

## Analyzing of the timber beams of ancient buildings strengthened with NSM stainless steel bars

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### Abstract

ANSYS11.0 which is a finite element software is applied to the numerical stimulation of 4 timber beams :L1 (not reinforced), L2(strengthened with one CFRP sheets), L3 (strengthened with NSM stainless steel bars), L4 (strengthened with one CFRP sheets and NSM stainless steel bars), it is analyzed that the performance indexes under different strengthening methods. The results show that the ultimate load of reinforced timber beams increases by 7.91%~36.10%,the ductility increases by 12.40%~27.55%, the stiffness increases by 4.61%~20.53%,L4 has the best effect in the 4 timber beams, all performances of it have been greatly improved. This method can be used and extended for the restoration of ancient buildings. Based the formula about the bearing capacity and existing theory, a formulas about the bearing capacity of the timber structural members strengthened with NSM stainless steel bars are presented in this dissertation, the comparison of the results obtained using the formulas and ANSYS, the error which is in the allowable range is 8.66%~11.03%, it verifies that using ANSYS software to the numerical stimulation of timber beams strengthened with NSM stainless steel bars is feasible.

### Keywords

timber beams; stainless steel bars; NSM; ANSYS .

### 1. Introduction

The wood components of ancient buildings heritage value is priceless, because there is no suitable materials and technology, its development has been lagging behind for a long time. Currently, there are the main methods of restoring the ancient buildings, such as, replacing the entire column and the entire beam, filling it with concrete. However, it violates the principle of "repaired old as the old". There are many traditional methods reinforcement, such as, adding nailing, bolting reinforcement, banding circumferential, etc. Those methods would undermine the original style of buildings, and bring harm and corrosion to the original components, which lost as the value of cultural heritage. For other traditional reinforcement methods, such as, embedded reinforcing, the dig and patching, etc. Although those methods can meet the principle of "repaired old as the old", its stress performance is far less than the original component, and can't resist pests and environmental erosion, the reason is that the timber beam become two parts after reinforcing. The wooden structure of ancient building maintenance and reinforcement of traditional scientific methods cannot meet the actual needs of the project. This paper presents an construction technology of strengthened with NSM, which it can reinforce and restore the component of ancient building with the more serious damage, and reserve the damaged components maximize, and restore its carrying capacity.

It is the way to engrave on the surface of wooden component with NSM stainless steel bars, then strengthen with one CFRP sheets. This method can be used to retain the part original components when the timber beams of ancient buildings are damaged and loss bearing capacity. And it can retain the value of cultural relics of ancient buildings, recover the load function, and improve the components of the corrosion resistance and durability. The difference of its appearance and the

original part is small, and it meet with the principle of "repaired old as the old". It is great significance to protect cultural relics of the state.

At present, the study on strengthening the wooden components is experimental research. For limiting conditions or large structural testing, there is very high cost and extraordinary difficulty. The time of experiment is long. And it is very difficult to obtain pure analytic solution in theory. So, the study on the computer emulation method needs more try and summary. It can adapt to the more complicated process, the more novel strengthened test. It can meet the needs of practical engineering by using the above methods. In this paper, by using the finite element software ANSYS11.0, the bending performance of embedded reinforcement of ancient building beams is numerically simulated, and the results are analyzed and discussed, put forward some conclusions and suggestions.

## 2. The construction technology for the timber beams strengthened with NSM stainless steel bars

### 2.1 The construction process for the timber beams strengthened with NSM stainless steel bars

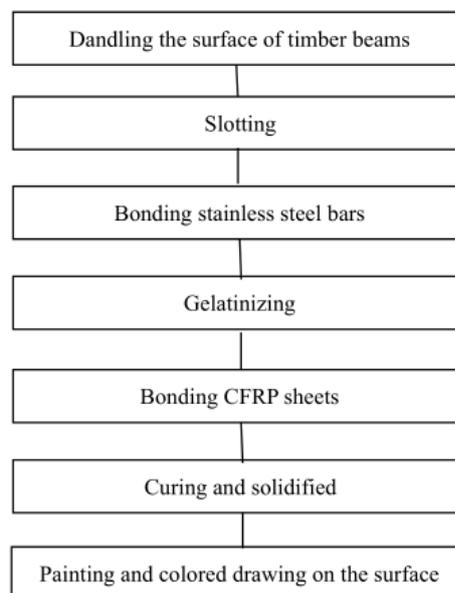


Fig.1 The construction process of timber beams strengthened with NSM stainless steel bars

