Study on the SEIR Differential Equation Model based on the Eradicating Ebola

Chunmei Chen [a], Ping Yang [b] and Zhengping Long [c]

College of science, the Rockets Force University of Engineering, Xi'an, Shaanxi 710025, China

[a]362437655@qq.com, [b]ccm1977@sohu.com, [c]erpmath@163.com

Abstract

Based on the analysis of spread law of Ebola virus, the paper studies the effective strategies. we build an optimized control model of Ebola virus and carry out solving processes by virtue of various mathematical methods. To solve the spread law problem of Ebola virus, firstly, we divide the ideal objects into four classes：S, I, E and R, and by improving SEIR model, we obtain a differential equation model with incubation and isolation consideration; secondly, through numerical solution and by virtue of MATLAB, we obtain the changing curve of relevant variables and compare it with real situation to test the stability of the model; finally, through the analysis on the influence of isolation intensity change on the spread of the epidemic, sensitivity is analyzed, further enhancing the reliability and effectiveness of the model.

Keywords

Eradicating Ebola, SEIR model, Differential Equation model, MATLAB.

1. Introduction

Life is the most precious gift granted by God.We can not forget the serious impact on 2,2000 thousand Western African people caused by the outbreak of Ebola virus in last March, which made numerous families lose their main labor force, more than 10,000 children lose their parents and a great number of survivors face social and psychological difficulties and have some problems, such as fatigue, arthropathy, hearing disorder and visual impairment. Due to the epidemic, more than five million children dropped out schools and stayed at homes, thus increasing the risk of suffering from measles, malaria, and so on. Ebola virus disease is a kind of severely lethal one, and its mortality is up to 90%. Its spread and infection approach is the direct contact with blood, body fluid and tissue of infectious animals or mankind.

Since the Ministry of Health of Guinea declared that the Ebola epidemic broke out in the country on 21 March, 2014, the virus has appeared in some other countries like Liberia and Sierra Leone, too. This virus is a lethal virus, whose death rate is very high, nearly 90%. Its spreading way is through body contact or blood, body fluid and tissue contact. In order to resist Ebola virus spread, we must find a scientific method to predict the trend of Ebola spread.[4]

2. SEIR model

2.1 Analysis on spreading mechanism of Ebola[1]

As for the mathematical modeling research concerning Ebola virus, there are also some documents in recent years. Based on the documents, we establish SEIR mathematical modeling of Ebola virus infection number. Hypothesis is conducted for model: due to shorter research time period, research objects are regarded as ideal people without migration or death phenomena caused by other reasons except for Ebola virus, the total number of people is kept at fixed level N ; suppose all healed people who have suffered from infectious disease possess permanent immunity and anybody who has suffered from the disease will become infectious immediately. In the case, all the people in the society are divided into four classes：

Susceptible: members in the class have not been infected with infectious disease, but they are short of immunity and vulnerable to infectious diseases;

Infective: members in the class have been infected with infectious disease, but they have not been isolated and can spread the disease to Class S members;

E: members in the class are virus carriers, but ultimately they will turn into Class Infective members;

Removal : They belong to elimination or death class, which is the number of Class Infective members who have been cured or died.

S(t), E(t), I (t) and R(t) represent the proportion between the above-mentioned four classes of members and total population in the city at t moment respectively,and their sum is 1:

$$S(t) + E(t) + I(t) + R(t) = 1 \quad (1)$$

In 2014, at the very beginning of the outbreak of the Ebola virus, because it is not the first time for Ebola virus to break out, the whole society has an enough awareness of the spread speed and perniciousness of Ebola virus and the government and the public take it seriously. Therefore, we think the epidemic enters the state of taking certain isolation measures even at the initial stage. so that we can assume that the government has set the isolation area. We define isolation intensity as p , daily contact rate of patient as λ ,daily removal rate as μ ,daily death rate as l and $\varepsilon(t)$ as outbreak rate, namely, the probability of change from E member to the one who has been infected, which is a variable $\varepsilon(t)$ that increases as time grows. Due to short incubation and high death rate of Ebola virus, so far, there is no effective removal measure, thus daily cure rate is not considered here.

