Model for Archives Websites’ Performance Evaluation in Our Country with Interval-valued Intuitionistic Fuzzy Information

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Abstract. In this paper, we investigate the multiple attribute decision making (MADM) problems for evaluating the archives websites’ performance with interval-valued intuitionistic fuzzy information. Then, we propose the interval-valued intuitionistic fuzzy Einstein Bonferroni mean (IVIFEBM) operator based on the Einstein operations and Bonferroni mean with interval-valued intuitionistic fuzzy information. Furthermore, we propose the interval-valued intuitionistic fuzzy Einstein geometric Bonferroni mean (IVIFEGBM) operator based on the IVIFEBM operator and geometric mean. Finally, an illustrative example for evaluating the archives websites’ performance with interval-valued intuitionistic fuzzy information is given.

Keywords: Multiple attribute decision-making (MAGM); Interval-valued intuitionistic fuzzy information; The Construction of Archives’ Websites; Performance evaluation

1. Introduction

Since the beginning of the 1980s, information Network of government affairs has been opened in succession in the many countries. Studying on the websites’ performance evaluation of government affairs have been the hot topic in academic circle at home and abroad. With the government information becoming into the open, the construction of archives websites was brought forward. However, we just find a few study of archives websites’ performance appraisal in our country. The study of archives websites’ performance evaluation in "appraised what", "how to appraise" has not formed the unified understanding and blindness and spontaneity coexist in the process of operating, which hold back archives websites’ performance evaluation in our country. How to makes justice, fairly and publicity appraisal of the construction of archives websites, and guides the construction of archives websites to the correct direction is a question that archives department is positively discussing and thinking. Based on this, the article thoroughly analyzes the questions of archives websites’ performance evaluation in our country and summarizes the core factors of the construction of archives websites [1].

Atanassov [2-3] introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy set [4]. The intuitionistic fuzzy set has received more and more attention since its appearance. Gau and Buehrer [5] introduced the concept of vague set. But Bustince and Burillo [6] showed that vague sets are intuitionistic fuzzy sets. Xu [7] developed some geometric aggregation operators with intuitionistic fuzzy information. Xu [8] developed some arithmetic aggregation operators with intuitionistic fuzzy information. Later, Atanassov and Gargov [9-10] further introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers. Xu [11-12] developed some aggregation operators with interval-valued intuitionistic fuzzy information.

The problem of evaluating archives websites’ performance with interval-valued intuitionistic fuzzy information is the multiple attribute decision making (MADM) problems [13-16]. In this paper, we investigate the multiple attribute decision making (MADM) problems for evaluating the archives websites’ performance with interval-valued intuitionistic fuzzy information. Then, we propose the
interval-valued intuitionistic fuzzy Einstein Bonferroni mean (IVIFEBM) operator based on the Einstein operations and Bonferroni mean with interval-valued intuitionistic fuzzy information. Furthermore, we propose the interval-valued intuitionistic fuzzy Einstein geometric Bonferroni mean (IVIFEGBM) operator based on the IVIFEBM operator and geometric mean. Finally, an illustrative example for evaluating the archives websites’ performance with interval-valued intuitionistic fuzzy information is given.

2. Preliminaries

In the following, we shall introduce some basic concepts related to interval intuitionistic fuzzy numbers.

Definition 1. Let $X$ be a universe of discourse, An IVFS $\tilde{A}$ over $X$ is an object having the form [9-10]:
$$\tilde{A} = \{ (x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) \mid x \in X \}$$
(1)

Where $\mu_{\tilde{A}}(x) \subseteq [0,1]$ and $\nu_{\tilde{A}}(x) \subseteq [0,1]$ are interval numbers, and
$$0 \leq \sup(\mu_{\tilde{A}}(x)) + \sup(\nu_{\tilde{A}}(x)) \leq 1, \forall x \in X$$

For convenience, let $\mu_{\tilde{a}}(x) = [a,b], \nu_{\tilde{a}}(x) = [c,d]$, so $\tilde{A} = ([a,b],[c,d])$.

