Analysis of Natural Characteristic of Simply Supported Beam under Axial Force

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Abstract

In order to solve the influence of axial force on the natural characteristics and stability of the Beam-Axis, using the transfer matrix method, it is concluded that the natural frequency of the beam can be reduced when subjected to axial pressure though compared the free mode of the beam and the constrained mode of the beam. Finally, by changing the length of the tension in the middle section, we can know the axial tension will inhibit the decline of the natural frequency of the beam, which lay the foundation for further research on the vibration characteristics of the beam.

Keywords

Beam; Axial Force; Free Mode; Transfer Matrix Method; Natural Frequency.

1. Introduction

In the practical application of engineering, the beam is the most basic component of most mechanical components and subjected to an axial force in the course of its work. For example, the crane rail can be seen as a simple supported beam model. When working on the guide rail, the two ends of the guide rail are subjected to maximum pressure due to the action of the friction respectively and the corresponding vibration characteristics can also be changed. Therefore, it is quite necessary to analyze the mode of a simply supported beam under tension and compression.

For the vibration problem of beam, T. F. Xu [1] assessed the series solution of natural vibration of the beam under axial force by using the Frobenius method and made the compassion for the variation of the natural frequencies under different boundary conditions. L.M.Guo [2] analyzed the vibration and stability of beams under compression at both ends by means of the transfer matrix method. Y.H.Li [3] gave a hasty calculation method of variable cross section beam in transverse vibration. Built on the practical application of engineering, this paper analyses the modal analysis of the beam at both ends through the transfer matrix method.

2. Free Modal Analysis

For a uniform beam, which simplified model of the free mode is shown in figure 1and the physical parameters of the material are shown in Table 1.

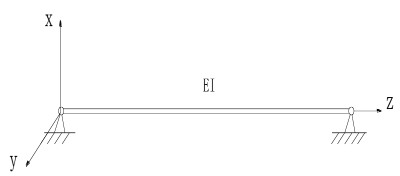


Fig.1 Rayleigh beam mode with two ends simply supported

Table 1 Physical parameters of the material

material	aluminum alloy
length	1.5m
diameter	0.05m
elastic modulus	7×10^{10}Pa
density	2800kg/m^2

For the beam-axis model, which following constitutive relation and displacement coordination condition

$$M = EI\frac{\partial\theta}{\partial z}, \theta = \frac{\partial w}{\partial z} \tag{1}$$

The free vortex motion equation can be got through Rayleigh beam theory, that is

$$-EI\frac{\partial^4 w}{\partial z^4} + \rho I\frac{\partial^4 w}{\partial z^2 \partial t^2} - 2\Omega\rho Ii\frac{\partial^3 w}{\partial z^2 \partial t} - \rho A\frac{\partial^2 w}{\partial t^2} = 0 \tag{2}$$

Where w is the transverse displacement. ρ ,A are the density and cross section area. For the general solution of the equation, which is

$$Y(z) = c_1 \sin\alpha z + c_2 \cos\alpha z + c_3 \sin\beta z + c_4 \cos\beta z \tag{3}$$

In which,

$$\begin{cases} \alpha_1 = \sqrt{\dfrac{\lambda^2}{2} + \sqrt{\dfrac{\lambda^4}{4} + \beta^4}} \\ \alpha_2 = \sqrt{-\dfrac{\lambda^2}{2} + \sqrt{\dfrac{\lambda^4}{4} + \beta^4}} \end{cases} \tag{4}$$

Where c_1, c_2, c_3, c_4 are arbitrary constants.

The displacement function of the beam is $\omega(z,t) = Y(z)e^{i\omega t}$, which differential equation of the real coordinate system is

$$\begin{cases} x(z,t) = \sin\dfrac{n\pi}{l} z\cos\omega t \\ y(z,t) = \sin\dfrac{n\pi}{l} z\sin\omega t \end{cases} \tag{5}$$

Considering boundary conditions

$$\begin{cases} \omega(0) = 0, \omega''(0) = 0 \\ \omega(l) = 0, \omega''(l) = 0 \end{cases} \tag{6}$$

Which can get mode function is

$$Y(z) = \sin\frac{n\pi}{l} z \tag{7}$$

The critical speed of the rotating shaft is

$$\omega_\gamma = \frac{n^2\pi^2}{\sqrt{1 - n^2\pi^2\delta}} \tag{8}$$

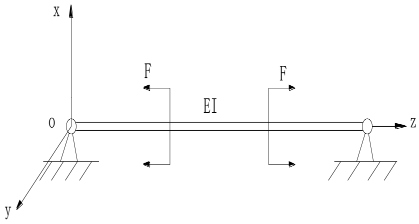
In which, $\delta = \dfrac{I}{Al^2}$ .It doesn't have critical speed when $n^2 > \dfrac{1}{\pi^2\delta}$. Thus, the top five order critical speeds of the beam are shown in Table 2.

