Some hesitant fuzzy aggregating operator with immediate probabilities

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Abstract

With respect to decision making problems by using probabilities, immediate probabilities and information that can be represented with hesitant fuzzy information, some new decision analysis are proposed. Then, we have developed some new probability aggregation operators with hesitant fuzzy information: probability hesitant fuzzy weighted average (P-HFWA) operator, immediate probability hesitant fuzzy ordered weighted average (IP-HFOWA) operator and probability hesitant fuzzy ordered weighted average (P-HFOWA) operator.

Keywords

decision making; hesitant fuzzy set; operational laws; probabilistic aggregating operators.

1. Introduction

Atanassov [1] introduced the concept of intuitionistic fuzzy set(IFS), which is a generalization of the concept of fuzzy set [2]. The intuitionistic fuzzy set has received more and more attention since its appearance[3-13]. Furthermore, Torra [14] proposed the hesitant fuzzy set which permits the membership having a set of possible values and discussed the relationship between hesitant fuzzy set and intuitionistic fuzzy set, and showed that the envelope of hesitant fuzzy set is an intuitionistic fuzzy set. Xia and Xu[15] gave an intensive study on hesitant fuzzy information aggregation techniques and their application in decision making. In order to aggregate the hesitant fuzzy information, they proposed a series of operators under various situations and discussed the relationships among them. Moreover, many scholars had developed some aggregation operators to solve the decision making problems with anonymity[16-18].

In this paper, we investigate the probabilistic decision making problems with hesitant fuzzy information, some new probabilistic decision making analysis methods are developed. Then, we have developed some new probability aggregation operators with hesitant fuzzy information: probability hesitant fuzzy weighted average (P-HFWA) operator, immediate probability hesitant fuzzy ordered weighted average (IP-HFOWA) operator and probability hesitant fuzzy ordered weighted average (P-HFOWA) operator.

2. Preliminaries

In the following, we shall introduce some basic concepts related to hesitant fuzzy sets and immediate probabilities.

2.1 Hesitant fuzzy sets

Atanassov[1] extended the fuzzy set to the IFS. However, when giving the membership degree of an element, the difficulty of establishing the membership degree is not because we have a margin of error, or some possibility distribution on the possibility values, but because we have several possible values. For such cases, Torra[14] proposed another generation of FS.

Definition 1[14]. Given a fixed set X , then a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a sunset of $[0,1]$.

To be easily understood, Xu express the HFS by mathematical symbol

$$E = \left(\left\langle x, h_E(x) \right\rangle \middle| x \in X \right), \tag{1}$$

where $h_E(x)$ is a set of some values in $[0,1]$, denoting the possible membership degree of the element $x \in X$ to the set E. For convenience, Xu call $h = h_E(x)$ a hesitant fuzzy element(HFE) and H the set of all HFEs.

Definition 2[15]. For a HFE h, $s(h) = \frac{1}{\#h}\sum_{\gamma \in h} \gamma$ is called the score function of h, where $\#h$ is the number of the elements in h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Based on the relationship between the HFEs and IFVs, Xia and Xu[16] define some new operations on the HFEs h, h_1 and h_2:

(1) $h^\lambda = \bigcup_{\gamma \in h} \left\{ \gamma^\lambda \right\}$;

(2) $\lambda h = \bigcup_{\gamma \in h} \left\{ 1 - (1-\gamma)^\lambda \right\}$;

(3) $h_1 \oplus h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \gamma_1 + \gamma_2 - \gamma_1 \gamma_2 \right\}$;

(4) $h_1 \otimes h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \gamma_1 \gamma_2 \right\}$.

2.2 Probabilistic aggregating operators

Merigó[19-20] developed the probabilistic weighted average (PWA) operator which is an aggregation operator that unifies the probability and the weighted average in the same formulation considering the degree of importance that each concept has in the aggregation.

Definition 3. An PWA operator of dimension n is a mapping PWA: $R^n \to R$, such that

$$\mathrm{PWA}(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} v_j a_j \tag{2}$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be the weight vector of $a_j\ (j = 1, 2, \cdots, n)$, and $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$, and a probabilistic weight $p_j > 0$, $\sum_{j=1}^{n} p_j = 1$, $v_j = \beta p_j + (1-\beta)\omega_j$ with $\beta \in [0,1]$ and v_j is the weight that unifies probabilities and WAs in the same formulation, then PWA is called the probabilistic weighted average(PWA) operator. The PWA operator is monotonic, commutative, bounded and idempotent[19-20].

Merigó[21] proposed the immediate probability (IP) which tries to include the decision maker's attitude in a probabilistic decision-making problem. The main advantage is that it is easy to apply it in almost all the probabilistic problems studied before such as in decision-making problems, actuarial sciences and statistics. Because the probabilistic information is objective but uncertain, we cannot then guarantee that the expected result is the result that will happen in the future. If we are in the situations of uncertainty (risk environments), each decision maker will have different attitudes towards the same problem.

