The Path Planning and Control of Aircraft

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Abstract

The design and control of aircraft arouse great interest all over the world. In this paper, a dynamic model is established and the optimal control method is applied to determine the optimal landing trajectory of an aircraft, the path planning of the elliptical orbit, and guidance control strategy for fuel consumption. And the error analysis and sensitivity analysis are also made for the optimization scheme used.

Keywords

path planning , trajectory optimization , parameter normalization , contour map.

1. Introduction

To achieve a soft landing within the predetermined region accurately, the path planning and control strategy is necessary when the aircraft is at high speed. It is of great importance to reduce the total fuel consumption of this landing process.

The models e：

eestablished in this paper are based on the following three hypotheses:

(1) There is no influence of other celestial bodies in space on the moon and the aircraft;

(2) The attitude transform of the aircraft is instantaneous.

(3) The moon is supposed to be a sphere without considering the impact of the rotation of the moon;

(4) When the aircraft is on the preparing landing orbit, there is no precession;

(5) The trust magnitude can be changed instantaneously.

2. The determination of the position and the corresponding velocity

2.1 The model establishment

(1) The model of the system

ConCThe landing coordinate system shown in Figure 1is established.

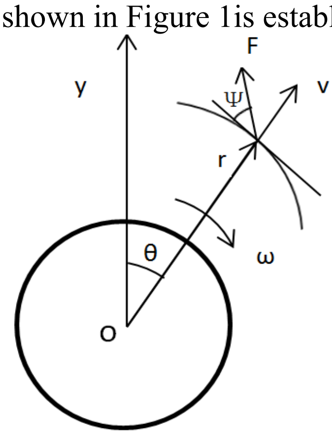

Fig. 1: model diagram

Assuming that the landing trajectory is in the longitudinal plane, the selenocentric coordinate is set to be the origin of coordinate O. Some symbols are listed as follows:

Table 1 Symbols

Symbols	The significance of the symbol
O_y	the starting braking points of the power down section
O_x	the starting direction of the movement of the aircraft
r	the selenocentric distance
θ	the polar angle
w	the angular velocity
m	quality of the aircraft
v	the velocity of the lander along the direction of r
F	the thrust of the retroengine(a fixed constant or 0)
I_{sp}	the specific impulse
μ	the moon's gravitational constant
$\emptyset$	The thrust direction angle between the engine thrust and the local horizontal line.
r_l	the moon radius,
h_0	the initial orbit height
w_0	the orbital angular velocity

The initial conditions of the power down section are determined by the pericynthion of the elliptical orbit later, while the terminal conditions are to achieve soft landing of the aircraft on the lunar surface. Setting the initial time $t_0 = 0$ and the terminal time t_f indefinite, the corresponding initial conditions are $r_0 = r_f + h_0, v_0 = 0$ and$w_0 = w_0$, and the terminal constraints are $r_f = r_l, v_f = 0$ and $w_f = 0$. The optimal orbit design is in the premise of satisfying the above conditions to adjust thrust magnitude and direction, which requires the following performance indexes reaching the maximum: $\bar{J} = \int_0^{t_f} \dot{m}\, dt$.

That is to transform it into a local minimum problem of this function under the multi-constrained conditions mentioned above.

(2) Parameterization method

Being discretized, the soft landing trajectory can be averagely divided into n sections and each node of each section is set a thrust direction $\emptyset$. Then, the thrust direction angle $\emptyset$ of the n+1 node and the terminal moment t_f can be provided as the parameters to be optimized. Then using the adaptive genetic algorithm to predict the optimal $\emptyset$ value of the next time point according to the $\emptyset$ value of the initial conditions at the initial time, optimal solutions of the $\emptyset$ values of every the same time interval can be calculated iteratively. The moment of each node can be obtained by the following formula: $t_i = \mathrm{t}_0 + i * \frac{\mathrm{t}_f - \mathrm{t}_0}{n}, (i = 0,1, \dots, n)$. Thus, the direction angle of the thrust of each node has a corresponding node time. If assuming that the thrust direction angle can be expressed as a polynomial:

$$\emptyset(\mathrm{t}) = \lambda_0 + \lambda_1 * t + \lambda_2 * t^2 + \lambda_3 * t^3$$

Then, the corresponding node moments and the thrust direction angles can be used to doing the fitting of this polynomial to obtain the coefficients of the polynomial $\lambda_i (i = 0,1, \dots, n)$, and then to gain the thrust direction angles of the entire landing trajectory.