Table 2 The critical angular velocity of the first five orders of the two ends simply supported beam rad/s

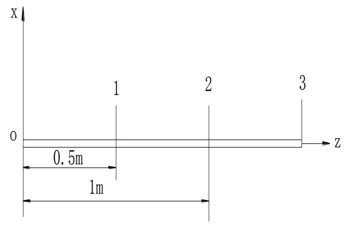
ω_1	ω_2	ω_3	ω_4	ω_5
9.863	39.493	89.103	158.787	248.880

3. Modal analysis of beam under axial force

In view of the above model, put pressure on the 1/3 and 2/3 of the shaft length. The natural frequencies of the beam is analyzed by the transfer matrix method. It consist of three parts, including the left part which are under pressure, the middle part which is bearing tension and the right part which are under pressure. The simplified model is shown in figure 2.



(a)



(b)

Fig.2 Simplified model of the beam subjected to an axial force

When applying an axial force, the steady vortex equation is

$$\frac{\partial^2}{\partial z^2}[EI(z)\frac{\partial^2 w}{\partial z^2}]+F_0\frac{\partial^2 w}{\partial z^2}+\rho A\frac{\partial^2 w}{\partial z^2}=0 \tag{9}$$

Where $w(z,t)$ is the lateral displacement of the Z, F_0 and E are the axial force, Young's modulus. I , ρ , A are the moment of inertia, density and cross section area. For $\lambda^2=\frac{F_0}{EI},\beta^4=\frac{\rho A\omega^2}{EI}$

The general solution of the equation is

$$\mathrm{W(z)=a_1\sin(\alpha_1 z)+a_2\cos(\alpha_1 z)+a_3 sh(\alpha_2 z)+a_4 ch(\alpha_2 z)} \tag{10}$$

Where a_1, a_2, a_3, a_4 are arbitrary constants.

$$\begin{cases}\alpha_1=\sqrt{\frac{\lambda^2}{2}+\sqrt{\frac{\lambda^4}{4}+\beta^4}}\\ \alpha_2=\sqrt{-\frac{\lambda^2}{2}+\sqrt{\frac{\lambda^4}{4}+\beta^4}}\end{cases} \tag{11}$$

State vector is $Z(z)=\{Y(z),\theta(z),M(z),Q(z)\}^T$. Through the transfer matrix calculation, it can be considered that

$$Z(z)=T\cdot[Y(0),Q(0),M(0),Q(0)]^T \tag{12}$$

Where T is the transfer matrix unit, which specific form is

$$T=T(z)=\begin{bmatrix}\alpha_1^2\alpha_2^2A & \alpha_1^2\alpha_2^2B & \frac{C}{EI} & \frac{D}{EI}\\ -\alpha_1^2\alpha_2^2D & \alpha_1^2\alpha_2^2A & -\frac{\mathrm{m}}{EI} & \frac{C}{EI}\\ EI\alpha_1^2\alpha_2^2C & EI\alpha_1^2\alpha_2^2D & b & m\\ -EI\alpha_1^2\alpha_2^2m & EI\alpha_1^2\alpha_2^2C & -d & b\end{bmatrix} \tag{13}$$

In which,

$$\begin{cases} \xi = \dfrac{1}{\alpha_1^2 + \alpha_2^2} \\ A = A(z) = \xi[\dfrac{\cos(\alpha_1 z)}{\alpha_1^2} + \dfrac{ch(\alpha_2 z)}{\alpha_2^2}] \\ B = B(z) = \xi[\dfrac{\sin(\alpha_1 z)}{\alpha_1^3} + \dfrac{sh(\alpha_2 z)}{\alpha_2^3}] \\ C = C(z) = \xi[\cos(\alpha_1 z) - ch(\alpha_2 z)] \\ D = D(z) = \xi[\dfrac{\sin(\alpha_1 z)}{\alpha_1} - \dfrac{sh(\alpha_2 z)}{\alpha_2}] \\ m = m(z) = \xi[\alpha_1 \sin(\alpha_1 z) + \alpha_2 sh(\alpha_2 z)] \\ b = b(z) = \xi[\alpha_1^2 \cos(\alpha_1 z) + \alpha_2^2 ch(\alpha_2 z)] \\ d = d(z) = \xi[\alpha_1^3 \sin(\alpha_1 z) - \alpha_2^3 sh(\alpha_2 z)] \end{cases} \tag{14}$$