In order to develop the analysis, Merigó[21] used in the same formulation the weights of the OWA operator and the probabilistic information and proposed the immediate probability OWA(IP-OWA) operator. It can be defined as follows.

Definition 4[21]. An IP-OWA operator of dimension n is a mapping IP-OWA: $R^n \to R$, that has an associated weight vector $w = (w_1, w_2, \cdots, w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$\text{IP-OWA}(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} \hat{p}_j a_{\sigma(j)} \tag{3}$$

where $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, such that $\alpha_{\sigma(j-1)} \geq \alpha_{\sigma(j)}$ for all $j = 2, \cdots, n$, each a_i has associated a probability p_i, p_j is the associated probability of $\alpha_{\sigma(j)}$, and $\hat{p}_j = \frac{w_j p_j}{\sum_{j=1}^{n} w_j p_j}$.

It's worth pointing out that IP-OWA operator is a good approach for unifying probabilities and OWAs in some particular situations. But it is not always useful, especially in situations where we want to give more importance to the probabilities or to the OWA operators. In order to show why this unification does not seem to be a final model, we could also consider other ways of representing $\hat{p}_j$. For example, we could also use $\hat{p}_j = \frac{w_j + p_j}{\sum_{j=1}^{n} (w_j + p_j)}$ or other similar approaches.

Another approach for unifying probabilities and OWAs in the same formulation is the probabilistic OWA (POWA) operator [19-24]. Its main advantage is that it is able to include both concepts considering the degree of importance of each case in the problem.

Definition 5. An POWA operator of dimension n is a mapping PWA: $R^n \to R$ that has an associated weight vector $w = (w_1, w_2, \cdots, w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$\text{POWA}(a_1, a_2, \cdots, a_n) = \sum_{j=1}^{n} \hat{p}_j a_{\sigma(j)} \tag{4}$$

where $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, such that $\alpha_{\sigma(j-1)} \geq \alpha_{\sigma(j)}$ for all $j = 2, \cdots, n$, each a_i has associated a probability p_i, with $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0,1]$, $\hat{p}_j = \beta p_j + (1-\beta) w_j$ with $\beta \in [0,1]$ and p_j is the associated probability of $\alpha_{\sigma(j)}$.

3. Some hesitant fuzzy aggregating operator with immediate probabilities

Based on the aggregation principle for HFEs, Xia and Xu[15] developed the hesitant fuzzy weighted averaging(HFWA) operator and hesitant fuzzy ordered weighted averaging (HFOWA) operator.

Definition 6. Let h_j $(j = 1, 2, \cdots, n)$ be a collection of HFEs. The hesitant fuzzy weighted averaging (HFWA) operator is a mapping $H^n \to H$ such that

$$HFWA(h_1,h_2,\cdots,h_n)=\overset{n}{\underset{j=1}{\oplus}}(\omega_j h_j)=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\left\{1-\prod_{j=1}^{n}(1-\gamma_j)^{\omega_j}\right\} \tag{5}$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ be the weight vector of $h_j\ (j=1,2,\cdots,n)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$.

Definition 7[15]. Let $h_j\ (j=1,2,\cdots,n)$ be a collection of HFEs, the hesitant fuzzy ordered weighted averaging (HFOWA) operator of dimension n is a mapping HFOWA: $H^n\to H$, that has an associated weight vector $w=(w_1,w_2,\cdots,w_n)^T$ such that $w_j>0$ and $\sum_{j=1}^{n}w_j=1$. Furthermore,

$$\begin{aligned}HFOWA(h_1,h_2,\cdots,h_n)&=\overset{n}{\underset{j=1}{\oplus}}(w_j h_{\sigma(j)})\\&=\bigcup_{\gamma_{\sigma(1)}\in h_{\sigma(1)},\gamma_{\sigma(2)}\in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)}\in h_{\sigma(n)}}\left\{1-\prod_{j=1}^{n}(1-\gamma_{\sigma(j)})^{w_j}\right\}\end{aligned} \tag{6}$$

where $(\sigma(1),\sigma(2),\cdots,\sigma(n))$ is a permutation of $(1,2,\cdots,n)$, such that $h_{\sigma(j-1)}\ge h_{\sigma(j)}$ for all $j=2,\cdots,n$.

Similarly, the PWA operator, IPOWA operator and POWA operator have usually been used in situations where the input arguments are the exact values. In the following, we shall extend these operators to accommodate the situations where the input arguments are hesitant fuzzy information. These operators include: probability hesitant fuzzy weighted average (P-HFWA) operator, immediate probability hesitant fuzzy ordered weighted average (IP-HFOWA) operator and probability hesitant fuzzy ordered weighted average (P-HFOWA) operator.