#### 2.1.1 Handling the surface of timber beams

a. When the beams have different level decay, first, cleaning the rotten and treat preservative. Second, producing the beams of corresponding size and shape with dry wood, last, patching structural adhesive up. b. The beams' horizontal crack depth exceeds beam's span or diameter 1/4 and the results that calculates the bearing capacity of this structure do not satisfy the requirements of stress, it needs wood and structural adhesive to bond crack with NSM. c. There are rotten wood and debris to be removed which is the part of the timber moldy and wormy beams, then spraying with chemicals to protect against decay and insect damage. The strips of wood and structural adhesives will be used to fill tightly the gap.

#### 2.1.2 Slotting

The wooden components tick in the slot which its width and depth is greater than the cross-section of stainless steel bars with NSM and the groove slightly longer than the length of stainless steel bars after repairing the partial of the damaged wooden beams.

#### 2.1.3 Bonding stainless steel bars

a. we NSM with stainless steel bars after injecting stressfully structural adhesive in the groove. b. The embedded wood compact the stainless steel strip and inject stressfully structural adhesive in the groove again.

### 2.1.4 Gelatinizing

a. Brushing structural adhesive for bonding CFRP after inlaying stainless steel bars and the structural adhesive fully dry. b. When the surface of the wooden beams should make full dry, the glue evenly and without omission paint wooden beams on the surface. The adhesive is greater than the area of the size of pasted CFRP. c. Solidifying structural adhesive. Structural adhesive dried sufficiently need one hour or so. After drying, the surface is checked whether it is smooth or not, if it has the projections, we should use gauze to polish the smooth surface. The polished surface need to be careful and moderate and protect the structure layer is worn.

### 2.1.5 Bonding CFRP sheets

a. According to the size of wooden beams, we cut CFRP. b. According to the prevailing temperature and the amount of the general standard glue stick, the need of glue can be determined. It is certain the glue is used out. The configuration ratio of the main agent and hardener of epoxy adhesive fulfill the norm. The paste adhesive is evenly stirred by an electric mixer in a plastic container. But you cannot surpass the usage of adhesive glue which has been determined. The glue evenly and exhaustively coat are smeared on the surface of the wooden timber, especially, the area of the coating glue is greater than the bonding CFRP. d. We spread CFRP on the surface of the wooden timber which has been coated by glue. The spread CFRP need to fulfill the direction of design, and every CFRP should lap along the direction of carbon fiber, and the width of the lap fulfill the norm of design. We put rolling pressure with special rubber or squeegee along the direction of the fibers or from the middle of CFRP to the two sides. But this process does not allow for scrolling back and forth. In order to avoid the phenomenon of folding wire and bending wire, the construction should be carried out along the fiber direction, and don't allow along the perpendicular direction of the fiber. This work is mainly to fully make glue impregnated CFRP. Finally, the bubbles can be also eliminated and the excess glue can be wiped, it makes CFRP and the surface of wooden beams fully bonded.

### 2.1.6 Curing and solidified

a. We should be carried out the nature conservancy in 24 hours when finish every construction. The conservation process should avoid external rigid interference. b. During the construction every process and after completion, it should avoid being contaminated or rained and the protection measure should be taken in advance. c. To meet the design requirements, it needs nature conservation for three to five days, and shall be constructed in average temperature above 5°C.

### 2.1.7 Painting and colored drawing on the surface

a. We would paint the pattern which is the same as the original appearance of the wooden structures after pasted CFRP when the pasted CFRP is qualified during the acceptance. b. The level of painting, the original color and the decorative themes aren't changed when we paint decorative painting on the surface of ancient architectural.

