Individual Travel Choice Equilibrium Model of Integrated Transport Corridors Based on Game Theory

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Abstract

In-depth analysis of the game among the individual travel choice in integrated transport corridors, establishing personal utility function of CES, using the Bayesian theorem estimate the parameter of individual utility function. Constructing the equilibrium model how to maximize individual choice utility with the decision variables. The model which based on personal transportation choice analysis demonstrated the impact of variables such as fares, service frequencies and vehicle capacity on the operation costs, benefit and the utility of travellers and reflected the role of travellers' choices in adjusting transport modes strategies, then designs the corresponding heuristic algorithm approximate Nash equilibrium.

Keywords

Integrated transport corridors, travel choice, game theory, utility function, heuristic algorithm.

1. Introduction

The first modern high-speed rail (HSR) – the route between Tokyo and Osaka with a maximum speed of 210 km/h – went into operation in 1964 in Japan. In 1976, British Railways opened an HSR line between London and Bristol. France commenced the operation of its first HSR between Paris and Lyon in 1981. Since then, many European countries have built HSR lines, including Spain, Germany, Italy, Belgium, and the Netherlands.1 In Asia, South Korea started its first HSR line between Seoul and Daegu in 2004 (which later was extended to Busan), and Taiwan started its HSR service between Taipei and Kaohsiung in 2007.

Yet, the most ambitious HSR development so far is in China: Its original plan, first elaborated in a National Development Plan in 2003, was to build a 12,000 route-km HSR network by 2020, based on a network of four vertical and four horizontal trunk lines. The stimulus package launched by China in 2008 to mitigate the impact of the global financial crisis has more than doubled the investment funds available for railways for the period 2008–2010, enabling its Ministry of Railways to accelerate the HSR construction. The total investment in the HSR network is about USD 300 billion. As a result, the completion dates of several projects have been brought forward, and it is now planned to complete construction of 42 HSR lines, amounting to 13,000 km HSR coverage, by 2012. This will give China the world’s largest and most modern (with a maximum speed of 350 km/h) HSR network.

With the improvement of the high-speed rail network, competing with other modes of transportation in integrated transport corridors will be increasingly fierce. The high-speed rail system can significantly impact the spatial structures and market shares of existing transportation modes. Most published literature studied how the new high-speed rail system competes with existing modes. Yao and Morikawa (2005) developed an integrated intercity travel demand model with a nested structure to estimate induced intercity travel demand, and concluded that intercity travel increases with decreasing travel time, travel cost, and access time, as well as improvement in service frequency. Chang and Chang (2004) proposed static traffic assignment methods to predict the market share of high-speed rail in the northwest−southeast corridor of Korea. Under given fare structures and capacity constraints of all competing transportation modes, they employed the time–space network to estimate the flow on each link formed by competing modes. Roman, Espino, and Martin (2007) analyzes potential competition of the high-speed rail with the air transport along the Madrid–Barcelona corridor in Spain by estimating disaggregated mode-choice models using information provided by mixed revealed and stated preferences database. They found that highspeed rail will hold minor market share (relative to air transport) in long distance trips, while high-speed rail may be more competitive on the shorter segments by capturing traffic from cars and buses. Allport and Brown (1993) suggested that the relative impact of high-speed rail is more significant in smaller cities than in large metropolitan areas. An interpretation is that smaller cities are not as well served by airlines due to infrequent flights, whereas the travel time to high-speed rail stations is relatively low (Kim, 2000).

However, the possibility for cooperation between high-speed rail system and other transportation mode does exist. For example, Givoni (2007), Givoni and Banister (2006) suggested that airlines can use railway services as additional spokes in their network of services from a hub airport to complement and substitute for existing aircraft services. They also examined the potential intermodal integration at Heathrow airport and assessed the benefits and limitations of it. Hsu and Chung (1997) estimated the spatial distribution of markets for high-speed and conventional rail services in a transportation corridor. High-speed rail services are shown to best serve medium-trip to long-trip markets, while conventional rail services for commuter trip markets and collection/distribution markets.

