Research of temperature variation in the egg heat conduction experiment

Rongzhen You

Thermal energy and Mechanical Engineering College, North China Electric Power University (Bao Ding), Bao Ding 071000, China

rzyou917@163.com

Abstract

In order to understand the change of physical form during the egg-heated process, an egg heat conduction experiment had been tried to analyses the thermal field in it. In this passage, the temperature variation regulation would be respectively analyzed by the analytical algorithm, lumped parameter algorithm, numeric calculation algorithm and MATLAB simulation tools. The lumped parameter algorithm and the numeric algorithm should simplify the egg shape as a ball, and than calculate the heat transfer. As for MATLAB simulation tools, it would generate a temperature variation curve which can be compared with the experiment curve. Through the comparison between the calculation curves and the experiment curve, we can get the characters of each method and analyze each method's errors.

Keywords

analytical algorithm, numeric algorithm, MATLAB simulation.

1. Introduction

In the egg heat conduction experiment, we cut a hole on the egg and put the thermocouple though the hole into the egg core. Then the egg was put into the boiling water, at the same time the thermocouple would measure the temperature of the egg core. After the experiment, we can gain a list of temperature data varied with time. Lastly, we can process the data by four methods mentioned above and analyze the accuracy of each method.

In order to decrease the calculation complexity, we can assume that the egg density is 1000 1000kg/m^3 . We can check relative information to find that the egg specific heat is $3310\text{J/}(\text{kg*k})$ and the egg surface heat transfer coefficient in the boiling water is $\text{h=1200W/}(\text{m}^2\text{*k})$.

2. Problem Analysis

Firstly, the analytical algorithm would be conducted. Since the egg was simplified as a sphere, we can calculate the thermal diffusivity and the Biot number and than we can formulate the dimensionless excess-temperature during the unsteady normal state in a sphere. Based on this formula, we can solve out the time for the egg core temperature needed to reach the 80℃.

Secondly, we would use the lumped parameter algorithm to analyze the heat conduction progress. The unsteady heat-conduction phenomenon that objects are heated in the liquid is very common in the engineering. In this process, the object surface exchange heat with outside liquid by convection heat transfer, so the object internal energy would be increased. Since we can see egg as a high thermal conductive object, the egg can be counted as an isothermal sphere in calculation. Than we formulate the temperature variation in egg with the lumped parameter equation to calculate the internal temperature of the eggs changing with time. Therefore we can draw the temperature profile with the calculation result.
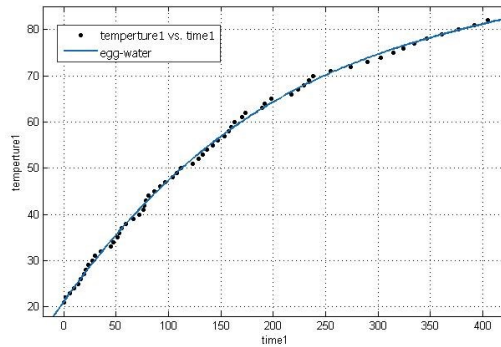
Thirdly, we would conduct the numeric algorithm. As for the egg seen as a sphere, we can mesh the sphere into 11 regions and respectively formulate the forward difference equations as well as the implicit back difference equation. After that, we would use the iterative method to solve the forward

difference equations and use the euler method to solve the implicit back difference equation. According to the calculation result, the temperature profile would be drawn and compared to the temperature profiles made by other methods.

Lastly, the MATLAB simulation tools would be used to simulate the dynamic process of heating egg. The temperature variation during the simulation can be recorded and protract the temperature profile.

3. Problem solution

3.1 Analytical algorithm

Firstly, we make the numerical fitting with the experimental data and get the practical heating process temperature profile.

Then we would conduct the analytical calculation and solve out the time needed to increase the egg core temperature to $82℃$.

Biot number:

$$Bi = \frac{hR}{\lambda}$$

$$Bi = \frac{1200 * 0.025}{0.7} = 42.86$$

Coefficient of cubical expansion:

$$a = \frac{\lambda}{\rho c}$$

$$a = \frac{0.7}{3310 * 1000} = 2.415 * 10^{-7}$$

Assume that the egg reach the standard conduction stage when the temperature of egg core reach $82℃$.

Characteristic root of transcendental equation：

$$1 - u_n \cot u_n = Bi \quad n = 1.2......$$

$$u_1 = 3.032$$

$$\frac{\theta_m}{\theta_0} = \frac{82 - 100}{20 - 100} = 0.025$$

$$2\frac{\sin u_1 - u_1 \cos u_1}{u_1 - \cos u_1 \sin u_1} = -2.0$$

In the egg core:

$$\eta = \frac{r}{l} = 0$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\frac{-2.0}{3.092 * \eta} \sin(3.092 * \eta) \exp(-3.092^2 F_0) = -2.0 * \exp(-3.0322 F_0) = 0.222$$

Solve out the Fourier number:

$$F_0 = 0.24$$

Solve out the time needed:

$$F_0 = \frac{at}{R^2} = \frac{1.51 * 10^{-7}}{0.022} \tau$$
$$\Rightarrow \tau = 635$$

