Mathematical Analysis of the Question of Safety Driving after Drinking

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Abstract

In this paper, we establish the mathematical models of differential equations of the alcohol concentration in liver, stomach and body fluid under the conditions of a short period of time and long time drinking, respectively, by exploiting three compartment model of pharmacoki netics and analyzing the changes of alcohol concentration in the intestines and stomach, liver and blood of human body after drinking. We obtain the corresponding trend curve of alcohol concentration with the time for three parts of human body using the software MATLAB, and make scientific and rational answers to some relevant practical problems, which validate the correctness, reasonableness and practicality of the proposed models.

Keywords

Pharmacokinetics; Three compartment model; Mathematical modeling; Alcoholicity.

1. Introduction

A large number of data indicates that tens of thousands of traffic accidents happen because of drunk driving each year in China. Fatal accidents in 50% above are related with drunk driving. The harm of drunk driving is shocking, and has become the first big "killer of traffic accident". So it is necessary to reflect the relationship between the alcoholicity in the blood and time by mathematical models [1-4]. In this paper, we establish the mathematical models of differential equations of the alcohol concentration in liver, stomach and body fluid under the conditions of a short period of time and long time drinking, respectively, by exploiting three compartment model of pharmacokinetics, and we obtain the corresponding trend curve of alcohol concentration with the time for three parts of human body using the software MATLAB. Thus it can provide the reasonable suggestions for the drivers after drinking long intervals.

2. The establishment of the model

In order to establish the mathematical model of alcoholicity in human body after drinking alcohol, we should neglect some secondary factors and make the following assumptions:

(1). Alcohol enters the body, and goes into the body fluid through the gastrointestinal absorption. Then it goes into the decomposition of the liver by the circulation of body fluid. CO2 and acetic acid generated in the process of liver decomposition enter into the blood, but we ignore the change of volume.

(2). The rate of gastrointestinal absorption of alcohol into the fluid, the rate of the liver which breaks down alcohol, and the transfer rate of alcohol in each part of the human body are all the constants, which are proportional to the mass concentration of alcohol without considering the individual differences.

(3). Body fluid accounts for 68% of body weight, and blood accounts for 7% of body weight. The alcoholicity in the blood is the same as other body fluids. We neglect occupied alcohol in the body volume.

(4). Drinking the alcohol in a short period of time is considered instantaneous. Drinking the alcohol in a long period of time is considered as drinking the alcohol at the constant rate during this time.

(5). The alcohol content of breathing, sweat and urine account for 5% of the total amount of alcohol.

(6). Herein we ignore the change of three-compartment volume, drugs, the effects of temperature and other factors

Alcohol enters the body, and goes into the body fluid through the gastrointestinal absorption. Then it goes into the decomposition of liver by the body fluid circulation. CO2 and acetic acid generated in the process of liver decomposition enters into the blood. After human body drinks alcoholic beverage, the alcohol concentration in the brain or kidney can reach the phase equilibrium with the alcohol concentration of blood in 10 minutes. In fact, the drinking process is divided into pre-15min stage and post-15min stage.

The stomach, liver, gastrointestinal fluid is simplified to the first room, the second room, the third room, respectively. Then the process of absorption and transfer of the alcohol in the first 15 minutes is shown as follows:

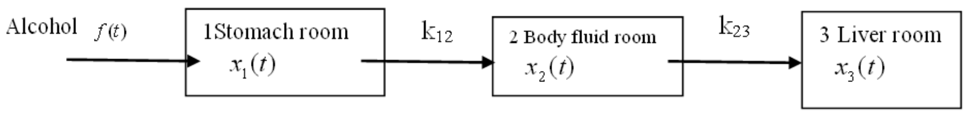

Fig. 1 The process of absorption and transfer of alcohol before 15 minutes

After 15 minutes, the alcohol concentration in the liver can reach the phase equilibrium with the alcohol concentration of blood. The process of absorption and transfer of the alcohol is shown as follows:

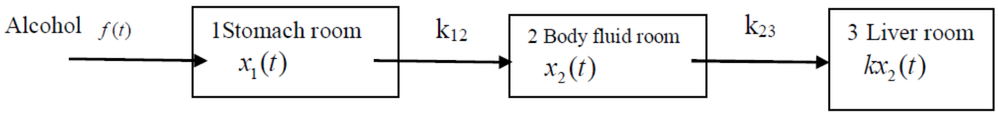

Fig. 2 The process of absorption and transfer of alcohol after 15 minutes

The meaning of the symbols is as follows [3, 5]:

k_{12}: The diffusion rate of alcohol from gastrointestinal to body fluids. $k_{12} = 1.6109$.

k_{23}: The diffusion rate of alcohol from body fluids to the liver. $k_{23} = 0.2012$.