In this paper we investigate the effects of individual travel choice among different transportation modes in integrated transport corridors, which is an important topic in contemporary transport research. As train speeds become faster, HSR is likely to impose significant competitive pressures on air transport. Janic (1993) argues that HSR can compete with air transport over a relatively large range of distances from 400 to over 2000 km. Rothengatter (2011) finds empirical evidence that fierce competition between air transport and HSR may occur on routes with distance up to 1000 km, mostly likely between 400 and 800 km. In China, routes between 400 and 800 km account for about 30% of domestic airline network (Fu et al., 2012). For example, all the flights between Zhengzhou and Xi'an (505 km) were cancelled by the airlines in March 2010 – 48 days after the opening of HSR service – due to very low demand. Even for the Wuhan–Guangzhou route – a much longer route (1069 km) – daily airline flights were reduced from 15 to 9, 1 year after the HSR entry (Fu et al., 2012).

Game theory, a relatively young science, studies the strategic interaction between individuals, organizations and countries and provides many insights for social science and biology. The interaction among different transportation modes that share a corridor is both competitive and cooperative since their network configurations are closely located, largely parallel to each other, and share some of the stations. Cooperation exists when they create value together by attracting more passengers to use the complementary services, which form when passengers transfer among the different systems in their trips. Competition occurs when passengers select among the different systems. Therefore, competitive threat and complementary opportunity exist simultaneously.

Early work by Fisk (1984) applied game theory to model the behaviors of transportation systems in which Nash non-cooperative game between intercity carriers and Stackelberg game of signal optimization problem were analyzed. Recent works extensively focused on competitive behaviors and decision variables of airline industry, including choice of hubs (Hansen, 1990; Martin & Roman, 2003) and stop location (Bhaumik, 2002), pricing strategy (Adler, 2001; Schipper, Nijkamp, & Rietveld, 2007), coalition(Shyr & Kuo, 2008), decision of service frequency (Adler, 2001; Bhaumik, 2002; Hansen, 1990; Schipper et al., 2007), and plane size (Adler, 2001; Bell, 2000; Bell & Cassir, 2002; Wie, 1995). Levinson (2005) studied the user equilibrium and congestion pricing of transport networks in a game theoretical approach Nagurney and Dong (2002) Sun and Gao (2007) Zhou, Lam, and Heydecker (2005) and Zubieta (1998) studied urban transit assignment and competition between different urban transit operators. Game theory not only offers useful concepts to model transportation problems but also introduces the potential sources of solution algorithm.

The paper is organized as follows. Next section sets up our theoretical model. The capacity of integrated transport corridors are sufficient large to meet the demand given their time tables. Integrated transport corridors systems compete to maximize individual utility on sets up the model of competition and route choice behavior. Establishing personal utility function of CES , Using the bayesian theorem estimate the parameter of individual utility function presented in Section 3. In Section 4, we propose a heuristic to solve Nash price equilibrium.. Finally, Section 5 provides concluding remarks.

2. Theoretical Model

2.1 Game Theoretical Model of Personal Travel Mode Choice

Assumptions one area existing I mode of travel choice (transport) for a origin-destination (OD) movement. A hybrid strategy takes shape to personal travel mode choice which according to different preferences and decision variables. One person's mixed strategy is a probability vector $p = \left\{ p_i \middle| i \in I \right\}^T$ that means traveler choice possible probability of each transport mode which is content to (Eq. (1)):

$$0 \le p_i \le 1 \text{ and } \sum_{i \in I} p_i = 1, i \in I \tag{1}$$

The vector $s_i (i \in I)$ is a alternative strategy of transportation modes strategy space. So a traveler's choice of travel mode mixed strategy can be expressed as (Eq. (2))

$$\sigma_i = \sum_{i \in I} p_i s_i \tag{2}$$

In formal terms of discrete choice modeling, the decision maker (n) is assumed to have some level of utility ($U_i(\sum p_i s_i \left| i \in I \right.)$) regarding an alternative in the behavior choice set which is defined generally as $\left\{ p_i \middle| i \in I \right\} \subseteq \prod_{i \in I} P_i$ in all modes of transportation combination strategy.