## 2.2 The chart of timber beams strengthened with NSM stainless steel bars

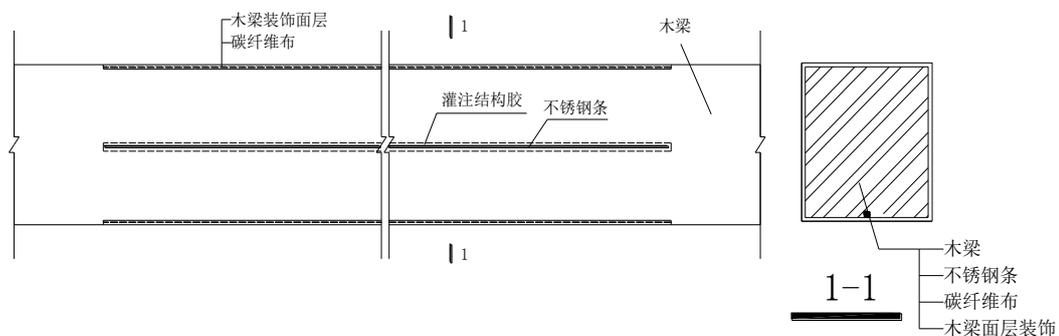


Fig.2 The chart of timber beams strengthened with NSM stainless steel bars

### 3. The scheme design

This paper introduces the model which designs the size of wooden beams of 2400mm\*80mm\*120mm. In order to avoid stress concentration, two cushion blocks are added in the upper of loading beam, and its size is 80mm\*80mm\*10mm. In particular, the pads need to set isotropic, the diameter of stainless steel is 8mm, the way of loading is two-loading, and the wooden beams of loading is at the position of third point. The finite element model of reinforcement scheme and specimen number is shown in Table 1 and Figure 3.

Table 1 Reinforcement scheme and specimen number

number	specimen number	Reinforcement scheme	rmark
1	L1	Reinforcement directly	see Fig. 2
2	L2	covering with a CFRP on the tension zone of beam	
3	L5	a entire stainless steel bar with NSM at the loading position and the diameter is 8mm	
4	L6	a entire stainless steel bar with NSM at the loading position and covered a CFRP	

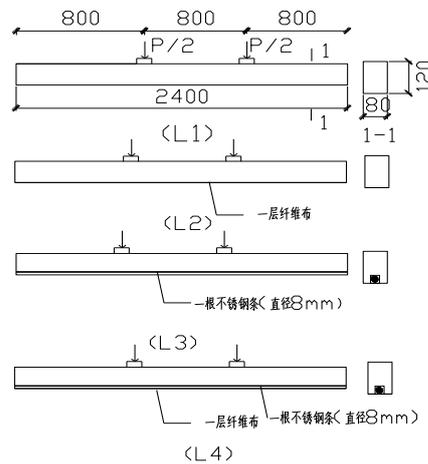


Fig.3 Finite element model

### 4. The material property

#### 4.1 Timber

It is considered that the anisotropic of the timber bring the effect to structure without regard to the peristalsis and relaxation behavior of timber. This paper defines three directions modulus of elasticity of wood and Poisson's ratio. The parameter [1] of wooden beams is shown in Table 2, Table 3.

Table2 The model parameters of timber beams

modulus of elasticity (MPa)			shear modulus (MPa)			Poisson's ratio		
EL	ET	ER	GLT	GLR	GRT	uTL	uLR	uRT
10000	275	650	275	650	210	0.02	0.3	0.035

Annotate: The unit of elastic modulus is MP above the Table. The Poisson ratio is non-dimensional. L-longitudinal; R- radial; T- tangential; LR- radial section; LT- tangential section RT-cross section

#### 4.2 CFRP

CFRP is orthotropic material, it uses the parameter of T300 carbon/epoxy composites in this book of "Encyclopedia of composite materials"[2]. CFRP of model parameters and performance indicators is shown in Table 4