$f(t)$: The rate of drinking alcohol.

$x_1(t)$: The alcohol concentration in the stomach ($mg / 100ml$).

$x_2(t)$: The alcohol concentration in the body fluid ($mg / 100ml$).

$x_3(t)$: The alcohol concentration in the liver ($mg / 100ml$).

v: The volume of body fluid, $v = 115539ml$.

k: After 10 minutes, the alcohol concentration in the liver can reach the phase equilibrium with the alcohol concentration of body fluid. k represents the ratio of the alcohol concentration of liver room and blood room, and the constant $k = x_3(t) / x_2(t)\ (t \geq 10\ \min)$.

m_0: The weight of alcohol in a bottle of beer, and $m_0 = 60000mg$.

T: The time of long time drinking (hour).

k_0: The rate of drinking alcohol in a long time ($mg / 100ml$).

β: The rate of liver breaking down alcohol, and $\beta = 0.185502$.

2.1 Model 1 (The model for drinking alcohol quickly)

Since one drinks a bottle of beer in a short time, $f(t)=0$, and the change rate of alcohol concentration in the stomach room is proportional to $x_1(t)$. The change rate of alcohol concentration in the body fluid room is linearly related to $x_1(t)$ and $x_2(t)$, respectively. Obviously, alcohol enters into the stomach in an extreme short time. So we have that $x_1(0) = \frac{m_0}{V}$ and $x_2(0) = x_3(0) = 0$ when $t = 0$.

First, we can construct the model of drinking alcohol in first 15 minutes:

$$\begin{cases} dx_1(t)/dt = -k_{12}x_1(t) \\ dx_2(t)/dt = -k_{23}x_2(t) + k_{12}x_1(t), \\ dx_3(t)/dt = k_{23}x_2(t) - \beta x_3(t) \end{cases} \tag{1}$$

Then we have

$$\begin{cases} x_1(t) = m_0 v^{-1} e^{-k_{23}t} \\ x_2(t) = v^{-1}(k_{12}-k_{23})^{-1} m_0 k_{12}(e^{-k_{23}t} - e^{-k_{12}t}) \\ x_3(t) = k_{23}v^{-1}(k_{12}-k_{23})^{-1} \\ m_0 k_{12}(\frac{1}{\beta-k_{23}}e^{(\beta-k_{23})t} - \frac{1}{\beta-k_{12}}e^{(\beta-k_{12})t} + \frac{1}{\beta-k_{12}} - \frac{1}{\beta-k_{23}})e^{-\beta t} \end{cases} \tag{2}$$

According to the corresponding data of alcoholicity in the blood, we have $x_2(0.25) = 15mg/100ml$. When $t = 15\min = 0.25h$, $x_3(0.25) = 0.4469mg/100ml$, $k = \frac{x_3(0.25)}{x_2(0.25)} = \frac{0.4469}{15} = 0.0298$.

After 15 minutes, the alcohol concentration in the liver can reach the phase equilibrium with the alcohol concentration of body fluid. From the computation of the differential equation above, we can obtain the initial condition:

$$x_1(0.25) = 49.3830mg/100ml,\ x_2(0.25) = 15mg/100ml,\ x_3(0.25) = 0.447mg/100ml.$$

Then we have

$$\begin{cases} dx_1(t)/dt = -k_{12}x_1(t) \\ dx_2(t)/dt = -k_{23}x_2(t) + k_{12}x_1(t) \\ x_3(t) = kx_2(t) \end{cases} \tag{3}$$

Exploiting the software MATLAB, we can obtain the curve of alcohol concentration of stomach room, body fluid room and liver room after 15 minutes as shown in Fig. 3.