For traveler, if had a kind of transportation mode i, its selection strategy $p_i^* = \left\{ p_i^* \middle| i \in I \right\}$ is the optimal policy response to the other modes of transportation combination strategy $p_{-i}^* = \left\{ p_k^* \middle| k \in I, k \ne i \right\}$, says it's the Nash equilibrium in the game, that is for all $i, j \in I$ and $i \ne j$ conforming to(Eq. (3))

$$U_i \succ U_j \tag{3}$$

So p_i^* can be expressed as the solution of optimization problem in the form (Eq. (4)):

$$\max_{i \in I} U_i (p_i^*, p_{-i}^*) \tag{4}$$

2.2 Personal Travel Mode Choice Utility Function Analysis

The passengers' choice among alternatives is based on the discrete choice theory of product differentiation (Anderson et al., 1996). The market share model permits passengers to participate in the game by choosing between the available alternatives or not traveling at all. The passengers choose an alternative based on the total trip time, the total price and the log of frequency (which acts as a proxy for level of service (Hansen, 1990; Pels et al., 2000)) on all modes.A representative consumer is assumed for each traveler class (business, leisure) to choose the travel alternative (mode and route) which yields the highest utility. Utility depends on the various characteristics of the alternative, including fare, travel time, distance, routing, etc.

According to travel personally, pay attention to choose the kind of travel way which could maximize their utility, namely, one can be achieved the optimal individual utility function after considering multi-factor variable. Here, We select CES utility function which has advantages of covering the linear and non-linear utility function at the same time, to analysis the personal utility can be represented as(Eq. (5)).

$$U_{in} = V_{in} + \varepsilon = -\frac{\beta_0}{\gamma}\ln\left[\sum_{n=1}^{N}\beta_n X_{in}^{-\gamma}\right] + \varepsilon \tag{5}$$

Variables of personal travel mode choice decision expect the price p , also including service frequency f_{in}, waiting time WT_{in} , travel speed V_{in}, comfort level S_{in} , travelling time TT_{in} . The data of fares and comfort can be obtained through questionnaire survey methods, the rest of the variables can be represented in the following formula in turn：

Service frequency uses the number of transport Q and the corresponding transport equipment ready time ZT and traveling time TT expressing as.

$$f_{in} = \frac{Q_{in}}{TT_{in} + ZT_{in}} \tag{6}$$

Waiting time can use the queue length L and the average service rate μ expressed as:

$$WT_{in} = {L_{in}}\big/{\mu_{in}} \tag{7}$$

The queue length L can be indicated as according to Pollaczek-Khintchine queuing theory:

$$L_{in} = \rho_{in} + \frac{\rho_{in}^2 + \lambda_{in} Var(1/f_{in})}{1-\rho_{in}} \tag{8}$$

whereby: ρ_{in} is the ratio of the average arrival rate and the average service rate, $Var(1/f_{in})$ is the variance of average service interval time, λ_{in} is the rate of average traveler arrival.

Travel time V can use travel distance D and travelling time TT represented as:

$$V_{in} = {D_{in}}/{TT_{in}} \tag{9}$$

So the expression of personal utility can be rewrite into:

$$U = -\frac{\beta_0}{\gamma}\ln\left[\beta_1 p_{in}^{-\gamma} + \beta_2 f_{in}^{-\gamma} + \beta_3 WT_{in}^{-\gamma} + \beta_4 L_{in}^{-\gamma} + \beta_5 V_{in}^{-\gamma} + \beta_6 S_{in}^{-\gamma}\right] + \varepsilon \tag{10}$$

Where β_0、β_1、β_2、β_3、β_4、β_5、β_6、γ are parameters, The utility function, as the transformation of γ, includes properties of linear(γ=-1) and logarithmic linear(γ=0). In spite of universal, but become extremely complex on the parameters estimated.