Definition 8. Let $h_j\ (j=1,2,\cdots,n)$ be a collection of HFEs, an P- HFWA operator of dimension n is a mapping P- HFWA: $Q^n\to Q$, such that

$$\text{P-HFWA}_{\hat{v}}(h_1,h_2,\cdots,h_n)=\overset{n}{\underset{j=1}{\oplus}}(\hat{v}_j h_j)=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2,\cdots,\gamma_n\in h_n}\left\{1-\prod_{j=1}^{n}(1-\gamma_j)^{\hat{v}_j}\right\} \tag{7}$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ be the weight vector of $a_j\ (j=1,2,\cdots,n)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$, and a probabilistic weight $p_j>0$, $\sum_{j=1}^{n}p_j=1$, $\hat{v}_j=\beta p_j+(1-\beta)\omega_j$ with $\beta\in[0,1]$ and $\hat{v}_j$ is the weight that unifies probabilities and HFWAs in the same formulation.

Definition 9. Let $h_j\ (j=1,2,\cdots,n)$ be a collection of HFEs, an IP-HFOWA operator of dimension n is a mapping IP-HFOWA: $Q^n\to Q$, that has an associated weight vector $w=(w_1,w_2,\cdots,w_n)^T$ such that $w_j>0$ and $\sum_{j=1}^{n}w_j=1$. Furthermore,

$$\begin{aligned}&\text{IP-HFOWA}_{\hat{p}}\left(h_1,h_2,\cdots,h_n\right)=\overset{n}{\underset{j=1}{\oplus}}\left(\hat{p}_j h_{\sigma(j)}\right)\\&=\bigcup_{\gamma_{\sigma(1)}\in h_{\sigma(1)},\gamma_{\sigma(2)}\in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)}\in h_{\sigma(n)}}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{\hat{p}_j}\right\}\end{aligned} \tag{8}$$

where $\left(\sigma(1),\sigma(2),\cdots,\sigma(n)\right)$ is a permutation of $\left(1,2,\cdots,n\right)$, such that $h_{\sigma(j-1)}\geq h_{\sigma(j)}$ for all $j=2,\cdots,n$, each a_i has associated a probability p_i, p_j is the associated probability of $\tilde{\alpha}_{\sigma(j)}$, and $\hat{p}_j=\dfrac{w_j p_j}{\sum_{j=1}^{n} w_j p_j}$.

It's worth pointing out that IP-HFOWA operator is a good approach for unifying probabilities and HFOWAs in some particular situations. But it is not always useful, especially in situations where we want to give more importance to the probabilities or to the HFOWA operators. In order to show why this unification does not seem to be a final model, we could also consider other ways of representing $\hat{p}_j$. For example, we could also use $\hat{p}_j=\dfrac{w_j+p_j}{\sum_{j=1}^{n}\left(w_j+p_j\right)}$ or other similar approaches.

Definition 10. Let $h_j\left(j=1,2,\cdots,n\right)$ be a collection of HFEs, an P-HFOWA operator of dimension n is a mapping P-HFOWA: $Q^n\rightarrow Q$, that has an associated weight vector $w=\left(w_1,w_2,\cdots,w_n\right)^T$ such that $w_j>0$ and $\sum_{j=1}^{n}w_j=1$. Furthermore,

$$\begin{aligned}&\text{P-HFOWA}_{\tilde{p}}\left(h_1,h_2,\cdots,h_n\right)=\overset{n}{\underset{j=1}{\oplus}}\left(\tilde{p}_j h_{\sigma(j)}\right)\\&=\bigcup_{\gamma_{\sigma(1)}\in h_{\sigma(1)},\gamma_{\sigma(2)}\in h_{\sigma(2)},\cdots,\gamma_{\sigma(n)}\in h_{\sigma(n)}}\left\{1-\prod_{j=1}^{n}\left(1-\gamma_{\sigma(j)}\right)^{\tilde{p}_j}\right\}\end{aligned} \tag{9}$$

where $\left(\sigma(1),\sigma(2),\cdots,\sigma(n)\right)$ is a permutation of $\left(1,2,\cdots,n\right)$, such that $h_{\sigma(j-1)}\geq h_{\sigma(j)}$ for all $j=2,\cdots,n$, each $\tilde{a}_i$ has associated a probability p_i, with $\sum_{i=1}^{n}p_i=1$ and $p_i\in\left[0,1\right]$, $\tilde{p}_j=\beta p_j+\left(1-\beta\right)w_j$ with $\beta\in\left[0,1\right]$ and p_j is the associated probability of $\tilde{\alpha}_{\sigma(j)}$.

4. Conclusion

In this paper, we investigate the probabilistic decision making problems with hesitant fuzzy information, some new probabilistic decision making analysis methods are developed. Then, we have developed some new probability aggregation operators with hesitant fuzzy information: probability hesitant fuzzy weighted average (P-HFWA) operator, immediate probability hesitant fuzzy ordered weighted average (IP-HFOWA) operator and probability hesitant fuzzy ordered weighted average (P-HFOWA) operator.

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