The Material Thickness on the Bragg Gap Position in Photonic Crystals Containing Dispersive Left-handed Materials

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Abstract

We investigate the effects of material thickness on the Bragg gap position in one-dimensional photonic crystals stacked by alternating layers of left-handed materials and right-handed materials. Due to the dispersive property of left-handed materials, the band-gap positions are no more inversely proportional to the lattice constants. Three photonic crystals with different material thickness are fabricated at microwave frequencies and the experimental results and the simulations agree extremely well with the theoretical expectation.

Keywords

The Bragg gap, The photonic crystals.

1. Introduction

Recently, the study of electromagnetic properties of left-handed materials (LHMs) has been the intriguing subject of great attention [1-9]. LHMs, whose permittivity and permeability are simultaneously negative, exhibit many unusual physical properties different from the conventional right-handed materials (RHMs). The emergence of LHMs introduces many unique photonic band-gaps [8-9] (PBG) to photonic crystals. The PBG of photonic crystals stacked by LHMs or RHMs generally originate from the Bragg scattering mechanism. The Bragg gaps can be identified through the index m in the Bragg conditions5

$$\psi_{peroid} = k_1 d_1 + k_2 d_2 = m\pi \tag{1}$$

where we suppose that the electromagnetic wave propagates along the normal direction of photonic crystals staked by alternating layers of two materials. The parameter denotes the phase shift of a period, ki and di (i.e. i=1, 2) are the propagation constants and the layer thickness of two layers, respectively, and m is the band-gap index which is integer, including zero, negative and positive numbers. According to Eq. (1), the photonic crystals stacked by two purely RHMs will only lead to the positive modes (m>0) of Bragg gaps, while the multilayered two purely LHMs can just possess the negative modes (m<0) of Bragg gaps. However, the zero modes (m=0) of Bragg gaps can only exist in alternating layers of LHMs and RHMs.

For nondispersive LHMs, the zero modes and negative modes of Bragg gaps show similar properties with that of the positive modes in conventional photonic crystals, and the positions of all band-gaps are inversely proportional to the lattice constant. However, the essence of LHMs is dispersive. In this letter, the effects of material thickness of the dispersive LHMs on Bragg gap positions in one-dimensional photonic crystals are investigated in detail, where the Bragg gaps including zero modes, negative modes and positive modes. Three photonic crystals stacked by alternating layers of LHMs and RHMs are fabricated to experimentally demonstrate the effects of LHMs thickness on Bragg gap positions.

2. The theoretical analysis

LHMs do not exist in nature, but the artificial LHMs have been realized by using periodic structures either with the unit cell of split-ring resonators (SRRs) and conducting wires2, 3 or with the unit cell of LC-loaded transmission line [11], [12]. All of these artificial LHMs can be describe by effective and [2], [11]as

$$\varepsilon_L = \varepsilon_1 - \frac{\omega_{pe}^2}{\omega^2} \tag{2}$$

$$\mu_L = \mu_1 - \frac{\omega_{om}^2}{\omega^2} \tag{3}$$

Here, the cutoff frequencies and losses are neglected. The frequencies and are the electric plasma frequency and magnetic plasma frequency, respectively. The parameters and represent relative permittivity and permeability of the host medium. For the sake of simplicity, we only consider the balance case [13], i.e. , where is a positive structure constant, that is to say, there are no single negative (negative- or negative-) bands between LH passband and RH passband. Thus, the effective propagation constant of LHMs is given by

$$\beta_L = \frac{\omega}{c}\sqrt{\varepsilon_L \mu_L} = \frac{\omega}{c} p(\varepsilon_1 - \frac{\omega_{pe}^2}{\omega^2}) \tag{4}$$

The propagation constant of conventional RHMs is given by

$$\beta_R = \frac{\omega}{c}\sqrt{\varepsilon_2 \mu_2} \tag{5}$$

where and represent relative permittivity and permeability of conventional RHMs.