Table 3 The strength of timber beams

bending strength (MPa)	compression strength parallel to grain (MPa)	tensile strength parallel to grain (MPa)	shear strength parallel to grain (MPa)
67.27	34.76	73.18	7.53

Table 4 The model parameters of CFRP

thickness (mm)	ensile strength (MPa)	modulus of elasticity (GPa)		shear modulus (GPa)	Poisson's ratio	
		E1	E2		$\gamma_1$	$\gamma_2$
0.167	3530	181	10.3	7.17	0.28	0.016

Annotate: E1 - longitudinal elastic modulus E2- lateral elastic modulus

G12- shear modulus in plane  $\gamma_1$ - longitudinal Poisson's ratio  $\gamma_2$ - transverse Poisson's ratio

### 4.3 Stainless steel bars

The stainless steel bar of this paper base on "stainless steel rod" (GB/T 1220-2007)<sup>[3]</sup>, its performance is shown in Table 5.

Table 5 The model parameters of stainless steel bars

name	modulus of elasticity	tensile strength	Poisson's ratio	density
stainless steel bars	206GPa	525MPa	0.3	7800kg/m

## 5. The foundation of finite element model

### 5.1 The selection of material element [4]

The unit of wooden beams and pad uses SOLID45. The unit has x, y, z direction of the three degrees of freedom. It has the functions of creep, expansion, plasticity, stress stiffening, large deformation, large strain and so on, and it supports anisotropic.

CFRP unit uses the SOLID46 of laminated farm SOLID45 to simulate performance. The unit has an equivalent lateral stiffness, there can be non-zero, strain and displacement in the lateral stress, failure criteria can also be determined. The unit has three degrees of freedom and they can move parallel along the x, y, z direction of the coordinate system.

The unit of stainless steel bar uses three-dimensional 10-node unit SOLID187, and each node has x, y, z direction three degrees of freedom. It has a secondary displacement and may simulate irregular model. The unit has some characteristics, such us, plastic, super elasticity, stress stiffening, creep, large deformation and large strain and so on.

### 5.2 Solid model and mesh generation

The model assumes that the material is reliable adhesion and no bonding slippage occurs among the material in the model of ANSYS. The solid elements bond together and form a whole by Boolean operations, then, it will be divided mesh. In this paper, the unit of boundary length 20mm is divided to the model mesh. The coincident nodes directly coupled the degree of freedom after dividing the mesh. As shown in Fig.4.

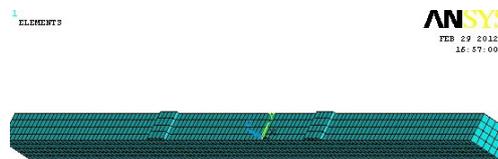


Fig.4 Finite element model after meshing

### 5.3 Loading model

Loading scheme is the two symmetrical loading, loading point is the third-point of wooden beams. The load applied to the wooden beams through pads, and to preventing the phenomenon of stress concentration. It compresses uniformly stress to the model of pressure face. The finite element model under load is shown in Fig.5.

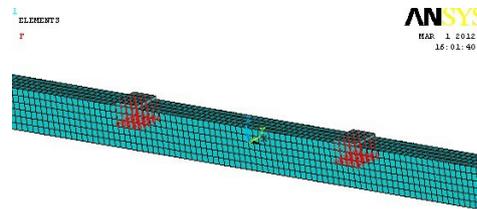


Fig.5 Finite element model under load

Then we add constraint on wooden beam which is from Charpy herein, and the constraint is shown in Fig.6.

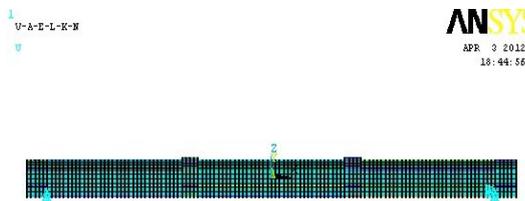


Fig.6 Finite element model after imposing constraints

### 5.4 Analysis result

#### 5.4.1 The ultimate load, rigidity, ductility

Table 6 The ultimate load and mid-span deflection of beams

The number of beam	Ultimate load (KN)	The maximum span deflection (mm)	Load increase rate (%)	Ductility increase rate (%)
L1	47.31	86.97	/	/
L2	51.05	97.75	7.91	12.40
L3	63.32	104.01	33.84	19.59
L4	64.39	110.93	36.10	27.55

By ANSYS finite element analysis, we can get the beam of the ultimate anti-bending load and the mid-span immunity under ultimate load. The table 6 gets the beams of the ultimate load and the ductility increased rate through analyzing the date.