Considering the accuracy of trajectory optimization, with the use of dynamic penalty function method to process the terminal constraints, the fitness function can be expressed in the following form:

$$\mathrm{f(x)} = -\sigma_1 [(v(t_f) - v_f)^2 + (\omega(t_f) r(t_f) - \omega_f r_f)^2]^{1/2} - \sigma_2 |r(t_f) - r_f| + m_0 + J$$

X is the individuals of population; $v(t_f)$, $\omega(t_f)$ and $r(t_f)$ are the state values of the terminal moment: σ_1, σ_2 are the penalty factors. With the optimization, they become larger, which makes the constraints gradually strengthened.

Using binary codes for this search, the crossover operation adopts the single-point crossover and the mutation operation employs the binary variation.

The pericynthion's distance of 15 km and the apocynthion's distance of 100 km are known. 'μ' is set as the standard gravitational parameter of the moon, and 'a' is set as the orbital semi-major axis of the elliptic orbit of the Chang'e 3. According to the velocity formula derived from the Vis Viva equation: $v = \sqrt{\mu(\frac{2}{r} - \frac{1}{a})}$. The velocity size v_n at the pericynthion and the velocity size v_n at the apocynthion can be obtained.

2.2 The solution of the model

As shown in figure 1, the direction pointing to the pericynthion is the Y-axis direction, the direction pointing to the velocity direction of the lander at the pericynthion is the X-axis direction. According to the documentation and the analysis, it can be deduced that

$$\varphi = \arctan(\frac{\lambda_v}{\lambda_\omega})$$

This is an optimal control problem with the unfixed end time and the end point constraint, without the path constraint. For this kind of problem, there are two kinds of methods such as the indirect method and the direct method. In this paper, the CVP method of the direct method (Control Variable Parametererization) is used to solve this problem. The CVP method only uses the control variables discretization, and the state variables are still in continuous form. In the process of calculation, the CVP method can obtain the control variables and state variables alternately, and tends to be optimal.

After the above transformation, the following parameters are obtained:

Controller:

$$\hat{u}(s) = \begin{bmatrix} \hat{F}(s) = \sum_i^n \sigma_{F,i}\chi_{[\zeta_{i-1},\zeta_i)}(s) \\ \hat{\psi}(s) = \sum_i^n \sigma_{\psi,i}\chi_{[\zeta_{i-1},\zeta_i)}(s) \\ v(s) = \sum_i^n \delta_i\chi_{[\zeta_{i-1},\zeta_i)}(s) \end{bmatrix}$$

The control parameters are selected as: $\{\sigma_{F,i}\}_{i=1}^n$, $\{\sigma_{\psi,i}\}_{i=1}^n$ and $\{\delta_i\}_{i=1}^n$, the minimization index function is：

$$\hat{Z} = \hat{m}(0) - \hat{m}(1)$$

Which satisfies the terminal constraints

$$\begin{cases} \hat{r}(1) = r_f \\ \hat{v}(1) \le v_f \\ \hat{\omega}(1) = 0 \end{cases}$$

and the whole process inequality constraints:

$$g = \int_0^{t_f} vL_\varepsilon(\hat{r})dt + \tau \ge 0$$

$$\mathrm{L}\ (\hat{r}) = \begin{cases} \hat{r}(s) - r_f, & if \quad \hat{r} \geq r_f + \varepsilon \\ \frac{(\hat{r}(s) - r_f -)^2}{4}, & if \quad r_f - \varepsilon < \hat{r} < r_f + \varepsilon \\ 0, & if \quad \hat{r} \leq r_f - \varepsilon \end{cases}$$

In this way, the optimal control problem of the soft landing is transformed into a static parameter optimization problem. The algorithm summarized as:

1. Given the number of points n_p and parameters ε and τ in the constraint transformation, select the monotonic sequence $\{\xi_k^p\}_{k=0}^{n_p}$ between the sequence interval;
2. Arbitrarily select a set of parameters $\{\sigma_{F,i}\}_{i=1}^n$,$\{\sigma_{\psi,i}\}_{i=1}^n$ and $\{\delta_i\}_{i=1}^n$;
3. Making the parameters into the system equations and constraints, the gradient of the index function about parameter can be calculated.
4. Use the optimization algorithm to update $\{\sigma_{F,i}\}_{i=1}^n$, $\{\sigma_{\psi,i}\}_{i=1}^n$ and $\{\delta_i\}_{i=1}^n$;
5. Judge whether the updated parameters meets the terminal constraints and the inequality constraints, and meantime judge whether the index function value meets the requirements. If so, output the results; otherwise, do the loop to get the results.