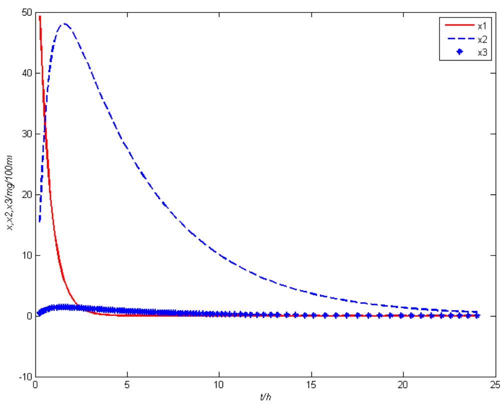

Fig. 3 Alcohol concentration of stomach room, body fluid room and liver room after 15 minutes

2.2 Model 2 (The model for drinking alcohol at average rate)

In fact, the model in the first 15 minutes is the case of drinking alcohol in a long time. For the convenience of analysis and computation, the drinking process is considered as uniform drinking of the same amount of alcohol, i. e., $f(t)=k_0$. With the increased frequency of drinking, alcohol concentration of the blood gradually increase. After one completes drinking beer in the time of T, the alcohol that one drinks at a short time before T time is not absorbed in a short time. So alcohol concentration will continue to increase in a period of time after the time of T. It will achieve maximum value at a moment, and then alcoholicity will gradually decrease. Setting $T = 2h$, three bottles of beer will be drunk at constant speed in the time of T. Then we have

$$k_0=\frac{3m_0}{TV}=\frac{3\times6\times10^4}{2\times1155.39}\text{mg}/100ml=7.789578\text{mg}/100ml$$

The initial condition is as: $x_1(0)=x_2(0)=x_3(0)$, then we can obtain

$$\begin{cases} dx_1(t)/dt=-k_{12}x_1(t)+k_0 \\ dx_2(t)/dt=-k_{23}x_2(t)+k_{12}x_1(t) \\ dx_3(t)/dt=k_{23}x_2(t)-\beta x_3(t) \end{cases} \quad (4)$$

After 15 minutes, the alcohol concentration in the liver can reach the phase equilibrium with the alcohol concentration of body fluid. According to the computation of the differential equations of the first 15 minutes, we can obtain the initial conditions:

$$x_1(0.25)=16.03, x_2(0.25)=3.3852, x_3(0.25)=0.10088.$$

Then we can obtain the corresponding mathematical model for the time between 15 minutes and the end of drinking as follows:

$$\begin{cases} dx_1(t)/dt=-k_{12}x_1(t)+k_0 \\ dx_2(t)/dt=-k_{23}x_2(t)+k_{12}x_1(t) \\ x_3(t)=kx_2(t) \end{cases} \quad (5)$$

Exploiting the software MATLAB, we can obtain the curve of alcohol concentration of stomach room, body fluid room and liver room from 15 minutes to 2 hours as shown in Fig. 4.

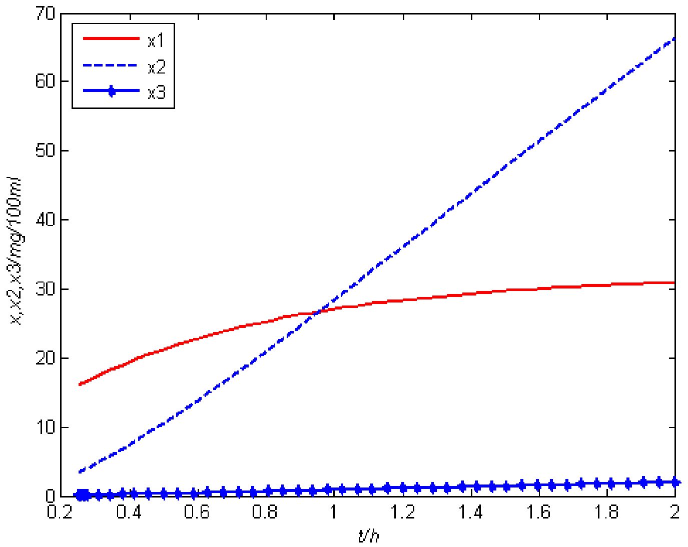

Fig. 4 Alcohol concentration of stomach room, body fluid room and liver room from 15 minutes to 2 hours

The process of drinking beer comes to an end after 2 hours. It is not difficult to obtain the initial conditions: $x_1(2) = 46.4259$, $x_2(2) = 93.5113$, $x_3(2) = 2.7866$. Thus we can obtain

$$\begin{cases} dx_1(t)/dt = -k_{12}x_1(t) \\ dx_2(t)/dt = -k_{23}x_2(t) + k_{12}x_1(t) \\ x_3(t) = kx_2(t) \end{cases} \tag{6}$$

Exploiting the software MATLAB, we can obtain the curve of alcohol concentration of stomach room, body fluid room and liver room after 2 hours as shown in Fig. 5.