The spread process of Ebola is shown in Fig.1.

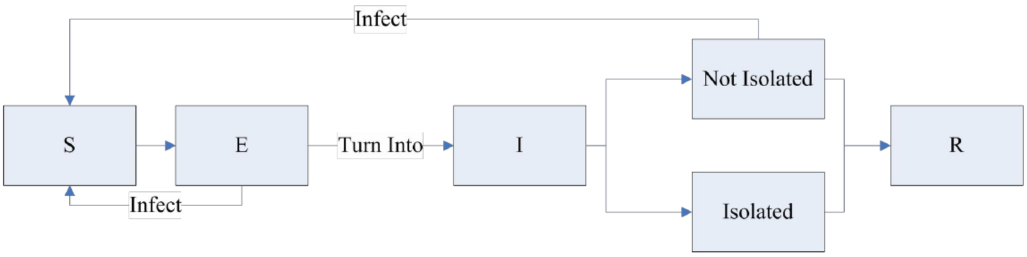

Fig.1 Diagram of the SEIR model

2.2 Modeling

Now, the change situation of various people are considered during the period from t to $t + \Delta t$.

Current patient number. The number of new patients during the period of Δt is equal to the difference between the number of patients who have turned into patients from E and the number of Removal during the period, namely:

$$\frac{dI}{dt} = E(t) \times \varepsilon(t) - I(t) \times l \quad (2)$$

Removal number:the removal number of people during the period of Δt, which is approximately the number of dead patients.

$$\frac{dR}{dt} = I(t) \times l \quad (3)$$

E number: There are two change sources of E number: one is that people are infected by E and not in outbreak period, the other is that people are infected by the patients who are not isolated and not in outbreak period.

$$\frac{dE}{dt} = I(t) \times (1 - \varepsilon(t)) \times \lambda \times (1 - p) \times S(t) + E(t) \times S(t) \times (1 - \varepsilon(t)) \times \lambda \quad (4)$$

Susceptible number:The reduction of susceptible number is due to the number of people who have become E.

$$\frac{dS}{dt} = -I(t)\times(1-\varepsilon(t))\times\lambda\times(1-p)\times S(t) - E(t)\times S(t)\times(1-\varepsilon(t))\times\lambda \tag{5}$$

In summary, we get the spread model of Ebola epidemic:

$$\begin{cases} \frac{dS}{dt} = -I(t)\times(1-\varepsilon(t))\times\lambda\times(1-p)\times S(t) - E(t)\times S(t)\times(1-\varepsilon(t))\times\lambda \\ \frac{dE}{dt} = I(t)\times(1-\varepsilon(t))\times\lambda\times(1-p)\times S(t) + E(t)\times S(t)\times(1-\varepsilon(t))\times\lambda - E(t)\varepsilon(t) \\ \frac{dI}{dt} = E(t)\times\varepsilon(t) - I(t)\times l \\ \frac{dR}{dt} = I(t)\times l \end{cases} \tag{6}$$

Obviously, the above four differential equations are added:

$$\frac{dI}{dt} + \frac{dS}{dt} + \frac{dE}{dt} + \frac{dR}{dt} = 0 \tag{7}$$

3. Solution of model

3.1 Determining parameter

$\varepsilon(t)$:Generally speaking, the incubation of Ebola virus is from 2 to 21 days,and when t =0, the transformation probability from E to Infetive is zero; when t=21, the transformation probability from E to Infective is 1, and it becomes larger as time grows.[2]

$$\varepsilon(t) = ae^{t-21} + b \tag{8}$$

where, $a = \frac{1}{1-e^{-21}}$, $b = \frac{-e^{-21}}{1-e^{-21}}$.

