Generalized Fuzzy Linguistic Weighted Bonferroni Mean Operator

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Abstract

In this paper, we investigate the multiple attribute decision making (MADM) problems with fuzzy linguistic information. Motivated by the ideal of generalized Bonferroni mean, we develop the generalized fuzzy linguistic Bonferroni Mean (GFLBM) operator for aggregating the fuzzy linguistic information. For the situations where the input arguments have different importance, we then define the generalized fuzzy linguistic weighted Bonferroni Mean(GFLWBM) operator, based on which we develop the procedure for multiple attribute decision making under the fuzzy linguistic environments.

Keywords

Multiple Attribute Group Decision Making; Fuzzy Linguistic Variables; generalized fuzzy linguistic Bonferroni Mean(GFLBM) operator; generalized fuzzy linguistic weighted Bonferroni Mean(GFLWBM) operator

1. Introduction

In the real world, human beings are constantly making decisions under linguistic environment [1-12]. For example, when evaluating the "comfort" or "design" of a car, linguistic terms like "good", "fair", "poor" are usually be used [1]. Sometimes, however, the decision makers are willing or able to provide only triangular fuzzy linguistic information because of time pressure, lack of knowledge, or data, and their limited expertise related to the problem domain [8]. Thus, Xu [8] developed some operators for aggregating triangular fuzzy linguistic variables, such as the fuzzy linguistic averaging (FLA) operator, fuzzy linguistic weighted averaging (FLWA) operator, fuzzy linguistic ordered weighted averaging (FLOWA) operator, and induced FLOWA (IFLOWA) operator, etc. Wei[13] investigated the multiple attribute group decision making problem with triangular fuzzy linguistic information, in which the attribute weights and expert weights take the form of real numbers, and the preference values take the form of triangular fuzzy linguistic variables and proposed some operators for aggregating triangular fuzzy linguistic variables, such as the fuzzy linguistic harmonic mean (FLHM) operator, fuzzy linguistic weighted harmonic mean (FLWHM) operator, fuzzy linguistic ordered weighted harmonic mean (FLOWHM) operator, and fuzzy linguistic hybrid harmonic mean (FLHHM) operator are proposed.

Yager [14] provided an interpretation of BM operator as involving a product of each argument with the average of the other arguments, a combined averaging and "anding" operator. Beliakov et al. [15] presented a composed aggregation technique called the generalized Bonferroni mean (GBM) operator, which models the average of the conjunctive expressions and the average of remaining. In fact, they extended the BM operator by considering the correlations of any three aggregated arguments instead of any two. However, both BM operator and the GBM operator ignore some aggregation information and the weight vector of the aggregated arguments. To overcome this drawback, Xia et al. [16] developed the generalized weighted Bonferroni mean (GWBM) operator as the weighted version of the GBM operator. Based on the GBM operator and geometric mean operator, they also developed the generalized Bonferoni geometric mean (GWBGM) operator. The fundamental characteristic of the GWBM operator is that it focuses on the group opinions, while the GWBGM operator gives more importance to the individual opinions. Because of the usefulness of the aggregation techniques, which reflect the correlations of arguments, most of them have been extended to fuzzy, intuitionistic fuzzy, or hesitant fuzzy environment [17–21].

In this paper, we investigate the multiple attribute decision making (MADM) problems with fuzzy linguistic information. Motivated by the ideal of generalized Bonferroni mean, we develop the generalized fuzzy linguistic Bonferroni Mean(GFLBM) operator for aggregating the fuzzy linguistic information. For the situations where the input arguments have different importance, we then define the generalized fuzzy linguistic weighted Bonferroni Mean(GFLWBM) operator, based on which we develop the procedure for multiple attribute decision making under the fuzzy linguistic environments.

2. Preliminaries

2.1 Fuzzy Linguistic Variables

Let $S=\{s_i \mid i=1,2,\cdots,t\}$ be a linguistic term set with odd cardinality. Any label, s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics: ①The set is ordered: $s_i > s_j$, if $i > j$; ② There is the reciprocal operator: $rec(s_i)=s_j$ such that $i=t+1-j$; ③ Max operator: $\max(s_i,s_j)=s_i$, if $s_i \ge s_j$; ④ Min operator: $\min(s_i,s_j)=s_i$, if $s_i \le s_j$. For example, S can be defined as [5]

$$\begin{aligned} S=\{&s_1=extremely\ poor, s_2=very\ poor, s_3=poor, s_4=medium\\ &s_5=good, s_6=very\ good, s_7=extremely\ good\}\end{aligned}$$

To preserve all the given information, we extend the discrete term set S to a continuous term set $\bar{S}=\{s_a \mid s_1 \le s_a \le s_q, a\in[1,q]\}$, where q is a sufficiently large positive integer. If $s_a \in S$, then we call s_a the original linguistic term, otherwise, we call s_a the virtual linguistic term. In general, the decision maker uses the original linguistic term to evaluate attributes and alternatives, and the virtual linguistic terms can only appear in calculation [5].