Using the optimal control software Dotcvp to solve the optimal parameter problem.

Initial parameter settings:

$r_0 = 1.753 \times 10^6 m$, $r_f = 1.738 \times 10^6 m$, $m_0 = 2400kg$, $\mu = 4.9 \times 10^{12} m^3/s^2$, $\omega_0 = 9,64 \times 10^{-4} rad/s$, $F_{max} = 7500N$, $F_{min} = 1500N$, $C = 2.94 \times 10^3 m/s$, $v_f = 50m/s$.

2.3 Results

Running results show that the total time of the main deceleration phase and the attitude adjustment stage is 454.8677s and the quantity of fuel is 1.4862t.

By changing the number of theta, trying to output plenty of the track cluster. This process by computer simulation. By computer simulation, the trajectory cluster under the non inertial reference system is available. It can be easily found that when theta is 0, the vertical velocity of the height of 3000m can be accepted within the allowable error. The flight trajectory map of the final deceleration stage is shown in figure 2.

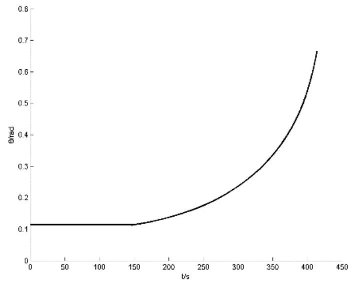

Fig. 2: The Flight path diagram of the main deceleration stage

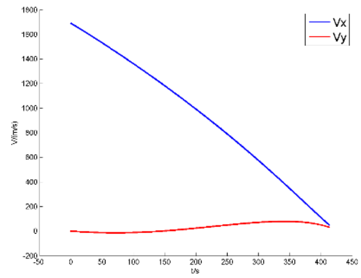

Fig 3: The V_y-V_x variation curve

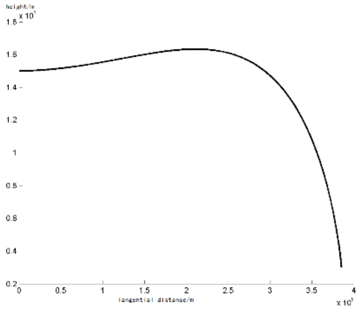

Fig 4: The Y-X track curve

Figure 3, 4 show the time-dependent speed and height changing curves when the device is operating. According to this and the longitude and latitude of the scheduled landing point, the specific position of the pericynthion and the apocynthion can be calculated.

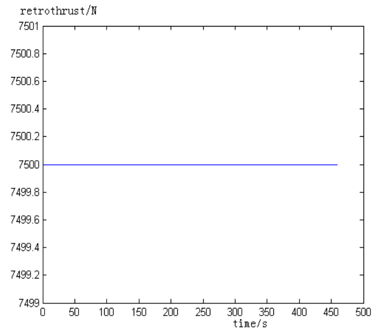

Fig 5: The retrotrust variation curve

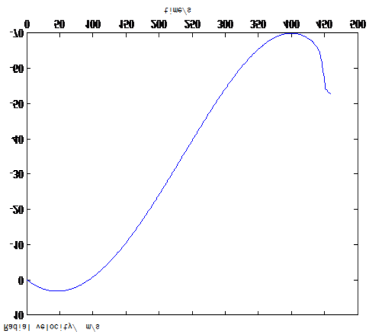

Fig 6: The radial velocity variation curve

From the figure, it can be seen that in the optimal strategy, when the engine thrust is at the maximum value, the fuel consumption is least and the direction angle of the thrust is similar to the linear increase. So the engine try to bring the aircraft down first in order to prevent its early falling. The level speed of the ship gradually reduced and eventually dropped to zero. The figure shows that the trajectory of the aircraft is rapidly changed into a straight line. Under the optimal engine control strategy, the fuel consumption is 909.2kg, and the total time is 454 seconds.

3. The optimal control strategy of the landing trajectory and the landing process of the aircraft

3.1 The coarse obstacle avoidance phase

The distance between the Chang'e 3 and the moon of the coarse obstacle avoidance section ranges from 2.4km to 100m. The main requirement is to avoid the large crater, realize hovering at a distance of 100m above the designed landing point, and initially determine the landing place.