λ :contact rate between one infective and other people, which is related to the alert level of whole society and various measures taken by government and public,such as wearing gauze mask, decreasing the time of staying at pubic places, spraying disinfectant and increasing isolation intensity, which can all reduce the value of contact rate effectively. In general, the value of varies along with the evelopment of Ebola. At the initial stage of Ebola,government and public do not take it seriously,is kept at a higher value; after it goes into outbreak period, the public find that the infectives become more and more, so they become worried and take various easures, which make controlled to some degree, but its effect is not evident, and suppose decays slowly in linear form; in peak period, after the implementation of control measures with high intensity, the effective contact rate of virus transmission decreases obviously, so we may think decays in exponential form according to day number; after that, it goes into attenuation phase,is kept around a lower value.

μ :Considering the fact that before the effective vaccine are developed, the cure rate of Ebola virus is nearly zero. The removal can be supposed to be composed of all dead patients, $\mu \approx 1$.

l: Considering the fact that medical and health conditions in African areas are almost alike, daily death rate is approximately equal to the total one:

$$l \approx \frac{accumulative\ death\ number}{accumulative\ patient\ number} \tag{9}$$

Based on the official website data of WHO[4], the death rate of Guinea, Sierra Leone and Liberia are as follows:

The death rate of Guinea: $l_1 = \frac{1944}{2975} = 0.65344$

The death rate of Sierra Leone: $l_1 = \frac{3276}{10740} = 0.30503$

The death rate of Liberia: $l_3 = \frac{3746}{8745} = 0.42836$

Total death rate: $l = \frac{8981}{22495} = 0.399433$

3.2 Case study

Judging from the result, we know the epidemic mainly occurs in Guinea, Liberia and Sierra Leone. Numerical solution is carried out with Formula (6) by means of MATLAB, then the relation between weekly infected number and time (week) is obtained. Curve takes positive values all the time 'as shown in Fig.2-Fig.7'.

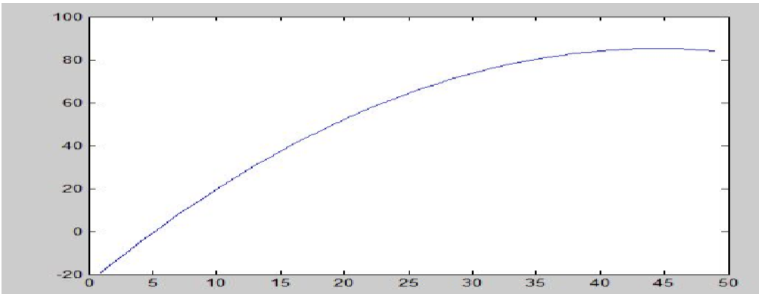

Fig.2 The cure relation between weekly infected number and time (week) of Guinea

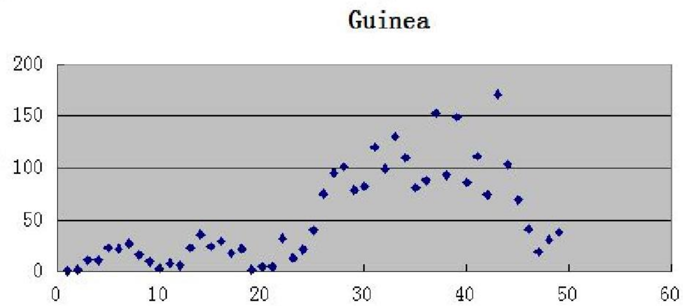

Fig.3 The scatter diagram showing relation between weekly infected number and time (week) of Guinea