Definition 1. Let $s_\alpha, s_\beta \in \bar{S}$, then we call

$$d(s_\alpha,s_\beta)=|\alpha-\beta| \tag{1}$$

the distance between s_α and s_β [8].

In the following we introduce the concept of fuzzy linguistic variable.

Definition 2. Let $\tilde{s}=[s_\alpha,s_\beta,s_\gamma]\in\tilde{S}$, where $s_\alpha,s_\beta,s_\gamma\in\bar{S}$, s_α, s_β and s_γ are the lower, modal and upper values of $\tilde{s}$, respectively, then we call $\tilde{s}$ a fuzzy linguistic variable, which is characterized by the following member function [8]

$$\mu_{\tilde{s}}(\theta)=\begin{cases}0, & s_1\le s_\theta\le s_\alpha\\ d(s_\theta,s_\alpha)/d(s_\beta,s_\alpha), & s_\alpha\le s_\theta\le s_\beta\\ d(s_\theta,s_\gamma)/d(s_\beta,s_\gamma), & s_\beta\le s_\theta\le s_\gamma\\ 0, & s_\gamma\le s_\theta\le s_q\end{cases} \tag{2}$$

Clearly, s_β gives the maximal grade of $\mu_{\tilde{s}}(\theta)\left(\mu_{\tilde{s}}(\theta)=1\right)$, s_α and s_γ are the lower and upper bounds with limit the field of the possible evaluation. Especially, if $s_\alpha = s_\beta = s_\gamma$, then $\tilde{s}$ is reduced to a linguistic variable.

Let $\tilde{S}$ be the set of all fuzzy linguistic variables. Consider any three fuzzy linguistic variables $\tilde{s}=\left(s_\alpha, s_\beta, s_\gamma\right)$, $\tilde{s}_1=\left(s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}\right)$, $\tilde{s}_2=\left(s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}\right)\in\tilde{S}$, and suppose that $\lambda\in[0,1]$, then we define their operational laws as follows:

(1) $\tilde{s}_1\otimes\tilde{s}_2=\left(s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}\right)\otimes\left(s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}\right)=\left(s_{\alpha_1\alpha_2}, s_{\beta_1\beta_2}, s_{\gamma_1\gamma_2}\right)$;

(2) $\tilde{s}^\lambda=\left(s_\alpha, s_\beta, s_\gamma\right)^\lambda=\left(s_{\alpha^\lambda}, s_{\beta^\lambda}, s_{\gamma^\lambda}\right)$.

(3) $\tilde{s}^{-1}=\left(s_\alpha, s_\beta, s_\gamma\right)^{-1}=\left(s_{1/\gamma}, s_{1/\beta}, s_{1/\alpha}\right)$

In the following, we introduce a formula for comparing fuzzy linguistic variables.

Definition 3. Let $\tilde{s}_1=\left(s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}\right)$, $\tilde{s}_2=\left(s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}\right)\in\tilde{S}$, then the degree of possibility of $\tilde{s}_1\ge\tilde{s}_2$ is defined as[8]

$$p\left(\tilde{s}_1\ge\tilde{s}_2\right)=\lambda\max\left\{1-\max\left[d\left(s_{\beta_2}, s_{\alpha_1}\right)/\left(d\left(s_{\beta_1}, s_{\alpha_1}\right)+d\left(s_{\beta_2}, s_{\alpha_2}\right)\right),0\right],0\right\}+ \\ (1-\lambda)\max\left\{1-\max\left[d\left(s_{\gamma_2}, s_{\beta_1}\right)/\left(d\left(s_{\gamma_1}, s_{\beta_1}\right)+d\left(s_{\gamma_2}, s_{\beta_2}\right)\right),0\right],0\right\} \quad (3)$$

where the value λ is an index of rating attitude. It reflects the decision maker's risk-bearing attitude. If $\lambda<0.5$, the decision maker is risk lover. If $\lambda=0.5$, the decision maker is neutral to risk. If $\lambda>0.5$, the decision maker is risk avertor.