3. Parameter Estimation

3.1 Establishing Personal Utility Function of CES

Considering the estimated demand factors such as sample size and parameters estimation precision, this paper will use Bayesian method which is developed based on Bayes' theorem method for statistical analysis to estimate parameters. Be different from classical statistical methods only use sample information, Bayesian analysis is a basic method that synthesis priori and sample information of unknown parameters, according to Bayes' theorem, then get the posteriori information to infer unknown parameters. So it can be described as follows:

$$\text{The joint posterior probability density} \propto \text{The prior probability density} \times \text{Likelihood function} \tag{11}$$

Where $\propto$ is a function by Pro Rata. The posteriori probability density centralize all the three kinds information of overall, sample and priori about the parameters, and have ruled out all the information has nothing to do with the parameters. Priori information through prior density into the posterior density, and all the sample information through likelihood function into the system.

Assumes the error term ε in Eq(10) obey normal distribution with mean is zero and variance is τ. p_{in}、f_{in}、WT_{in}、V_{in}、S_{in} are exogenous variables and independent of ε. So get likelihood function as:

$$U_{in}(\theta|q) \propto \tau^{n/2} \exp\left[-\left(\tau/2\right)\sum_{i=1}^{n}\left(U_i + \frac{\beta_0}{\gamma}\ln\left[\beta_1 p_{in}^{-\gamma} + \beta_2 f_{in}^{-\gamma} + \beta_3 WT_{in}^{-\gamma} + \beta_4 L_{in}^{-\gamma} + \beta_5 V_{in}^{-\gamma} + \beta_6 S_{in}^{-\gamma}\right]\right)^2\right] \tag{12}$$

For unknown parameters' priori information $p(\theta) = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \gamma, \tau)$, Assumptions, γ is subject to normal distribution and τ is subject to Gamma distribution; For parameters β_i, it's assumed to be uniform distribution, given the observed datas, according to the Eq(11), the joint posterior probability density for the parameter vector can be represented as:

$$p(\theta|_{in}、f_{in}、WT_{in}、V_{in}、S_{in}) \propto p(\theta) \times U_{in}(\theta|q) \tag{13}$$

Using integral method, we can get parameters' posterior density. For example, If interested in parameters β_i, γ, we can do integrate of τ in the Eq (13) to get the joint posterior density of β_i, γ. If only interested in parameters β_i, we also can do integral of γ. So, we can get single parameter of marginal posterior density.

Because involves the posterior distribution of multiple integral, Bayesian analysis is complicated. But utilize *WinBUGS* or *OpenBUGS* can simplify complex numerical problems, make setting posterior distribution simulation parameters and initial value becomes very convenient. Any changes of the model only modify less code, So we can make all unknown parameters as random variables, then finding out the parameters of this probability model with Monte carlo simulation as the foundation.

3.2 Personal Travel Mode Choice Equilibrium Model

After get estimated parameters, we can build individual travel choice equilibrium model as follows:

$$Max\left\{U_{in} = -\frac{\beta_0}{\gamma}\ln\left[\beta_1 p_{in}^{-\gamma} + \beta_2 f_{in}^{-\gamma} + \beta_3 WT_{in}^{-\gamma} + \beta_4 L_{in}^{-\gamma} + \beta_5 V_{in}^{-\gamma} + \beta_6 S_{in}^{-\gamma}\right] + \varepsilon\right\} \tag{14}$$

$$p_{in}^{H} \geq p_{in} \geq p_{in}^{L} \qquad \forall i \in I \tag{15}$$

$$f_{in} = \frac{Q_{in}}{TT_{in} + ZT_{in}} \qquad \forall i \in I \tag{16}$$

$$WT_{in} = {}^{L_{in}}\!\big/_{\mu_{in}} \qquad \forall i \in I \tag{17}$$

$$V_{in} = {}^{D_{in}}\!\big/_{TT_{in}} \tag{18}$$

$$L_{in} = \rho_{in} + \frac{\rho_{in}^2 + \lambda_{in} Var(1/f_{in})}{1 - \rho_{in}} \tag{19}$$