Definition 2. Let $\tilde{a} = ([a,b],[c,d])$ be an interval-valued intuitionistic fuzzy number, a score function $S$ of an interval-valued intuitionistic fuzzy value can be represented as follows [11-12]:
$$S(\tilde{a}) = \frac{a-c+b-d}{2}, S(\tilde{a}) \in [-1,1].$$
(2)

Definition 3. Let $\tilde{a} = ([a,b],[c,d])$ be an interval-valued intuitionistic fuzzy number, an accuracy function $H$ of an interval-valued intuitionistic fuzzy value can be represented as follows [11-12]:
$$H(\tilde{a}) = \frac{a+b+c+d}{2}, H(\tilde{a}) \in [0,1].$$
(3)

to evaluate the degree of accuracy of the interval-valued intuitionistic fuzzy value $\tilde{a} = ([a,b],[c,d])$, where $H(\tilde{a}) \in [0,1]$. The larger the value of $H(\tilde{a})$, the more the degree of accuracy of the interval-valued intuitionistic fuzzy value $\tilde{a}$.

Based on the score function $S$ and the accuracy function $H$, in the following, Xu[11] give an order relation between two interval-valued intuitionistic fuzzy values, which is defined as follows:

Definition 4. Let $\tilde{a}_1 = ([a_1,b_1],[c_1,d_1])$ and $\tilde{a}_2 = ([a_2,b_2],[c_2,d_2])$ be two interval-valued intuitionistic fuzzy values, $s(\tilde{a}_1) = \frac{a_1-c_1+b_1-d_1}{2}$ and $s(\tilde{a}_2) = \frac{a_2-c_2+b_2-d_2}{2}$ be the scores of $\tilde{a}$ and $\tilde{b}$, respectively, and let $H(\tilde{a}_1) = \frac{a_1+c_1+b_1+d_1}{2}$ and $H(\tilde{a}_2) = \frac{a_2+c_2+b_2+d_2}{2}$ be the accuracy degrees of $\tilde{a}$ and $\tilde{b}$, respectively, then if $S(\tilde{a}) < S(\tilde{b})$, then $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a} < \tilde{b}$; if $S(\tilde{a}) = S(\tilde{b})$, then if $H(\tilde{a}) = H(\tilde{b})$, then $\tilde{a}$ and $\tilde{b}$ represent the same information, denoted by $\tilde{a} = \tilde{b}$; (2) if $H(\tilde{a}) < H(\tilde{b})$, $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a} < \tilde{b}$.

Bonferroni [1] originally introduced a mean type aggregation operator, called Bonferroni mean, which can provide for aggregation lying between the max, min operators and the logical “or” and “and” operators, which was defined as follows:

Definition 5[17]. Let $p,q \geq 0$ and $a_i (i = 1,2,\cdots,n)$ be a collection of non-negative real numbers. Then the aggregaton functions:
\[
BM^{p,q}(a_1, a_2, \ldots, a_n) = \left( \frac{1}{n(n-1)} \sum_{i,j=1, i\neq j}^n a_i^p a_j^q \right)^{\frac{1}{p+q}}
\]  

Is called the Bonferroni mean (BM) operator.

\textbf{Definition 6}[18]. Let \( p, q \geq 0 \) and \( a_i (i = 1, 2, \ldots, n) \) be a collection of non-negative real numbers with the weight vector \( w = (w_1, w_2, \ldots, w_n)^T \) such that \( w_j > 0 \) (\( j = 1, 2, \ldots, n \)), and \( \sum_{j=1}^n w_j = 1 \). If

\[
NWBM^{p,q}(a_1, a_2, \ldots, a_n) = \left( \sum_{i,j=1, i\neq j}^n \frac{w_i w_j}{1-w_i} a_i^p a_j^q \right)^{\frac{1}{p+q}}
\]  

Then \( NWBM^{p,q} \) is called the normalized weighted Bonferroni mean (NWBM) operator.

\section{Interval-Valued Intuitionistic Fuzzy Einstein Bonferroni mean (IVIFEBM) operator}

In this section, we shall propose the interval-valued intuitionistic fuzzy Einstein Bonferroni mean (IVIFEBM) operator based on the Einstein operations and Bonferroni mean with interval-valued intuitionistic fuzzy information.

\textbf{Definition 7}. Let \( \tilde{a}_i = \left[ [a_i, b_i], [c_i, d_i] \right] \) (\( j = 1, 2, \ldots, n \)) be a collection of interval-valued intuitionistic fuzzy values, and let \( p, q > 0 \). If

\[
IVIFEBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left( \sum_{i,j=1, i\neq j}^n \frac{w_i w_j}{1-w_i} \tilde{a}_i^p \tilde{a}_j^q \right)^{\frac{1}{p+q}}
\]  

Then \( IVIFEBM^{p,q} \) is called the interval-valued intuitionistic fuzzy Einstein Bonferroni mean (IVIFEBM) operator.