From Definition 3, we can easily get the following results easily:

(1) $0\le p\left(\tilde{s}_1\ge\tilde{s}_2\right)\le 1, 0\le p\left(\tilde{s}_2\ge\tilde{s}_1\right)\le 1$;

(2) $p\left(\tilde{s}_1\ge\tilde{s}_2\right)+p\left(\tilde{s}_2\ge\tilde{s}_1\right)=1$. Especially, $p\left(\tilde{s}_1\ge\tilde{s}_1\right)=p\left(\tilde{s}_2\ge\tilde{s}_2\right)=0.5$.

2.2 Bonferroni mean

Bonferroni [27] originally introduced a mean type aggregation operator, called Bonferroni mean, which can provide for aggregation lying between the max, min operators and the logical "or" and "and" operators, which was defined as follows:

Definition 5[27]. Let $p,q\ge 0$ and $a_i\,(i=1,2,\cdots,n)$ be a collection of non-negative real numbers. Then the aggregaton functions:

$$BM^{p,q}\left(a_1,a_2,\cdots,a_n\right)=\left(\frac{1}{n(n-1)}\sum_{\substack{i,j=1\\ i\ne j}}^{n}a_i^p a_j^q\right)^{\frac{1}{p+q}} \quad (4)$$

is called the Bonferroni mean (BM) operator.

Beliakov et al. [14] further extended the BM operator by considering the correlations of any three aggregated arguments instead of any two.

Definition 2. Let $p,q,r\ge 0$ and $a_i\,(i=1,2,\cdots,n)$ be a collection of nonnegative numbers. If

$$GBM^{p,q,r}\left(a_1,a_2,\cdots,a_n\right)=\left(\frac{1}{n(n-1)(n-2)}\sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} a_i^p a_j^q a_k^r\right)^{\frac{1}{p+q+r}} \tag{5}$$

then $GBM^{p,q,r}$ is called the generalized Bonferroni mean (GBM) operator.

In particular, if $r=0$, then the GBM operator reduces to the BM operator. However, it is noted that both BM operator and the GBM operator do not consider the situation that $i=j$ or $j=k$ or $i=k$, and the weight vector of the aggregated arguments is not also considered. To overcome this drawback, Xia et al. [15] defined the weighted version of the GBM operator.

Definition 2. Let $p,q,r\geq 0$ and $a_i\left(i=1,2,\cdots,n\right)$ be a collection of nonnegative numbers with the weight vector $w=\left(w_1,w_2,\cdots,w_n\right)^T$ and $w_j>0$, $\sum_{j=1}^{n} w_j=1$. If

$$GWBM^{p,q,r}\left(a_1,a_2,\cdots,a_n\right)=\left(\frac{1}{n(n-1)(n-2)}\sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n} w_i w_j w_k a_i^p a_j^q a_k^r\right)^{\frac{1}{p+q+r}} \tag{6}$$

then $GWBM^{p,q,r}$ is called the generalized weighted Bonferroni mean (GWBM) operator.

3. Generalized Fuzzy Linguistic Weighted Bonferroni Mean Operator

Based on the well-known generalized Bonferroni Mean operator, in the following, we shall develop the generalized fuzzy linguistic Bonferroni Mean(GFLBM) operator to deal with fuzzy linguistic information.

Definition 5. Let $\tilde{s}_i=\left[s_{\alpha_i},s_{\beta_i},s_{\gamma_i}\right]\left(i=1,2,\cdots,n\right)$ be a set of fuzzy linguistic variables, and let $p,q,r\geq 0$. If

$$\begin{aligned}
&\mathrm{GFLBM}^{p,q,r}\left(\tilde{s}_1,\tilde{s}_2,\cdots,\tilde{s}_n\right)\\
&=\left(\frac{1}{n(n-1)(n-2)}\mathop{\oplus}_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(\tilde{s}_i{}^p\otimes\tilde{s}_j{}^q\otimes\tilde{s}_k{}^r\right)\right)^{\frac{1}{p+q+r}}\\
&=\left[\left(\frac{1}{n(n-1)(n-2)}\mathop{\oplus}_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(\left(s_{\alpha_i}\right)^p\otimes\left(s_{\alpha_j}\right)^q\otimes\left(s_{\alpha_k}\right)^r\right)\right)^{\frac{1}{p+q+r}},\right.\\
&\quad\left(\frac{1}{n(n-1)(n-2)}\mathop{\oplus}_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(\left(s_{\beta_i}\right)^p\otimes\left(s_{\beta_j}\right)^q\otimes\left(s_{\beta_k}\right)^r\right)\right)^{\frac{1}{p+q+r}},\\
&\quad\left.\left(\frac{1}{n(n-1)(n-2)}\mathop{\oplus}_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n}\left(\left(s_{\gamma_i}\right)^p\otimes\left(s_{\gamma_j}\right)^q\otimes\left(s_{\gamma_k}\right)^r\right)\right)^{\frac{1}{p+q+r}},\right]
\end{aligned} \tag{7}$$