Judging from the data above the Table 6, we can get the conclusion that the flexural ultimate bearing capacity of the beams can be improved by pasting CFRP in the tension zone or NSM the stainless steel bars. The ultimate capacity improved by 7.91% through pasting CFRP on the zone of reinforced wood beams. It is significantly to improve the bearing capacity with NSM stainless steel bars. The ultimate bearing capacity has been greatly improved, by 33.84%. The ultimate bearing capacity of wood beam improve greater by 36.10% after embedding CFRP on the tension zone. So, the effect of NSM with stainless steel bars is greater and more loading improvement than reinforced with CFRP. This reason is that the stainless steel bar has greater physical strength and elastic modulus than the CFRP, and the stainless steel bar bear more load when it NSM on the surface of the beam and becomes one of the original structure. It can obviously improve the integrity of the beams and closely combine the stainless steel bar with the original wooden beam when we NSM with stainless steel bar, while paste CFRP on the wooden beams.

For ductility, the paper uses it to compare with the mid-span immunity of the ultimate load. The mid-span flexibility of every beams is increased by 12.4%-27.55% under the ultimate load. In particular,

the improvement of L4 is the most. Namely, the common reinforcement of stainless steel bars and CFRP can remarkably improve the ultimate bearing capacity and the ductility of wooden beams. So, this is a worthy application method to reinforce and restore the ancient building.

Table 7 Improve effectiveness of stiffness

Serial number	Mid-span flexibility (mm)	The improvement of rigidity (%)
L1	-28.20	/
L2	-26.90	4.61
L3	-22.45	20.39
L4	-22.41	20.53

The stiffness compares with the every beams of mid-span flexibility in the same load. The paper takes 15KN to reflect the effect of improving the stiffness. The Table 7 reflect the comparison of mid-span flexibility in 4 group beams. They are increased by 4.61%- 20.53%. The improvement of L4 is the most, is 20.53%. So, the effect of improving stiffness is different on the different ways of reinforcement. And, the NSM with stainless steel bars is better effect than CFRP, the effect of increased stiffness is more apparent.

5.4.2 The load-strain curves

We take the measuring position of the mid-span cross section at the every beams as shown in Fig.7. And it gets the load-strain curves as shown in Fig.8-11 by ANSYS analysis.