The \( IVIFEBM^{p,q} \) operator has the following properties.

\textbf{Theorem 1}. (Idempotency) If all \( \tilde{a}_j \) (\( j = 1, 2, \ldots, n \)) are equal, i.e. \( \tilde{a}_j = \tilde{a} \) for all \( j \), then

\[
IVIFEBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \tilde{a}
\]

\textbf{Theorem 2}. (Boundedness) Let \( \tilde{a}_j \) (\( j = 1, 2, \ldots, n \)) be a collection of IVIFNs, and let

\[
\tilde{a}^- = \min_j \tilde{a}_j \quad \text{and} \quad \tilde{a}^+ = \max_j \tilde{a}_j
\]

Then

\[
\tilde{a}^- \leq IVIFEBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq \tilde{a}^+
\]

\textbf{Theorem 3}. (Monotonicity) Let \( \tilde{a}_j \) (\( j = 1, 2, \ldots, n \)) and \( \tilde{a}'_j \) (\( j = 1, 2, \ldots, n \)) be two set of IVIFNs, if \( \tilde{a}_j \leq \tilde{a}'_j \), for all \( j \), then

\[
IVIFEBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq IVIFEBM^{p,q}(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n)
\]

\textbf{Theorem 4}. (Commutativity) Let \( \tilde{a}_j \) (\( j = 1, 2, \ldots, n \)) and \( \tilde{a}'_j \) (\( j = 1, 2, \ldots, n \)) be two set of IVIFNs, then

\[
IVIFEBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = IVIFEBM^{p,q}(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n)
\]

where \( \tilde{a}'_j \) (\( j = 1, 2, \ldots, n \)) is any permutation of \( \tilde{a}_j \) (\( j = 1, 2, \ldots, n \)).

The \( IVIFEBM^{p,q} \) operator is proposed based on the arithmetic average operator, and it is a basic aggregation technique, which focuses on the group opinions. Another basic average is the geometric
mean, and it gives more importance to the individual opinions. Then, in the following, we shall propose the interval-valued intuitionistic fuzzy Einstein geometric Bonferroni mean (IVIFEGBM) operator.

Definition 8. Let $\tilde{a}_j = \left[\left[a_{ij}, b_{ij}\right], \left[c_{ij}, d_{ij}\right]\right]$ $(j = 1, 2, \ldots, n)$ be a collection of interval-valued intuitionistic fuzzy values, and let $p, q > 0$. If

$$IVIFEGBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \frac{1}{n} \prod_{j=1}^{n} \left( p\tilde{a}_j \oplus q\tilde{a}_j \right)$$

Then $IVIFEGBM^{p,q}$ is called the interval-valued intuitionistic fuzzy Einstein geometric Bonferroni mean (IVIFEGBM) operator.

The $IVIFEGBM^{p,q}$ operator has the following properties.

Theorem 5. (Idempotency) If all $\tilde{a}_j (j = 1, 2, \ldots, n)$ are equal, i.e. $\tilde{a}_j = \tilde{a}$ for all $j$, then

$$IVIFEGBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \tilde{a}$$

Theorem 6. (Boundedness) Let $\tilde{a}_j (j = 1, 2, \ldots, n)$ be a collection of IVIFNs, and let

$$\tilde{a}^- = \min_j \tilde{a}_j, \quad \tilde{a}^+ = \max_j \tilde{a}_j$$

Then

$$\tilde{a}^- \leq IVIFEGBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq \tilde{a}^+$$

Theorem 7. (Monotonicity) Let $\tilde{a}_j (j = 1, 2, \ldots, n)$ and $\tilde{a}_j' (j = 1, 2, \ldots, n)$ be two set of IVIFNs, if $\tilde{a}_j \leq \tilde{a}_j'$, for all $j$, then

$$IVIFEGBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq IVIFEGBM^{p,q}(\tilde{a}_1', \tilde{a}_2', \ldots, \tilde{a}_n')$$

Theorem 8. (Commutativity) Let $\tilde{a}_j (j = 1, 2, \ldots, n)$ and $\tilde{a}_j' (j = 1, 2, \ldots, n)$ be two set of IVIFNs, then

$$IVIFEGBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = IVIFEGBM^{p,q}(\tilde{a}_1', \tilde{a}_2', \ldots, \tilde{a}_n')$$

Where $\tilde{a}_j' (j = 1, 2, \ldots, n)$ is any permutation of $\tilde{a}_j (j = 1, 2, \ldots, n)$.