then $\mathrm{GFLBM}^{p,q,r}$ is called the generalized fuzzy linguistic Bonferroni Mean(GFLBM) operator.

Considering that the input arguments may have different importance, here we define the generalized fuzzy linguistic weighted Bonferroni Mean(GFLWBM) operator.

Definition 5. Let $\tilde{s}_i = \left[s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i}\right](i=1,2,\cdots,n)$ be a set of fuzzy linguistic variables, and let $p,q,r \geq 0$. $w=(w_1,w_2,\cdots,w_n)^T$ is the weight vector of $\tilde{s}_i = \left[s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i}\right](i=1,2,\cdots,n)$, where w_i indicates the importance degree of $\tilde{s}_i = \left[s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i}\right]$, satisfying $w_i > 0(i=1,2,\cdots,n)$, and $\sum_{i=1}^{n} w_i = 1$. If

$$
\begin{aligned}
&\mathrm{GFLWBM}^{p,q,r}(\tilde{s}_1,\tilde{s}_2,\cdots,\tilde{s}_n) \\
&= \left(\frac{1}{n(n-1)(n-2)} \mathop{\oplus}_{\substack{i,j,k=1 \\ i\neq j\neq k}}^{n} \left(w_i w_j w_k \tilde{s}_i^{\,p} \otimes \tilde{s}_j^{\,q} \otimes \tilde{s}_k^{\,r} \right) \right)^{\frac{1}{p+q+r}} \\
&= \left[\left(\frac{1}{n(n-1)(n-2)} \mathop{\oplus}_{\substack{i,j,k=1 \\ i\neq j\neq k}}^{n} \left(w_i w_j w_k \left(s_{\alpha_i}\right)^p \otimes \left(s_{\alpha_j}\right)^q \otimes \left(s_{\alpha_k}\right)^r \right) \right)^{\frac{1}{p+q+r}}, \right. \\
&\quad \left(\frac{1}{n(n-1)(n-2)} \mathop{\oplus}_{\substack{i,j,k=1 \\ i\neq j\neq k}}^{n} \left(w_i w_j w_k \left(s_{\beta_i}\right)^p \otimes \left(s_{\beta_j}\right)^q \otimes \left(s_{\beta_k}\right)^r \right) \right)^{\frac{1}{p+q+r}}, \\
&\quad \left. \left(\frac{1}{n(n-1)(n-2)} \mathop{\oplus}_{\substack{i,j,k=1 \\ i\neq j\neq k}}^{n} \left(w_i w_j w_k \left(s_{\gamma_i}\right)^p \otimes \left(s_{\gamma_j}\right)^q \otimes \left(s_{\gamma_k}\right)^r \right) \right)^{\frac{1}{p+q+r}}, \right]
\end{aligned}
\tag{8}
$$

then $\mathrm{GFLWBM}^{p,q,r}$ is called the generalized fuzzy linguistic weighted Bonferroni Mean(GFLWBM) operator.

Some special cases can be obtained as the change of the parameters as follows.

If $r=0$, then the generalized fuzzy linguistic weighted Bonferroni Mean(GFLWBM) operator reduces to the fuzzy linguistic weighted Bonferroni Mean(FLWBM) operator.

