepsilon} P^* B D^{-1} B^T P^* e^{-2\mu h} < 1 \tag{17}$$

Here P is positive order symmetrical answer of Lyapunov's equation (8) while $Q = I$, we call system (1) is exponential stability and its convergent rate is μ.

4. Analyze of oil-water separation in the process of control

The water in the crude oil in the unite station obtains dissociate water and emulsification water. Two part of "heat-electricity-chemistry combine to dehydration" is often used. We draw a system chart as fig 1to analyze easily. In the chart u_1, u_2 is control variation of sedimentation dehydration and electrical dehydration system X_1, X_2 is state variation of sedimentation dehydration and electrical dehydration system Z_{12}, Z_{21} is relevancy effect of two systems. It is not difficult to see, the process of oil-water separation in the unite station is a typical complex industry system. It has the characteristic of nonlinear, time-change, strong coupling and uncertainty.

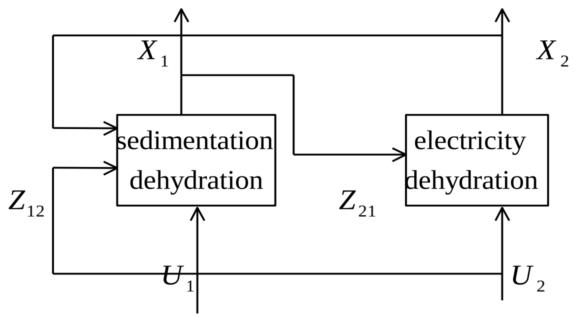

Fig. 1 reduced figure of oil-water separation control process

Let state equation describe the process of oil-water separation as follows:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}\boldsymbol{x}(\boldsymbol{t}-\boldsymbol{h}) \tag{18}$$

$$y(t) = Cx(t) \tag{19}$$

Evaporator model using the relation of pressure change and flux in paper [2] is derived:

$$\frac{dh_{d1}}{dt} = \frac{F_d \beta_1 Q_{so} - Q_{dw1}}{2L_d\sqrt{h_{d1}(2R_d - h_{d1})}} \tag{20}$$

$$\frac{dP_d}{dt} = \frac{1}{2C_d V_d}\left(Q_{so} - \sum Q_{dwj} - K_{vd}\sqrt{(P_d - P_{02})u_d P_o}\right) \tag{21}$$

5. Classify Model

We classify model with the method of linearization in the near work dot to easy apply. The model is

$$\dot{x}(t)=\begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} x(t)+\begin{bmatrix} 0 & b_{12} \\ b_{21} & b_{22} \end{bmatrix} x(t-h) \tag{22}$$

We obtain math model of electrical evaporation system using oil-water separation process in the unite station in daqing oil field. We obtain practical model with model obtained 4 and measure data in the factory as follows:

$$\dot{\boldsymbol{x}}(\boldsymbol{t})=\begin{bmatrix} -2 & -1 \\ 0 & -2 \end{bmatrix} \boldsymbol{x}(\boldsymbol{t})+\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \boldsymbol{x}(\boldsymbol{t}-\boldsymbol{h}) \tag{23}$$

Choosing $\alpha = 0$ for the convenience, Reference to [3], Q matrix is $\boldsymbol{diag}[2 \quad 1]$. According to the method of this paper, we calculate the result below

When $D = 0.5$, $Q, \varepsilon = 1$ obtain $\boldsymbol{P} = \begin{bmatrix} 0.538 & -0.149 \\ -0.149 & 0.361 \end{bmatrix}$ .When $\boldsymbol{D} = \boldsymbol{I}_2, \varepsilon = 0.5\lambda_{\min}$, obtain $\boldsymbol{P} = \begin{bmatrix} 0.538 & -0.149 \\ -0.149 & 0.361 \end{bmatrix}$.

According to theorem 1, System above is delayed independent close stability. P Matrix mentioned in this paper have less conservation (P matrix in the reference [3] is unit matrix.).

6. Conclusions

By choosing even universal Lyapunov’s function, we obtain sufficient condition of independent stability and exponential stability of delayed system. By choosing a set of suitable parameters, we can reduce the conservation of condition. As a result, this method provides a new way to ameliorate stability in robustness bounds and robustness control for uncertain system with delayed state.

Analyze oil-water separation process of control system in the unite station. Obtain variation relevancy chart with oil-water separation process. Clear classify the relation among variation.

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