4. Model for Archives Websites’ Performance Evaluation in Our Country with Interval-Valued Intuitionistic Fuzzy Information

The following assumptions or notations are used to represent the MADM problems for evaluating archives websites’ performance with interval-valued intuitionistic fuzzy information. Let $A = \{A_1, A_2, \ldots, A_n\}$ be a discrete set of alternatives. Let $G = \{G_1, G_2, \ldots, G_n\}$ be a set of attributes. The information about attribute weights is completely known. Let $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ be the weight vector of attributes, where $\omega_j > 0, j = 1, 2, \ldots, n$. Suppose that $\tilde{R} = (\tilde{R})_{nn} = ([a_{ij}, b_{ij}],[c_{ij}, d_{ij}])_{nn}$ is the interval-valued intuitionistic fuzzy decision matrix, where $[a_{ij}, b_{ij}]$ indicates the degree that the alternative $A_i$ satisfies the attribute $G_j$ given by the decision maker, $[c_{ij}, d_{ij}]$ indicates the degree that the alternative $A_i$ doesn’t satisfy the attribute $G_j$ given by the decision maker, $[a_{ij}, b_{ij}] \subseteq [0, 1]$, $[c_{ij}, d_{ij}] \subseteq [0, 1]$, $b_{ij} + d_{ij} \leq 1$, $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.

In the following, we apply the IVIFEGBM operator to MADM problems for evaluating the archives websites’ performance with interval intuitionistic fuzzy information. The method involves the following steps:
Step 2. Utilize the IVIFEBM operator
\[ \tilde{r}_i = ([a_i, b_i], [c_i, d_i]) \]
= IVIFEBM^{p,q}(\tilde{a}_{i1}, \tilde{a}_{i2}, \cdots, \tilde{a}_{in}) \]
\[
= \left( \sum_{m,k=1}^{n} \frac{w_m w_k a_{im}^p a_{ik}^q}{1 - w_m} \right)^{1/(p+q)}, i = 1, 2, \cdots, m. \tag{16}
\]

To derive the overall preference values \( \tilde{r}_i (i = 1, 2, \cdots, m) \) of the alternative \( A_i \).

Step 3. Calculate the scores \( S(\tilde{r}_i) (i = 1, 2, \cdots, m) \) of the collective overall values \( \tilde{r}_i (i = 1, 2, \cdots, m) \) to rank all the alternatives \( A_i (i = 1, 2, \cdots, m) \) and then to select the best one(s) (if there is no difference between two scores \( S(\tilde{r}_i) \) and \( S(\tilde{r}_j) \), then we need to calculate the accuracy degrees \( H(\tilde{r}_i) \) and \( H(\tilde{r}_j) \) of the collective overall preference values \( \tilde{r}_i \) and \( \tilde{r}_j \), respectively, and then rank the alternatives \( A_i \) and \( A_j \) in accordance with the accuracy degrees \( H(\tilde{r}_i) \) and \( H(\tilde{r}_j) \).

Step 4. Rank all the alternatives \( A_i (i = 1, 2, \cdots, m) \) and select the best one(s) in accordance with \( S(\tilde{r}_i) \) and \( H(\tilde{r}_i) (i = 1, 2, \cdots, m) \).

Step 5. End.

5. Numerical example

Let us suppose there is an investment company, which wants to invest a sum of money for archives websites’ construction. There is a panel with five possible archives websites \( A_i (i = 1, 2, \cdots, 5) \) to invest the money. In order to evaluate archives websites’ performance of five archives websites, the investment company must take a decision according to the following four attributes: (1) \( G_1 \) is the network infrastructure (The network infrastructure is application system, including the support platform with the website file network infrastructure standard, network structure, outer net, intranet); (2) \( G_2 \) is the hardware environment (Hardware environment is archives facility, including the main hard equipment and hardware maintenance procedures); (3) \( G_3 \) is the software environment (Software environment website file is the effective operation of the guarantee, including the operating system, database system, network management system and the business software); (4) \( G_4 \) is the operations management (Operations management refers to the process of operation website file in the implementation of the management, mainly including process management, security system management, business operation management, personnel management and system management). The five possible archives websites \( A_i (i = 1, 2, \cdots, 5) \) are to be evaluated using the interval intuitionistic fuzzy information by the decision maker under the above four attributes whose weighting vector \( \omega = (0.25, 0.18, 0.35, 0.22) \), as listed in the following matrix:

\[
\begin{array}{cccc}
G_1 & G_2 & G_3 & G_4 \\
A_1 & \begin{bmatrix} [0.3, 0.4], [0.4, 0.5] \end{bmatrix} & \begin{bmatrix} [0.5, 0.6], [0.1, 0.3] \end{bmatrix} & \begin{bmatrix} [0.4, 0.5], [0.3, 0.4] \end{bmatrix} & \begin{bmatrix} [0.4, 0.6], [0.2, 0.4] \end{bmatrix} \\
A_2 & \begin{bmatrix} [0.3, 0.6], [0.3, 0.4] \end{bmatrix} & \begin{bmatrix} [0.4, 0.7], [0.1, 0.2] \end{bmatrix} & \begin{bmatrix} [0.5, 0.6], [0.2, 0.3] \end{bmatrix} & \begin{bmatrix} [0.6, 0.8], [0.1, 0.2] \end{bmatrix} \\
\tilde{R} = A & \begin{bmatrix} [0.2, 0.5], [0.4, 0.5] \end{bmatrix} & \begin{bmatrix} [0.2, 0.3], [0.4, 0.6] \end{bmatrix} & \begin{bmatrix} [0.3, 0.5], [0.3, 0.4] \end{bmatrix} & \begin{bmatrix} [0.1, 0.3], [0.5, 0.6] \end{bmatrix} \\
A_4 & \begin{bmatrix} [0.4, 0.5], [0.3, 0.5] \end{bmatrix} & \begin{bmatrix} [0.5, 0.8], [0.1, 0.2] \end{bmatrix} & \begin{bmatrix} [0.2, 0.5], [0.3, 0.4] \end{bmatrix} & \begin{bmatrix} [0.4, 0.7], [0.1, 0.2] \end{bmatrix} \\
A_5 & \begin{bmatrix} [0.5, 0.6], [0.2, 0.4] \end{bmatrix} & \begin{bmatrix} [0.6, 0.7], [0.1, 0.3] \end{bmatrix} & \begin{bmatrix} [0.3, 0.4], [0.1, 0.3] \end{bmatrix} & \begin{bmatrix} [0.6, 0.7], [0.1, 0.3] \end{bmatrix}
\end{array}
\]

Then, we utilize the approach developed to evaluate archives websites’ performance of five archives websites.
Step 1. Utilize the decision information given in matrix $\bar{R}$, and the IVIFE BM operator, we obtain the overall preference values $\bar{r}_i$ of the archives website $A_i (i=1,2,\ldots,5)$.

$$
\bar{r}_1 = ([0.401,0.514],[0.220,0.395]), 
\bar{r}_2 = ([0.432,0.679],[0.172,0.280]), 
\bar{r}_3 = ([0.220,0.428],[0.377,0.497]), 
\bar{r}_4 = ([0.416,0.655],[0.183,0.322]), 
\bar{r}_5 = ([0.528,0.630],[0.134,0.294])
$$

Step 2. Calculate the scores $S(\bar{r}_i) (i=1,2,\ldots,5)$ of the overall preference values $\bar{r}_i (i=1,2,\ldots,5)$

$$
S(\bar{r}_1) = 0.150, 
S(\bar{r}_2) = 0.329, 
S(\bar{r}_3) = -0.113 
S(\bar{r}_4) = 0.283, 
S(\bar{r}_5) = 0.365
$$

Step 3. Rank all the archives website $A_i (i=1,2,3,4,5)$ in accordance with the scores $S(\bar{r}_i) (i=1,2,\ldots,5)$ of the overall preference values $\bar{r}_i (i=1,2,\ldots,5)$: $A_5 > A_2 > A_4 > A_1 > A_3$, and thus the most desirable archives website is $A_5$.

6. Conclusion

In this paper, we investigate the multiple attribute decision making (MADM) problems for evaluating the archives websites’ performance with interval-valued intuitionistic fuzzy information. Then, we propose the interval-valued intuitionistic fuzzy Einstein Bonferroni mean (IVIFE BM) operator based on the Einstein operations and Bonferroni mean with interval-valued intuitionistic fuzzy information. Furthermore, we propose the interval-valued intuitionistic fuzzy Einstein geometric Bonferroni mean (IVIFEGBM) operator based on the IVIFE BM operator and geometric mean. Finally an illustrative example for archives websites’ performance assessment with interval-valued intuitionistic fuzzy information has been given to show the developed approach and to demonstrate its feasibility and practicality.

Acknowledgements

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References