If $r=0, q=0$, the generalized fuzzy linguistic weighted Bonferroni Mean(GFLWBM) operator reduces to the following:

$$
\begin{aligned}
&\mathrm{GFLWBM}^{p,0,0}(\tilde{s}_1,\tilde{s}_2,\cdots,\tilde{s}_n) \\
&= \left(\frac{1}{n(n-1)(n-2)} \mathop{\oplus}_{\substack{i,j,k=1 \\ i\neq j\neq k}}^{n} \left(w_i w_j w_k \tilde{s}_i^{\,p} \otimes \tilde{s}_j^{\,0} \otimes \tilde{s}_k^{\,0} \right) \right)^{\frac{1}{p+0+0}} = \left[\left(\frac{1}{n} \mathop{\oplus}_{i=1}^{n} \left(w_i \left(s_{\alpha_i}\right)^p \right) \right)^{\frac{1}{p}} \right]
\end{aligned}
$$

4. An Approach to Multiple Attribute Decision Making under Fuzzy Linguistic Environment

In this section, we shall utilize the generalized fuzzy linguistic weighted Bonferroni Mean(GFLWBM) operator to multiple attribute decision making with fuzzy linguistic information. For a multiple attribute decision making problems with fuzzy linguistic information, let $A=\{A_1,A_2,\cdots,A_m\}$ be a discrete set of alternatives, $G=\{G_1,G_2,\cdots,G_n\}$ be the set of attributes, whose weight vector is $\omega=(\omega_1,\omega_2,\cdots,\omega_n)$,with $\omega_j \geq 0$, $j=1,2,\cdots,n$, $\sum_{j=1}^{n}\omega_j = 1$. Suppose that $\tilde{R}=(\tilde{r}_{ij})_{m\times n}$ is decision matrix, where $\tilde{r}_{ij} \in \tilde{S}$ is a preference value, which take the form of fuzzy linguistic variable, given by the decision maker for the alternative $A_i \in A$ with respect to the attribute $G_j \in G$.

Then, we utilize the generalized fuzzy linguistic Bonferroni Mean(GFLWBM) operator to develop an approach to multiple attribute decision making problems with fuzzy linguistic information, which can be described as following:

Step 1. Utilize the decision information given in matrix $\tilde{R}$, and the GFLWBM operator(in general, we can take $p = q = r = 1$)

$$\tilde{r}_i = \left[s_{\alpha_i}, s_{\beta_i}, s_{\gamma_i}\right] = \mathrm{GFLWBM}^{p,q,r}\left(\tilde{r}_{i1}, \tilde{r}_{i2}, \cdots, \tilde{r}_{in}\right)$$
$$= \left(\frac{1}{n(n-1)(n-2)} \mathop{\oplus}_{\substack{m,s,t=1 \\ m \neq s \neq t}}^{n} \left(w_m w_s w_t \tilde{s}_{im}{}^{p} \otimes \tilde{s}_{is}{}^{q} \otimes \tilde{s}_{it}{}^{r}\right)\right)^{\frac{1}{p+q+r}}$$
$$i = 1, 2, \cdots, m. \quad (9)$$

to derive the overall fuzzy linguistic preference values $\tilde{r}_i \left(i = 1, 2, \cdots, m\right)$ of the alternative A_i.

Step 2. To rank these collective overall preference values $\tilde{r}_i \left(i = 1, 2, \cdots, m\right)$, we first compare each $\tilde{r}_i$ with all the $\tilde{r}_j \left(j = 1, 2, \cdots, m\right)$ by using Eq. (3). For simplicity, we let $p_{ij} = p\left(\tilde{r}_i \geq \tilde{r}_j\right)$, then we develop a complementary matrix as $P = \left(p_{ij}\right)_{m \times m}$, where $p_{ij} \geq 0$, $p_{ij} + p_{ji} = 1$, $p_{ii} = 0.5$, $i, j = 1, 2, \cdots, n$.

Summing all the elements in each line of matrix P, we have

$$p_i = \sum_{j=1}^{m} p_{ij}, i = 1, 2, \cdots, m.$$

Then we rank the collective overall preference values $\tilde{r}_i \left(i = 1, 2, \cdots, m\right)$ in descending order in accordance with the values of $p_i \left(i = 1, 2, \cdots, m\right)$.

Step 3. Rank all the alternatives $A_i \left(i = 1, 2, \cdots, m\right)$ and select the best one(s) in accordance with the collective overall preference values $\tilde{r}_i \left(i = 1, 2, \cdots, m\right)$.

Step 4. End.

5. Conclusion

In this paper, we investigate the multiple attribute decision making (MADM) problems with fuzzy linguistic information. Motivated by the ideal of generalized Bonferroni mean, we develop the generalized fuzzy linguistic Bonferroni Mean (GFLBM) operator for aggregating the fuzzy linguistic information. For the situations where the input arguments have different importance, we then define the generalized fuzzy linguistic weighted Bonferroni Mean(GFLWBM) operator, based on which we develop the procedure for multiple attribute decision making under the fuzzy linguistic environments.

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