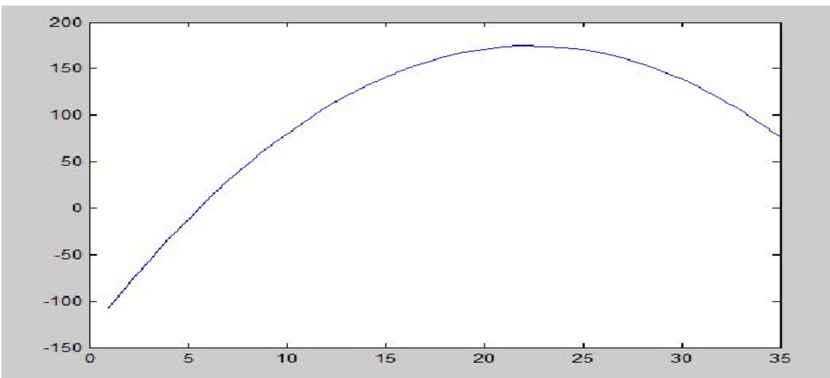

Fig.4 The cure relation between weekly infected number and time (week) of Liberia

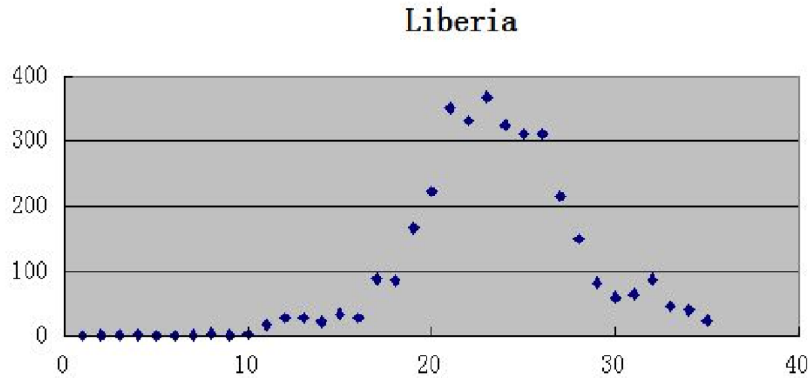

Fig.5 The scatter diagram showing relation between weekly infected number and time (week) of Liberia

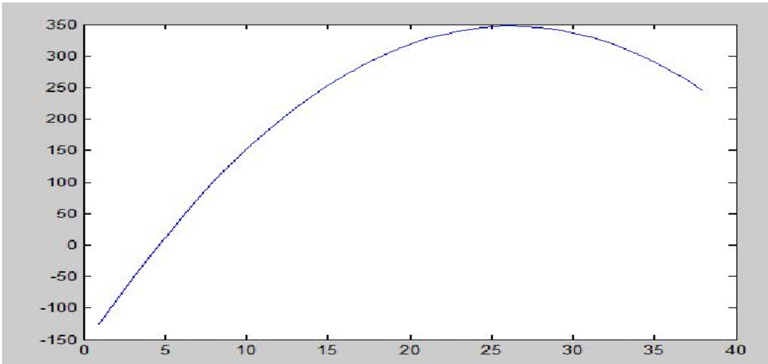

Fig.6 The cure relation between weekly infected number and time (week) of Sierra Leone

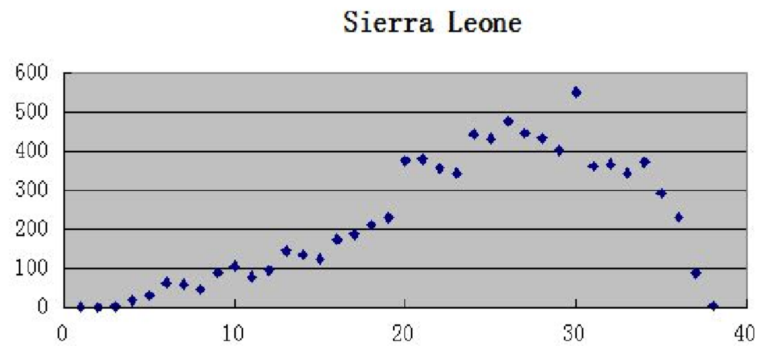

Fig. 7: The scatter diagram showing relation between weekly infected number and time (week) of Sierra Leone

So far, we have gotten the respective epidemic transmission situation of three countries, as shown in Table 1.