It can be seen that, there is a big error between the analytical calculation results with reality.
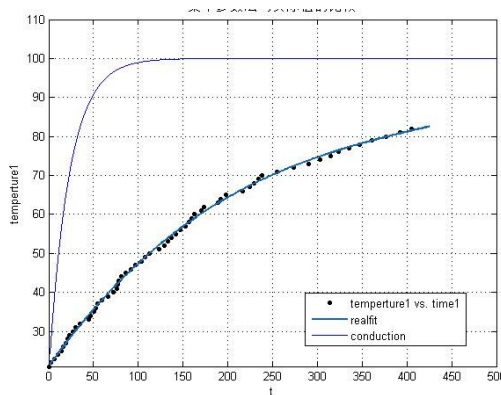
3.2 Lumped parameter algorithm

In the process of heating, we can consider that the egg inner thermal conductivity is much greater than the convective heat transfer coefficient on the egg surface. Therefore, we can think that the egg inner temperature is basically consistent and formulate the lumped parameter equations.
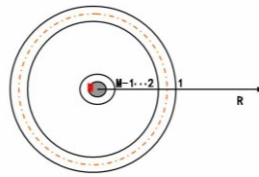
$$\rho c V \frac{dt}{d\tau} = -hA(t - t_\infty)$$

Boundary condition: $\tau = 0, \theta = t_0 - t_\infty = \theta_0$

Temperature expression:

$$t = \exp\left(-\frac{hA\tau}{\rho c V}\right)(t_0 - t_\infty) + t_\infty$$

It can be seen that, there is a big error between the lumped parameter curve with the reality curve. So the lumped parameter model can't describe the temperature variation well.

3.3 Numeric algorithm

  We can mesh the sphere into 11 regions.

$$\frac{\partial t}{\partial \tau} = \frac{a}{r^2}(\frac{\partial}{\partial r}(r^2 \frac{\partial t}{\partial r}))$$
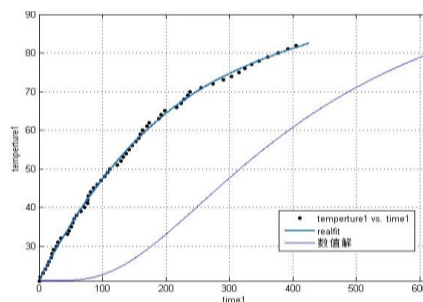
Boundary condition:
$$\begin{cases} \tau = 0, t = t_0 \\ r = 0, \dfrac{\partial t}{\partial r} = 0 \\ r = r_3, -\lambda \dfrac{\partial t}{\partial r} = h(t - t_\infty) \end{cases}$$

$$\nabla r = \frac{0.15}{M}$$

Establish the forward difference equations and solve the equations with iterative method.

| 节点	1	2	3	4	5	6	7	8	9	10	11
温度℃	99.52762	97.3078	94.81871	92.17543	89.50782	86.95221	84.64266	82.70237	81.236	80.32323	80.01448

  Profile:

From temperature profile above, we can find that the numeric algorithm curve has the similar trend with reality curve. In the early period, the numeric algorithm curve has a retard. Because we assume that the inner egg is an absolutely heat conduction model, but in reality the egg white is heat convection model and the egg core is heat conduction model. The convective heat transfer coefficient of egg white is mach greater than the heat conduction coefficient, which may influence the temperature increase rate in the early period. What's more, we assume that the egg surface heat transfer coefficient in the boiling water is $1200W/(m^2*k)$, but in reality the heat transfer coefficient is variational.

3.4 MATLAB simulation

Mesh the ellipse area at first, set the boundary condition and conduct the iteration calculation:

From the temperature figure above, we can find that the time needed to increase the egg core temperature to 82℃ is around 500 second, which is similar to the reality time. Therefore the MATLAB simulation method can describe the temperature variation in the egg well.

121
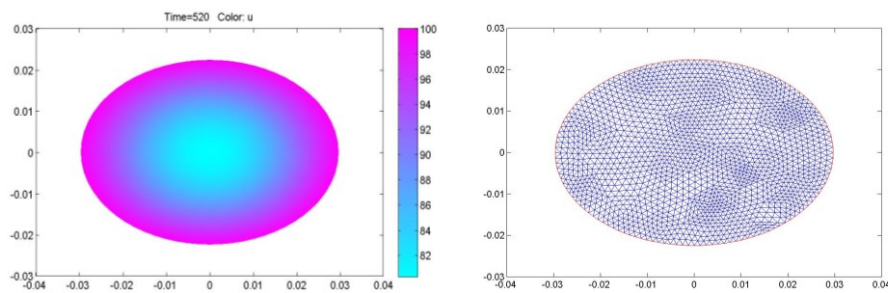
4. Conclusion

From what we discuss above, we can conclude that the MATLAB simulation is the most suitable method to describe the temperature variation among the four methods mentioned above. The simulation figure can vividly show the process of temperature variation, and the numeric algorithm can similarly describe the temperature variation in the egg core. Therefore these two method三 can be popularized to many areas to solve the complex geometry shape materials' heat transfer problem.

References

[1] W. Q. Tao, Heat Transfer[M], Beijing, Higher Education Press, 2005
[2] C. W. Jiang, J. P. Fu and L. T. Zhao, "Development of heat transfer virtual experiment based on MATLAB", China Electric Power Education, 2008.
[3] C. F. Li, "The application of mathematical model in the heat transfer", Hebei Normal University, 2005.
[4] C. X. Zhou, "Study on heat-induced gel formation and gel properties of egg", Nanjing Agricultural University, 2012.