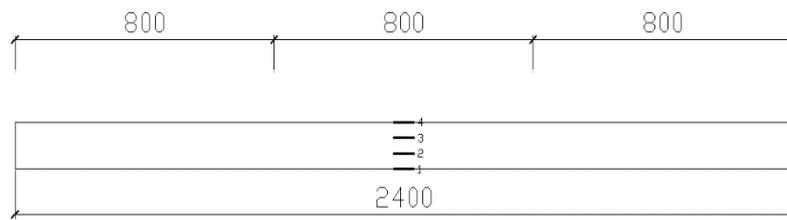


Fig.7 The strain gauge of timber beams

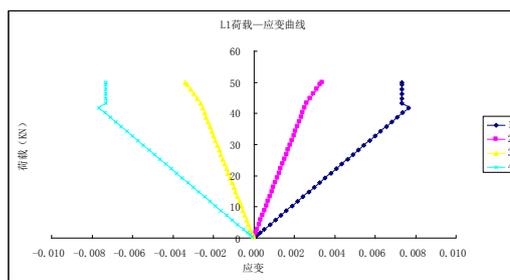


Fig.8 Load-strain curves of L1

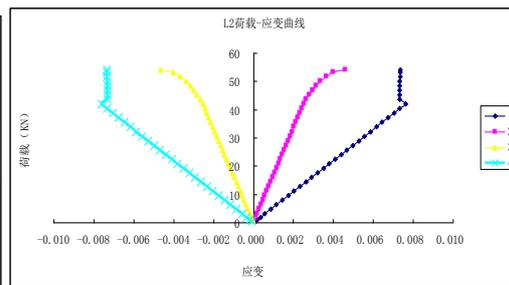


Fig.9 Load-strain curves of L2

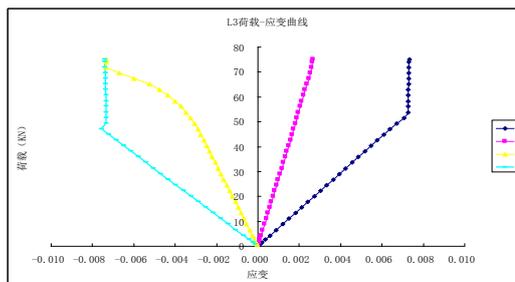


Fig.10 Load-strain curves of L3

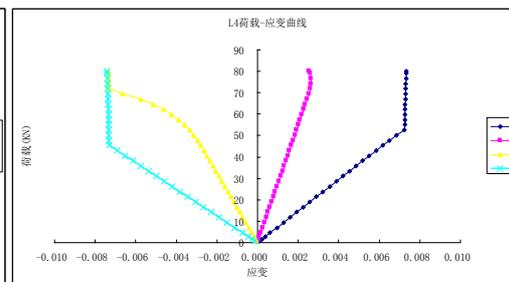


Fig.11 Load-strain curves of L4

Judging from the load-strain curves above the figures, we can see the wooden beams showing significant elasticity and plasticity during loading, at low loads, the wood is elasticity, at high loads, the timber is plasticity. As shown in the figures, the reinforced beams on the zone of bearing load is

larger plasticity than non-reinforced beams. So, it is indicated that the reinforced wooden beams significantly improve the deformation property of the wooden beams.

## 6. Theoretical analysis the bending performance of wooden beams

### 6.1 The basic assumption

a. The cross-section should accord with the strain assumption of plane section after the wooden beams in bending. b. The model is linear elastic when the beams has tensile elasticity, the model is ideal elastic and plastic when the beams has compression. c. The repaired wooden beams is substantially uniform material. We don't consider the natural defect of the timber, and the parallel grain tensile and compressive modulus of elasticity is the basically same. d. CFRP is an ideal linear elastic model. e. The stainless steel bar is an ideal elastic-plastic model. f. The stress of CFRP and stainless steel bar is approximate distribution, and the thickness of CFRP and structural adhesive don't affect the clear height of the wooden beams. f. Between CFRP materials and timber, the timber and the stainless steel bar, stainless steel bar and the CFRP, those bonding is reliable, is not relative slip.

### 6.2 The capacity calculation of NSM reinforcement

#### 6.2.1 The flexural failure mode

According to the different strength of compressive and tensile, the different strength of CFRP and stainless steel bar, the flexural failure of wood beam has the following three modes. a. The wood on the tension zone is pulled off. b. CFRP or stainless steel bar is pulled off, but the failure mode does not exist in reality. c. The timber on compression zone is crushed.

#### 6.2.2 The capacity calculation of NSM reinforcement

a. The wood on the tension zone is pulled off. When the wood on the tension zone is pulled off, the stress-strain diagram is shown in Fig.12