For boundary condition $Y(0)=0, M(0)=0, Y(l)=0, M(l)=0$, which can be got that

$$\begin{bmatrix} Y(l) & M(l) \end{bmatrix}^T = H \cdot \begin{bmatrix} Y(0) & M(0) \end{bmatrix}^T \tag{15}$$

$$H = P_1 \cdot T_3 \cdot T_2 \cdot T_1 \cdot P_0 \tag{16}$$

$$P_1 = \begin{bmatrix} \alpha_{31}^2 \alpha_{32}^2 A_3 & \alpha_{31}^2 \alpha_{32}^2 B_3 & \dfrac{C_3}{(EI)_3} & \dfrac{D_3}{(EI)_3} \\ (EI)_3 \alpha_{31}^2 \alpha_{32}^2 C_3 & (EI)_3 \alpha_{31}^2 \alpha_{32}^2 D_3 & b_3 & m_3 \end{bmatrix} \tag{17}$$

$$P_0 = \begin{bmatrix} \alpha_{11}^2 \alpha_{12}^2 B_1 & \alpha_{11}^2 \alpha_{12}^2 A_1 & (EI)_1 \alpha_{11}^2 \alpha_{12}^2 D_1 & (EI)_1 \alpha_{11}^2 \alpha_{12}^2 C_1 \\ \dfrac{D_1}{(EI)_1} & \dfrac{C_1}{(EI)_1} & m_1 & b_1 \end{bmatrix}^T \tag{18}$$

Which can get the natural frequencies equation of beam through

$$\mathrm{Det(H)} = \mathrm{h}_{11} \cdot \mathrm{h}_{22} - \mathrm{h}_{13} \cdot \mathrm{h}_{23} = 0 \tag{19}$$

When applying different forces, the top three order natural frequencies of the beam axis are calculated by MATLAB and the specific data is shown in Table 3.

Table 3 Natural frequency values under different force

F	ω_1	ω_2	ω_3
0	9.863	39.493	89.103
100	9.601	38.431	88.326
200	8.532	37.389	87.517
300	6.256	36.425	86.429

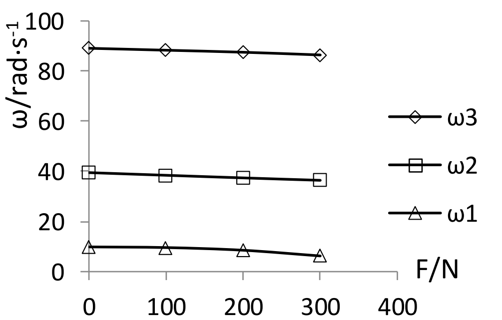


Fig. 3 The variation of natural frequency under different axial force

From the above data shows, the natural frequencies of the beam will be affected when different pull-pressure is applied. With the increase of the applied force, the natural frequencies will be reduced and it is especially obvious that the effect of the first-order natural frequency.

The natural frequencies equation is calculated again by MATLAB when the length of the tension shaft is changed to the 1/2 of cabinet minister. After that, the data obtained are shown in Table 4.

Table 4 Natural frequency values under different force

F	ω_1	ω_2	ω_3
0	9.863	39.493	89.103
100	9.701	38.931	88.826
200	9.232	38.489	88.217
300	8.856	37.925	87.829

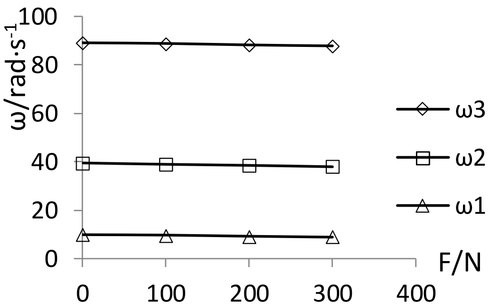


Fig. 4 The variation of natural frequency under different axial force

According to the shown in the date and the figure 3, the natural frequencies of the shaft will be changed when the tension length changes. The natural frequencies will decrease along with the pressure increases, but the trend is slow. Therefore, the axial tension can prevent the decline of the natural frequency.

4. Simulation analysis

Using finite element software [8]to simulate and analyze the model, which get the difference in the 4%-6% between theoretical value and simulation analysis values for the first two order and the difference of the third order natural frequency is about 7%.

5. Conclusion

Axial force will change the natural frequencies of the beam. Along with the increase of the force, the natural frequencies will be reduced when the shaft consists of three parts.

The natural frequencies of the beam will decrease with the increase of the force when the length of the shaft is modified. But compared to the former, the increase of the tension length will make the natural frequency decline slowly.

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