Where Eq(15) is the floating range of ticket prices under government guidance; Eq (16) - (19), express the calculating formula of relevant decision variables as mentioned in Section(2). Under the equilibrium state of travel choice way, individual can't through changes of travel choice get greater utility, equilibrium is decided by individual optimal choice of mode of travel. Reality, the individual choice behavior through the past experience or others' choice to judgment or update its own strategy. Therefore, travel choice of equilibrium state is the result of multistage evolution.

4. Solving for Nash Equilibrium

4.1. The Nash Equilibrium

A Nash equilibrium is a profile of strategies such that each player's strategy is an optimal response to the other players' strategies (Tirole, 1988). The equilibrium is determined by the condition that all firms choose the action that is a best response to the anticipated play of their opponents. Nash equilibria can be regarded as the result of learning or evolution. Suppose that both players simultaneously adjust their utility each period by choosing a best response to their opponent's utility. One way to interpret firms' adjustment process with either alternating or simultaneous adjustment is that player making a move expects that his opponent's action in the future will be the same as it is now.

A Nash equilibrium, and only a Nash equilibrium, can have the property that the players can predict it, predict that their opponents predict it, and so on (Fudenberg & Tirole, 1991).

4.2. The Heuristic

If the profit function is a continuous and differentiable function, the mathematical model can be solved using standard optimization techniques, such as partial differentiation with respect to prices. However, while the profit function (2) is continuous with respect to the ticket prices, it is not differentiable everywhere due to degeneracy as explained in the previous section.

Recall that a solution of the Nash equilibrium problem consists of the value of five variables. The heuristic starts by generating an initial solution with a simple rule. In the iterations of optimization, the heuristic adjusts combination strategy for the players bringing the solution closer to equilibrium. We propose the following heuristic to solve for the Nash equilibrium for this non-cooperative game.

Step 1 Initialization algorithm. Set the number of iterations $k = 0, \forall i \in I$, p_i^0 is the initial proportion for each mode of transportation, So get the initial mixed strategy for regional travel mode $\sigma_i^0 = \sum_{i \in I} p_i^0 s_i$; At the same time, gives the elements of vector $\varepsilon = \{\varepsilon_i | i \in I\}$ to a small positive number.

Step 2 $\forall i \in I$, In the case of combination strategy is $P_i^k = \{p_i^k, p_{-i}^k | i \in I\}$,get the optimal strategy $\tilde{p}_i^k$ of travel mode choice i by solving the equation $U_{in} = 0$,and satisfied $\tilde{\sigma}_i^k = \left\{\sum_{i \in I} \tilde{p}_i^k s_i \middle| i \in I\right\}$.

Step 3 Make $P_i^{k+1} = (\tilde{P}_i^k + P_i^k)/2$, and set $k = k+1$.

Step 4 After the above iterations, Tested whether meet $-\varepsilon_i \le p_i^{k+1} - p_i^k \le \varepsilon_i$, If meet, Outputting variable values and the optimal value, corresponding algorithm ended; Otherwise return to step2 continue to calculate.

5. Conclusion

This paper analyzes the potential competition and cooperation among the existing mode of transportation in a integrated transport corridor by studying the game relationship. Similar situations also occur in other countries, such as South Korean (Chang & Chang, 2004; Park & Ha, 2006) and Japan (Yao & Morikawa, 2005), also Taiwan (Chiung-Wen Hsu&Yusin Lee,2010).