Consider a 1-D infinite periodic structure with alternating layers of LHMs and RHMs mentioned above. The parameters dL and dR are the widths of two inclusion layers in a period respectively, and a=dL +dR is the lattice constant. Then, the phase shift of a period cascaded by LHMs and RHMs can be written as

$$\psi_{peroid} = \beta_L d_L + \beta_R d_R = \frac{\omega}{c} p(\varepsilon_1 - \frac{\omega_{pe}^2}{\omega^2}) d_L + \frac{\omega}{c}\sqrt{\varepsilon_2 \mu_2}\, d_R \tag{6}$$

which implies that the phase shift of a period will ascend as the frequency increasing.

The mth mode of Bragg gaps at the frequency can be written as

$$\psi_{peroid}\Big|_{\omega_0} = \frac{\omega_0}{c} p(\varepsilon_1 - \frac{\omega_{pe}^2}{\omega_0^2}) d_L + \frac{\omega_0}{c}\sqrt{\varepsilon_2 \mu_2}\, d_R = m\pi \tag{7}$$

If LHMs thickness increases as ( ), the phase shift of a period at becomes

$$\psi_{peroid}\Big|_{\omega_0} = m\pi + \frac{\omega_0}{c} p(\varepsilon_1 - \frac{\omega_{pe}^2}{\omega_0^2}) \triangle d \tag{8}$$

Therefore, the effects of the LHMs thickness dL on the mth gap position are predominant by the coefficient . If , i.e. LHMs exhibit LH attribute with both negative and , is obtained and the position of the mth band-gap is required to mount up to satisfy the Bragg scattering conditions (Eq. (7) with), while for , i.e. , the mth gap position is needed to shift down. However, for , i.e. the mth band-gap position locates at the balanced point [11], [13] of LHMs, the mth gap position will never vary by increasing or decreasing LHMs thickness .

In practice, the phase shift of RHMs in a period must be larger than zero. That is to say, the coefficient is required to be less than zero for the modes of Bragg gaps, therefore the center frequencies of the modes of Bragg gaps will always move up with increasing LHMs thickness . When the phase shift of RHMs in a period is less than , the center frequencies of the modes will move down as LHMs thickness increasing. If the mth gap occurs at the balanced point of LHMs, the position of the mth gap is independent of LHMs thickness . Furthermore, if the phase shift of RHMs in a period is larger than but less than , the positions of the gaps will be always proportional to the LHMs thickness dL, while the gaps positions are inversely proportional to the LHMs thickness dL, where is integer, including zero and positive numbers. In addition, increasing RHMs thickness will render all Bragg gaps move down for ever.

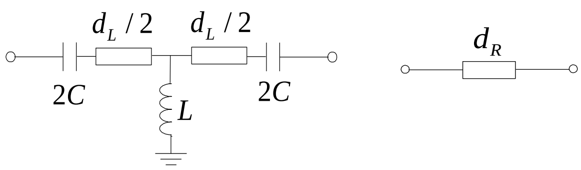
If the host medium of LHMs is neglected, Eq. (6) can be written as

$$\psi_{peroid} = \beta_L d_L + \beta_R d_R = -\frac{\omega_{pe}\omega_{om}}{c\omega} d_L + \frac{\omega}{c}\sqrt{\varepsilon_2 \mu_2} d_R \tag{9}$$

Thus, all Bragg gaps will move up with increasing LHMs thickness . However, it should be noticed that the purely LHMs have not been constructed so far because the host media are unavoidably distributed in the whole artificial structure [2], [11].

Consequently, the band-gap positions are no more inversely proportional to the lattice constant for dispersive LHMs, which break the traditional rules of photonic crystals stacked by RHMs.

3. Simlations and experiments



(a) (b)

Fig. 1 The equivalent circuit model of (a) a CRLH-TL unit with the loaded lumped-element: series capacitors (C) and shunt inductor (L) and (b) a RH-TL unit.