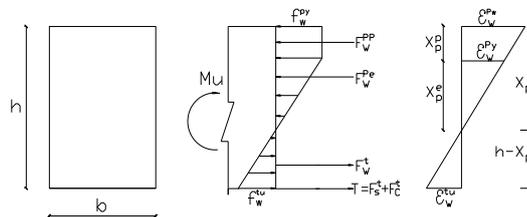


Fig.12 stress-strain diagram of timber on the tension zone of destruction

On variables d and t

$$\epsilon_w^{pw} < \epsilon_w^{pu}, \epsilon_c < \epsilon_c^u$$

According to the geometric relations

$$\frac{f_w^{py}}{f_w^{tu}} = \frac{\epsilon_w^{py}}{\epsilon_w^{tu}} = \frac{x_p^e}{h-x_p} = n(n \text{ is fixed value}) \tag{1}$$

$$x_p^p + x_p^e = x_p \tag{2}$$

Obtained by the hydrostatic balance

$$F_w^{pp} + F_w^{pe} = T + F_w^t \tag{3}$$

In which

$$F_w^{pp} = b f_w^{py} x_p^p; \quad F_w^{pe} = \frac{1}{2} b x_p^e f_w^{py}; \quad F_w^t = \frac{1}{2} b f_w^{tu} (h - x_p) \tag{4}$$

$$T = F_s^t + F_c^t = \sigma_s A_s + \rho_c A_c = E_s \varepsilon_w^{tu} A_s + E_c \varepsilon_w^{tu} A_c \quad (5)$$

Taking moments on the neutral axis, the expression of the ultimate bearing capacity is

$$M_u = F_w^{pp} (x_p - \frac{1}{2} x_p^p) + F_w^{pe} \frac{2}{3} x_p^e + F_w^t \frac{2}{3} (h - x_p) + F_c^t (h - x_p) + F_s^t (h - x_p - d/2) \quad (6)$$

The above formula can also be used to calculate the bearing capacity of un-reinforced wooden beams, we just let  $A_c = 0$ , we can get the bearing capacity of un-reinforced wooden beams.

b. The timber on compression zone is crushed. The stress-strain diagram is shown in Fig.13.

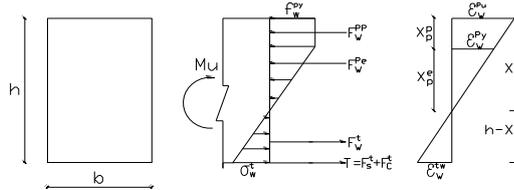


Fig.13 stress-strain diagram of timber on the compression zone of destruction

Obtained by the hydrostatic balance

$$F_w^{pp} + F_w^{pe} = T + F_w^t \quad (7)$$

In which

$$F_w^{pp} = b f_w^{py} x_p^p = b(1 - n) x_p f_w^{py} \quad (8)$$

$$F_w^{pe} = \frac{1}{2} b x_p^e f_w^{py} = \frac{1}{2} b n x_p f_w^{py} \quad (9)$$

$$F_w^t = \frac{1}{2} b (h - x_p) \sigma_w^t = E_w \varepsilon_w^{pu} \frac{b(h - x_p)^2}{2x_p} \quad (10)$$

$$T = F_s^t + F_c^t = \sigma_s A_s + \rho_c A_c = E_s \varepsilon_w^{tu} A_s + E_c \varepsilon_w^{tu} A_c = (E_s \varepsilon_w^{pu} A_s + E_c \varepsilon_w^{pu} A_c) \frac{h - x_p}{x_p} \quad (11)$$

Taking moments on the neutral axis, the expression of the ultimate bearing capacity is

$$M_u = F_w^{pp} (x_p - \frac{1}{2} x_p^p) + F_w^{pe} \frac{2}{3} x_p^e + F_w^t \frac{2}{3} (h - x_p) + F_s^t (h - x_p - d/2) + F_c^t (h - x_p) \quad (12)$$

The reinforced wooden beams alone NSM with stainless steel bars, we take  $F_c^t = 0$ .

In the formulas:  $F_w^{pp}, F_w^{pe}$  - The resultant force of compressive wooden beams on the elastic and plastic zone.  $F_w^t, F_c^t$  - The resultant force of tension wooden beams and CFRP.  $f_w^{py}, f_w^{tu}$  - The equivalent maximum compressive stress and ultimate tensile stress of the wood.  $\varepsilon_w^{pw}, \varepsilon_w^{tw}$  - The strain values outermost compression, tension wood beams.  $\varepsilon_w^{py}, \varepsilon_w^{pu}$  - The maximum elastic compressive strain, ultimate compressive strain of compression wood beams.  $\varepsilon_w^{tu}, \varepsilon_c^u$  - Tension wood beams ultimate tensile strain, tension CFRP ultimate tensile strain.  $\sigma_t, \varepsilon_t$  - CFRP stress, tensile strain values.  $b, h$  - The width and height of wooden beam section.  $x_p$  - The height of wooden beam compression zone.