The travel choice behavior is limited by factors both subjective and objective, according to their own preferences and different decision variables, Building game model of travel choice decisions based on individual utility maximization which describes the combination strategy of individual choice behavior under integrated transport corridors, then estimated parameters of CES utility function by Bayesian theorem. and design the corresponding heuristic algorithms to game model, this algorithm has strong robustness, regardless of the decision variables changes, the system still has strong robustness and is suitable for whole process of dynamic adjustment. Nash equilibrium as a result of personal decision multistage evolution has guiding significance to choose travel mode rationally.

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References

[1] Adler, N. (2001). Competition in a deregulated air transportation market. European Journal of Operational Research, 129(2), 337–345.

[2] Allport, R. J., & Brown, M. (1993). Economic benefits of the European high-speed rail network. Transportation Research Record, 1381, 1–11.

[3] Bell, M. G. H. (2000). A game theory approach to measuring the performance reliability of transport networks. Transportation Research Part B: Methodological, 34(6), 533–545.

[4] Bell, M. G. H., & Cassir, C. (2002). Risk-averse user equilibrium traffic assignment: An application of game theory. Transportation Research Part B: Methodological, 36(8), 671–681.

[5] Bhaumik, P. K. (2002). Regulating the domestic air travel in India: An umpire's game. Omega, 30(1), 33–44.

[6] Bierlaire, M., & Frejinger, E. (2008). Route choice modeling with network-free data. Transportation Research Part C: Emerging Technologies, 16(2), 187–198.

[7] Chang, I., & Chang, G. L. (2004). A network-based model for estimating the market share of a new high-speed rail system. Transportation Planning & Technology, 27(2), 67–90.

[8] Chiung-Wen Hsu, Yusin Lee & Chun-Hsiung Liao (2010). Competition between high-speed and conventional rail systems: A game theoretical approach. Expert Systems with Applications, 37 (2), 3162–3170.

[9] Chu, C. P., & Tsai, J. F. (2008). The optimal location and road pricing for an elevated road in a corridor. Transportation Research Part A: Policy and Practice, 42(5), 842–856.

[10] Economides, N., & Wildman, S. (1995). Monopolistic competition with twopart tariffs. Stern Business School Discussion Paper EC-95-10, New York University.

[11] Fisk, C. S. (1984). Game theory and transportation systems modelling. Transportation Research Part B: Methodological, 18(4-5), 301–313.

[12] Fudenberg, D., & Tirole, J. (1991). Game theory. MIT Press. Givoni, M. (2007). Role of the railways in the future of air transport. Transportation Planning and Technology, 30(1), 95–112.

[13] Givoni, M., & Banister, D. (2006). Airline and railway integration. Transport Policy, 13(5), 386–397.

[14] Hansen, M. (1990). Airline competition in a hub-dominated environment: An application of noncooperative game theory. Transportation Research Part B: Methodological, 24(1), 27–43.

[15] Hayes, B. (1987). Competition and two-part tariffs. The Journal of Business, 60(1), 41–54.

[16] Henn, V. (2000). Fuzzy route choice model for traffic assignment. Fuzzy Sets and Systems, 116(1), 77–101.

[17] Ho, H. W., & Wong, S. C. (2006). Two-dimensional continuum modeling approach to transportation problems. Journal of Transportation Systems Engineering and Information Technology, 6(6), 53–68.

[18] Horner, M. W., & Groves, S. (2007). Network flow-based strategies for identifying rail park-and-ride facility locations. Socio-Economic Planning Sciences, 41(3), 255–268.

[19] Hotelling, H. (1929). Stability in competition. Economic Journal, 39(153), 41–57. Hsu, C. I., & Chung, W. M. (1997). A model for market share distribution between high-speed and conventional rail services in a transportation corridor. The Annals of Regional Science, 31(2), 121–153.

[20] Janic, M., 1993. A model of competition between high speed rail and air transport. Transportation Planning and Technology 17 (1), 1–23.