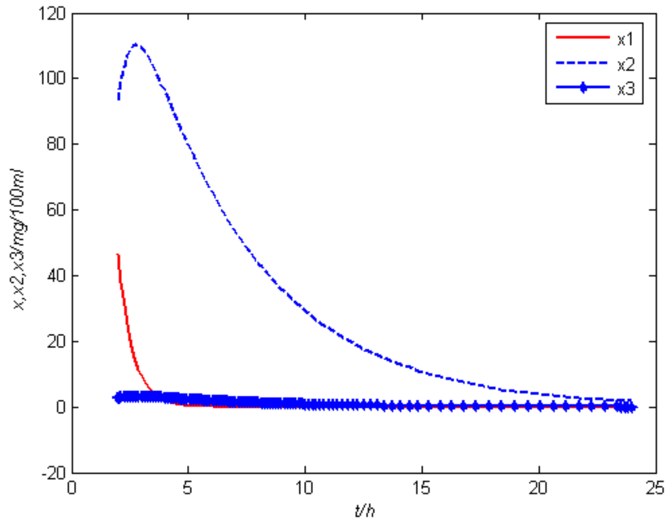

Fig. 5 Alcohol concentration of stomach room, body fluid room and liver room after 2 hours

3. Analysis and verification of the model

Question 1: How long does alcohol concentration of body fluid reach the maximum for the method of drinking in a short time and long time, respectively?

Solution: From Fig.1, we easily find that alcohol concentration of body fluid reaches the maximum at the time of $t_1 = 1.375h$. Likewise, it is easy to check that alcohol concentration of body fluid reaches the maximum at the time of $t_2 = 3.68h$. Moreover, $t_2 = 3.68h > 2h$, which satisfies the medical theory that alcohol concentration will continue to rise in a period of time after completing drinking beer.

Question 2: The body weight of a normal adult male is 70kg. He drinks a bottle of beer at 8 a.m. When he is being examined at 2 o'clock in the afternoon, his alcohol concentration meets the driving standard. However, he drinks the same one bottle of beer at 7 o'clock in the evening. After 6 hours, he drives. But at this time, the traffic police checks him again and finds that he is drunk driving. Why are two test results not the same under the same time interval when he drinks the same amount of wine? How long intervals does he ensure to compliance with the driving standards for the second drinking?

Solution: From Fig. 3, we can find that when the traffic police check him the first time, alcohol concentration of body fluid

$$x_2(6) = 19.8mg / 100ml < 20mg / 100ml.$$

It is easy to check that he doesn't break the traffic regulations. When he accepts the second check, the alcohol residue of first drink $x_2(17) = 3.5mg/100ml$. At this time, the alcohol concentration of body fluid contains the alcohol residue of first drink after 17 hours and the alcohol residue of second drink after 6 hours, i.e.,

$$23.3mg / 100ml > 20mg / 100ml.$$

These results are consistent with the facts and validate the correctness of the proposed model.

Question 3: How long can he drive after drinking three bottles of beer with the fast means?

Solution: From Figs 4-5, we can find that driving during in the following 12 hours is a drunken driving behavior, after he drinks three bottles of beer one time. Moreover, driving during in the time quantum from 0.35h and 4.5h is a drunken driving behavior. Alcohol of body will basically evacuate after 24 hours. These are consistent with medical knowledge.

4. Conclusion

In this paper, we construct the mathematical models of differential equations of the alcohol concentration in liver, stomach and body fluid under the conditions of a short period of time and long time drinking, respectively, by exploiting three compartment model of pharmacokinetics. Moreover, the proposed models consider two cases: the forth and back of the balance of alcohol concentration between body fluid and liver, which are very scientific, reasonable and reliable. We have analyzed the change curve of alcohol concentration in the liver with the time. It can help to prevent alcohol liver and other liver diseases. Scientific and rational answers to some relevant practical problems are very good, which validate the correctness, reasonableness and practicality of the proposed models.

Acknowledgements

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References

[1] X.L. Zhang, F. Lin. Mathematical Modeling and experiment [M]. Beijing: Chemical Industry Press, 2013. (In Chinese).

[2] Q.W. Sun. Metabolism of Alcoholic Beverage in Human Body and Its Optimum Drink. Liquor-making Science and Technology (8) 2006 126-127. (In Chinese).

[3] H.J. Xu. The Mathematical Models on the Issue of Drinking and Safety Driving. Journal of Xinxiang University (27) 2010 13-16. (In Chinese).

[4] W. Li et al. Mathematical Modeling of Drunken Driving. COLLEGE MATHEMATICS (25) 2009 154-158. (In Chinese).

[5] X.B. Fang et al. Mathematical Analysis of Alcoholic Metabolism. Chinese journal of engineering mathematics (21) 2004 155-158. (In Chinese).