To experimentally demonstrate the effects of LHMs thickness on Bragg gap positions, we respectively fabricate three photonic crystals with different LHMs thickness, where LHMs and RHMs are implemented by the composite right/left-handed transmission line (CRLH-TL) and the conventional right-handed transmission line (RH-TL). The unit cells of the CRLH-TL and the RH-TL are illustrated in Fig. 1(a) and (b), respectively. The photonic crystals are designed on a Teflon substrate of the thickness h=0.5 mm, relative permittivity and relative permeability =1. In the lossless case, the ABCD matrix of the CRLH-TL and the RH-TL can be obtained from the equivalent circuit model in Fig. 1, and the Bloch propagation constants can be respectively determined by using Bloch–Floquet theorem. The resulting dispersion relations11, 12 are

$$\cos(\beta_{CRLH} d_L) = \cos(k_1 d_L)(1 - \frac{1}{4\omega^2 LC}) + \sin(k_1 d_L)(\frac{1}{2\omega C Z_1} + \frac{Z_1}{2\omega L}) - \frac{1}{4\omega^2 LC} \tag{10}$$

$$\beta_{RH} d_R = k_2 d_R \tag{11}$$

where and (i.e. i=1, 2) are the wave numbers and the characteristic impedances of the host transmission line of the CRLH-TL and the RH-TL, and the parameters and are the Bloch propagation constants of the CRLH-TL and the RH-TL, respectively. The host microstrip transmission line of the CRLH-TL is with the width wL=1.37 mm for the characteristic impedance and the length dL=6 mm, and the loaded lumped-element components are chosen as =3.3 pF and =8.2 nH for the balanced condition13 . The RH-TL is a section of microstrip transmission line with the width wR=5 mm for the characteristic impedance and the length dR=18.8 mm.

According to Eqs. (10) and (11), the dispersion characteristics of the CRLH-TL and the RH-TL can be calculated, as provided in Fig. 2. The dispersion diagram of the CRLH-TL exhibits the LH passband in the lower frequency range, while the RH transmission properties are dominant in the upper frequency range. The transition frequency between the LH passband and the RH passband is about 2.25 GHz. The LH property is attributed to shunt inductors and series capacitors , and the pertinent cutoff frequency of lower edge is 0.48 GHz. Consequently, the CRLH-TL can be considered as LHMs from 0.48GHz to 2.25GHz, while the RH-TL can be severed as RHMs in the entire frequency range.

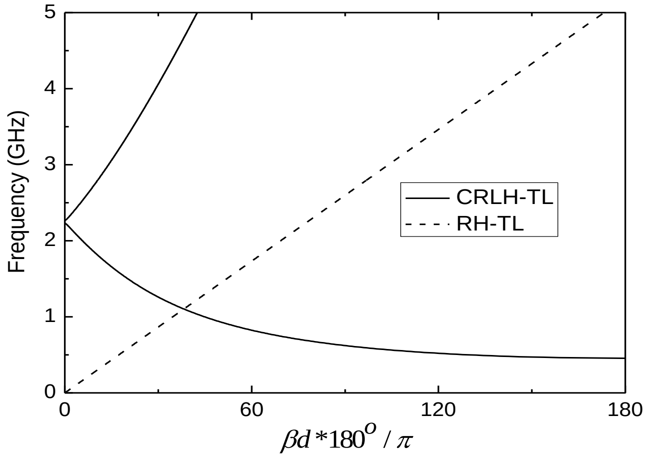


Fig. 2 The calculated dispersion diagrams of the CRLH-TL (solid line) with =3.3 pF, =8.2 nH, wL=1.37 mm and dL=6 mm and the RH-TL (dash line) with wR=5 mm and dR=18.8 mm.

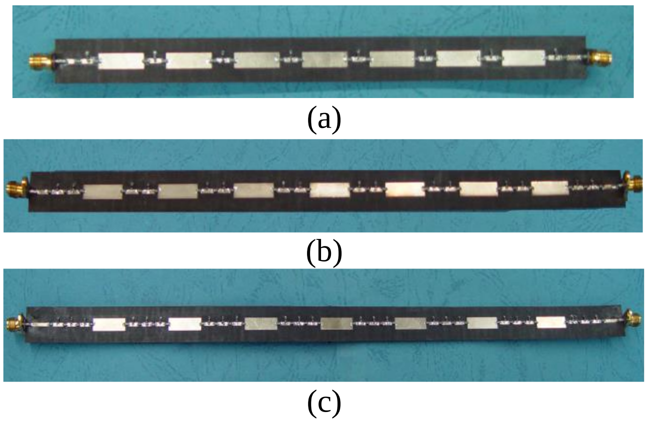


Fig. 3 (Color online) The photographs of three fabricated photonic crystals (a) (CRLH1-RH)6, (b) (CRLH2-RH)6 and (c) (CRLH3-RH)6.