[21] Jiang, C., Zhang, A. 2012. Effects of High-speed Rail and Airline Cooperation Under Hub Airport Capacity Constraint. Working Paper at the Centre for Transportation Studies, Sauder School of Business, University of British Columbia.

[22] Kim, K. S. (2000). High-speed rail developments and spatial restructuring: A case study of the Capital region in South Korea. Cities, 17(4), 251–262.

[23] Leurent, F. (1993). Cost versus time equilibrium over a network. European Journal of Operational Research, 71(2), 205–221.

[24] Levinson, D. (2005). Micro-foundations of congestion and pricing: A game theory perspective. Transportation Research Part A: Policy and Practice, 39(7-9), 691–704.

[25] Martin, J. C., & Roman, C. (2003). Hub location in the South-Atlantic airline market: A spatial competition game. Transportation Research Part A: Policy and Practice, 37(10), 865–888.

[26] Nagurney, A., & Dong, J. (2002). A multiclass, multicriteria traffic network equilibrium model with elastic demand. Transportation Research Part B:Methodological, 36(5), 445–469.

[27] Nakamura, K., & Kockelman, K. M. (2002). Congestion pricing and roadspace rationing: An application to the San Francisco Bay Bridge corridor. Transportation Research Part A: Policy and Practice, 36(5), 403–417.

[28] Park, Y., & Ha, H. K. (2006). Analysis of the impact of high-speed railroad service on air transport demand. Transportation Research Part E: Logistics and Transportation Review, 42(2), 95–104.

[29] Rothengatter, W., 2011. Competition between airlines and high-speed rail. In: Macario, R., van de Voorde, E. (Eds.), Critical Issues in Air Transport Economics and Business. Routeledge, Oxford, UK.

[30] Roman, C., Espino, R., & Martin, J. C. (2007). Competition of high-speed train with air transport: The case of Madrid–Barcelona. Journal of Air Transport Management, 13(5), 277–284.

[31] Schipper, Y., Nijkamp, P., & Rietveld, P. (2007). Deregulation and welfare in airline markets: An analysis of frequency equilibria. European Journal of Operational Research, 178(1), 194–206.

[32] Shyr, O. F., & Kuo, Y. P. (2008). Applying TOPSIS and cooperative game theory in airline merging and coalition decisions. Journal of Marine Science and Technology, 16(1), 8–18.

[33] Sun, L. J., & Gao, Z. Y. (2007). An equilibrium model for urban transit assignment based on game theory. European Journal of Operational Research, 181(1), 305–314.

[34] Tirole, J. (1988). The theory of industrial organization. Cambridge Mass: MIT Press. Tseng, P. (1993). Dual coordinate ascent methods for non-strictly convex minimization. Mathematical Programming, 59(1), 231–247.

[35] Von Neumann, J., & Morgenstern, O. (1947). Theory of games and economic behavior. Princeton, NJ: Princeton.

[36] Wang, J. Y. T., Yang, H., & Lindsey, R. (2004). Locating and pricing park-and-ride facilities in a linear monocentric city with deterministic mode choice. Transportation Research Part B: Methodological, 38(8), 709–731.

[37] Wie, B. W. (1995). A differential game approach to the dynamic mixed behavior traffic network equilibrium problem. European Journal of Operational Research, 83(1), 117–136.

[38] Yao, E., & Morikawa, T. (2005). A study of on integrated intercity travel demand model. Transportation Research Part A: Policy and Practice, 39(4), 367–381.

[39] Zhou, J., Lam, W. H. K., & Heydecker, B. G. (2005). The generalized Nash equilibrium model for oligopolistic transit market with elastic demand. Transportation Research Part B: Methodological, 39(6), 519–544.

[40] Zubieta, L. (1998). A network equilibrium model for oligopolistic competition in city bus services. Transportation Research Part B: Methodological, 32(6), 413–422.