Use the matlab software to figure out the contour map of the elevation map by the information searched and grid it to easily gain the fit landing grid. To make the results more accurate, the variance of each grid should be obtained, from which to select the grid with a smaller variance and a shorter distance away from the original location of the lander.

The result analysis is shown in Figure 7, and the landing point evaluation map is shown in Figure7:

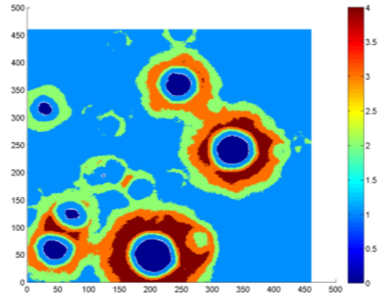

Fig. 7: Transform the elevation map into a contour map

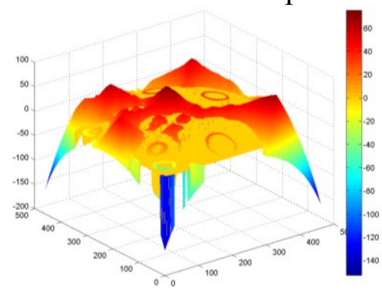

Fig 8: Comprehensive evaluation map

Based on the above analysis, the best landing area should be selected in the area of the horizontal axis of 800~1200m and the vertical axis of 1000~1400m. And according to the analysis of variance, variance as 0.0086, that is, the $1000 \le x \le 1200$-and-$1200 \le y \le 1400$ area should be found, so the lander controls the moving direction according to its own analysis results, moving a distance of 100 m along the Y -axis direction.

3.2 The fine obstacle avoidance

The fine obstacle avoidance segment is 100m to 30m away from the lunar surface, requiring the Chang'e 3 to hover at a distance of 100m from the lunar surface, take pictures of the 100m-range region near the landing site and get a three-dimensional digital elevation map.

To avoid craters and other rugged areas, meshing roughness (the variance) δ and the horizontal distance R from the original location of the lander can be used for analysis and determination in order to select the best landing sites. The color map of the comprehensive evaluation index is shown in Figure10.

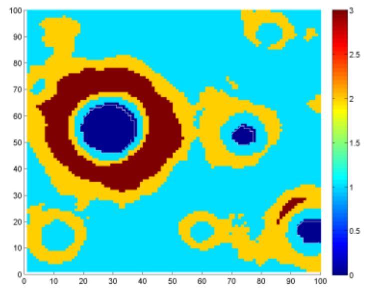

Fig 9: Transform the elevation map into a contour map

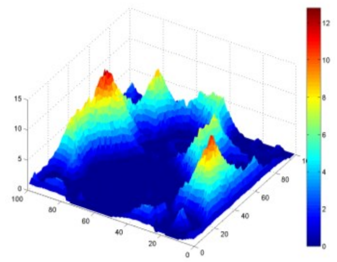

Fig 10: Comprehensive evaluation map

Agree with the ideas of the coarse obstacle avoidance, taking into account the lander's volume of 1 to 2 cubic meters and the error caused by the movement of the lander, the final choice of the scheduled landing point is the 4m*4m area. That is, the figure is about to be split into several 4m*4m sub-blocks, the variance of each block can be calculated. According to the results, it can be seen that the optimal landing zone lies in the 50≤x≤54-and-28≤y≤32 area, so the moving direction and distance of the lander is to move 2 m along the X-axis direction and 20 m along the opposite direction of the Y-axis. Considering the above factors, it can be calculated that the time of the last three stages is about 306s. Simplifying the force of the lander at this stage, the equation can be list as follows:

$$v_e m(t) = (m_0 - \int m(t)dt)\, g'.$$

Here the quality of consumption is 0.4131t, so the total fuel consumption of the whole process is 1.8993t.

4. Conclusions

Set the $\varepsilon_1(J1)$ to be the relative error caused by the terrestrial gravitational perturbation and the $\varepsilon_2(J2)$ to be the relative error caused by the oblateness of the moon, then $\varepsilon_1(J1) = O(10^{-4})$, $\varepsilon_2(J2) = O(10^{-5})$.

The difference between the actual position and the estimated position ranges from a few meters to several hundred meters, with the relative error ranging from $O(10^{-4})$ to $O(10^{-5})$.

So, this model can be extended to solve some of the problems with multiple variables and multiple constraints.

References

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