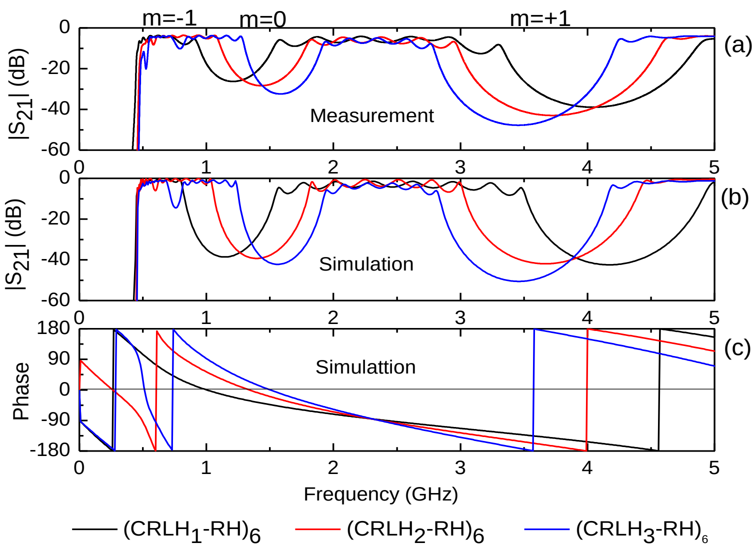


Fig. 4 (Color online) The transmission properties of the photonic crystals (CRLH1-RH)6, (CRLH2-RH)6 and (CRLH3-RH)6. (a) the measured and (b) the simulated transmission magnitudes of three photonic crystals, and (c) the simulated phase delays of a period of three photonic crystals.

The photographs of three fabricated periodic structures (CRLH1-RH)6, (CRLH2-RH)6 and (CRLH3-RH)6 using real lumped-element components are provided in Fig. 3, where the subscripts

"1", "2" and "3" represent the number of CRLH cell in a period, and "6" represents the period number, respectively. As the number of CRLH cell in a period increases, LHMs gradually become thicker for three periodic structures. All the structures are simulated by using the advanced design systems (ADS) of agilent and measured by the vector network analyzer. Figures 4(a) and (b) show the measured and simulated transmission magnitudes of the proposed photonic crystals. The corresponding phase delays of a period ( ) for three photonic crystals are depicted in Fig. 4(c). It is seen that the m=0 gaps coincide well with the phase shifts frequency points and the slightly differences are generated from the finite period of photonic crystals. Similarly, the m=+1 gaps perfectly correlate with the phase shifts . It is surprising that the phase shifts frequency points for three photonic crystals are respectively 1.03 GHz, 1.38 GHz and 1.57 GHz, while the phase shifts frequency points are 4.57 GHz, 4.0 GHz and 3.58 GHz, respectively. That is to say, as the thickness of LHMs increase, the m=0 gaps mount up but the m=+1 gaps drop down. In addition, the gaps have the similar performances as the m=0 gaps but with narrower bandwidth and lower gap levels. These behaviors agree extremely well with the theoretical expectation for the case that the phase shift of the RH-TL in a period is larger than zero but less than in the certain frequency range (see Fig. 2).

4. Conclution

To summarize, we have investigated the effects of material thickness on Bragg gap positions in photonic crystals stacked by LHMs and RHMs. Due to the dispersive property of LHMs, if the phase shift of RHMs in a period is larger than but less than , the positions of the gaps will be always proportional to the LHMs thickness, while the positions of the gaps are inversely proportional to the LHMs thickness, where is integer, including zero and positive numbers. Although the theory has been deduced by a 1D photonic crystal and validated at microwave frequencies, the rules also make sense for 2D or 3D photonic crystals in infrared or optic bands.

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