$x_p^e, x_p^p$  -The height of wooden compressive beams on elastic and plastic zone.  $A_C$  - Sectional area of CFRP.  $A_S$ -Sectional area of stainless steel bar.  $a_E, a_F$  -The ratio of CFRP elastic modulus, stainless steel bar and elasticity modulus of wood.  $E_w, E_c, E_s$  - Elastic modulus of wood, CFRP and stainless steel bar.  $\sigma_c, \varepsilon_c$ -CFRP stress and strain.  $n$  - The ratio of compressive wood beam maximum elastic strain and tensile wood beam ultimate tensile strain in the ideal elastic-plastic model.  $m$  - The ratio of compressive wood beam maximum elastic strain and compressive wood beam ultimate compressive strain in the ideal elastic-plastic model.  $\sigma_s$  -The stress of stainless steel bar.  $d$  - The diameter of stainless steel bar.

### 6.3 ANSYS calculated value compares with theoretical value

Table 8 Comparison the ultimate load of formulas with ANSYS

number	ANSYS analysis (KN)	theoretical calculation (KN)	error (%)
L1	47.31	43.54	8.66
L2	51.05	45.98	11.03
L3	63.32	68.68	7.80
L4	64.39	70.15	8.21

By ANSYS simulation analysis and calculated theoretical formula, the ultimate bearing capacity of every timber beams is shown in Table 8. Judging from the above table, the value of ANSYS analysis and theoretical calculation, those errors is in the 8.66% ~ 11.32%, the difference is not large, and it is within the allowable range. So, it shows wooden beams reinforced with ANSYS simulation is feasible.

## 7. Conclusion

In this paper, the relevant values are processed and analyzed by ANSYS finite element software, and the ultimate bearing capacity of the ANSYS analysis values are compared with theoretical calculated, the conclusions are as follows.

- Reinforcing ancient buildings wooden beams with NSM, it can great improve the ultimate bearing capacity, ductility and stiffness of the wooden beams. The ultimate bearing capacity of L3 and L4 increases by 33.84% and 36.10%, the ductility increases by 19.59% and 27.55%, the rigidity increases by 20.39% and 20.53. It is more pronounced effect than reinforced wooden beams with CFRP.
- Load - strain curve is divided into two phases, the elastic and plastic phase, the curve is linear when it is into elastic phase. The wooden beams stiffness decreases gradually when it is into the plastic phase, while the immunity increases rapidly.
- According to the different compressive and tensile strength of wooden beams, stainless steel bars and CFRP, deduce the bearing capacity theoretical formula of reinforced wooden beams with NSM. And this paper compares the ANSYS analysis of beams in every group with the theoretical calculated values, the error is with 8.66% ~ 11.03%, and within the allowable range. It verify the ANSYS software to simulate the reinforced wood beams.
- Due to the discreteness of timber is large, the ancient building is affected by many environmental factors, there are also a lot of their own shortcomings and the anisotropy, the physical and mechanical properties is the large variability, the ancient building technique of wooden structure is substantial complex.

In this paper, ANSYS models and theoretical models are the ideal model, because it don't consider the impact of intermediate adhesive strength and structural thickness, etc., so we cannot completely simulate actual conditions. In order to verify the results of the analysis of this topic, a lot of work need to be done further.

## Reference

- [1] Yi Sici. Wood Science[M].Beijing: China Forestry Publishing House,1996

[2] Wo Dingzhu. Composite materials[M]. Chemical Industry Publishing House.2002:599

[3] (GB/T1220-2007).Stainless steel rods[S].Beijing: Chinese standards Publishing House.2007

[4]Xue Shouyi. Finite element method[M]. China Building Materials Industry Press.2005