Table 1 Predicted infection peak period and infected number

Country	Country	Predicted infection peak period (week)	Weekly infected number
Guinea	March 8	37	82
Liberia	March 25	22	153
Sierra Leone	April 2	24	347

According to Table 1, when infection peak period comes, daily infected number will reach the maximum,while along with the enforcement of active treatment and prevention and control, the infected number will decrease gradually. Comparing predicted data with the official data announced by WHO, we know the result is credible.

4. Conclusion

All the parameters in the model are not negative. According to the model, for all the solutions of initial values which are not negative in the system, when $t \geq 0$, they exist and are not negative.Based on the solution of the formula, the transmission features of Ebola virus among three Western African countries are shown as in Fig. 8-Fig. 9,which describes the change law of I and E along with time at the moments of different initial values respectively.The two figures show in the initial period of virus outbreak, higher incidence appears, while with the advancement of time, the incidence will decrease gradually and disappear in the end. The only difference is that during E period, there is a delay, namely, it reaches the highest point first and then decrease gradually.The phenomenon also accords with the characteristics of Ebola virus, such as short incubation, fast attack and high death rate.[3]

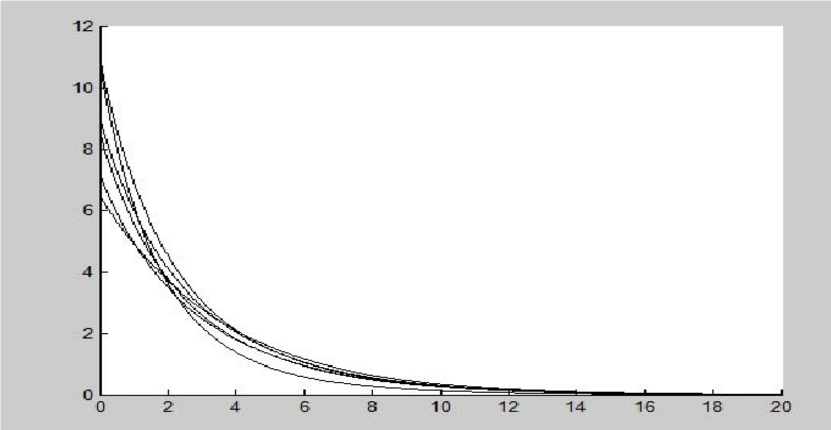

Fig. 8 Changing curve of I along with time

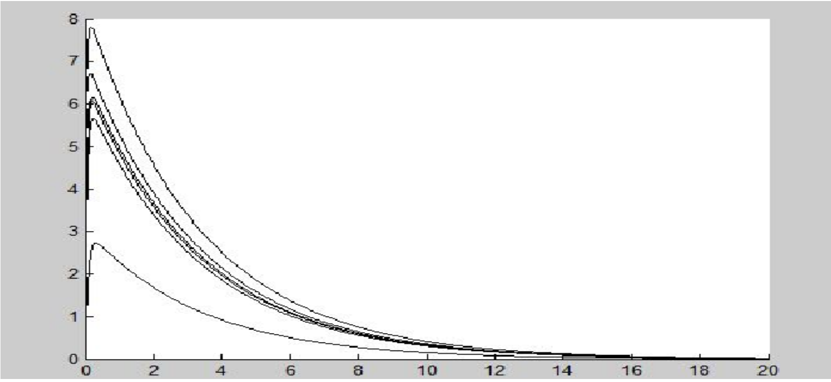

Fig. 9 Changing curve of E along with time

References

[1] ZHANG Hui, XU Wen-xiong:*Asymptotical stability analysis of a SEIS epidemic model with infectious force in both latent period and infected period*(Journal of Shaanxi Normal University),Vol.36(2008)No.6 ,p.5-9.

[2] Shoukui Si, Xijing Sun: *Mathematical Mo*deling (National defence industry press,China 2011), p.258-276.

[3] By Jing Xiang:*Stability Analysis and Optimal Control for a Epidemic Model*(Ph.D., Northeastern University, shen yang 2011), p.45-47.

[4] Information on http://www.who.int