Apparent Permeability Model of Nanometer Pore in the Shale Reservoir

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Abstract

The size of pore radius is primarily within the range of a few nanometers to a few tens of nanometers because of the very small pore structure, which can’t satisfy the basic assumption of the classic Darcy law. The current research of the flow mathematical model for the shale gas reservoir focuses mainly on the correction of the viscous flow model and ignores the problem the transition flow between the viscous flow and molecular flow. Based on the pressure, temperature, pore structure and gas properties of the gas reservoir, this paper considered the effect of gas slippage and influence of Knudsen diffusion. Due to the above factors, in this paper, apparent permeability is considered as a function of the pore radius and gas pressure and different flow models of apparent permeability and slippage is calculated and analyzed. The law of the ratio of apparent permeability to true permeability and slippage with the gas pressure and porous configuration of different models is analyzed. The results show: the smaller the porosity of the shale reservoir is and the lower the gas pressure, the ratio of apparent permeability to true permeability is smaller.

Keywords

Shale Gas; Nanometer Pores; Apparent Permeability; Slippage; Knudsen Diffusion.

1. Introduction

The pore size of conventional sandstone and carbonate rock reservoirs is between 1 and 100 micrometers, while the pore throat diameter of shale gas reservoir is between 5 and 1000 nanometers [1] or in a much smaller scale between 5 and 50 nanometers [2]. Because of the pore radius being at nanoscale sizes, the permeability of matrix is between micro-Darcy and nano-Darcy in the shale gas reservoir, which need fracturing reconstruction to get economic output. The flow characteristics of gas reservoir have many flow patterns from microcosmic flow in the nanometer pores to Darcy flow in the hydraulic fracture. Especially when the collisions between gas molecules in the matrix and pipe wall are much more frequent, the gas flow show the characteristic of molecular micro seepage pattern, but not follows the Darcy flow. The slippage and Knudsen diffusion can explain that the actual production of shale gas reservoir is higher than forecasting production.

In view of various flow patterns such as viscous flow, slip flow, transition flow and molecular flow, based on the definition of apparent permeability, different models of apparent permeability of shale gas reservoir comparative analyzed and the ratio of apparent permeability to true permeability of different models and the law of the slip factor with gas pressure and pore radius is calculated [3].

In this paper, apparent permeability analytical models as a function of gas pressure and pore radius are discussed and analyzed. Meanwhile, the suitability of different analytical models of apparent permeability is assessed under different pressures and pore radiiuses from theoretical formula and graphic method of analysis.
2. Knudsen diffusion and division of flow pattern

The speed of fluid at pore wall is considered as zero in the fluid mechanics of continuous medium [4]. The pore radius is between 1 and 100 micrometers in the conventional reservoir, which can satisfy the condition of Darcy law. In the shale reservoir, the pore radius is between 1 and 200 nanometers [5] and the fluid mechanics of continuous medium should not be applied. Many molecules strike pore wall and tend to slide on the pore wall [6].

Jones and Owens[7] carried out an experiment to derive the formula of gas static slip factor and made a comparison with the result got by Klinkenberg[8]; Sampath[9], Keighin and Florence [10,11] established an empirical formula similar to the equation derived by Jones and Owens. Ertekin et al.[12] introduce dynamic slip factor as a function of pressure which was closely related to Knudsen diffusion coefficient and further developed the slip factor. Based on the work of Beskok and Karniadakis, Michel et al.[13] developed slip factor by introducing Knudsen diffusion coefficient. Civan, Sakhaee-Pour and Bryant [14,15,16] determined slip factor and derived apparent permeability as a function of Knudsen number. Javadpour [17] deduced apparent the permeability formula as a function of pressure, temperature took into account the influence of Knudsen diffusion and slip factor.

Klinkenberg was the first person that proposed gas slippage effect in petroleum industry. The apparent permeability formula that he raised is:

\[ k_a = k_\infty (1 + \frac{b_k}{p_{avg}}) \]

Here, \( k_a \) is apparent permeability (mD); \( k_\infty \) is equivalent of fluid permeability (mD); \( p_{avg} \) is the average gas pressure under trial conditions; \( b_k \) is slip factor. The equation of slip factor is:

\[ b_k = \frac{4c\lambda p}{r} \]

Knudsen number is a vital parameter that decides the flow pattern (includes viscous flow, slip flow, transition flow and molecular flow). In math, the definition of Knudsen [18] number is:

\[ k_n = \frac{\lambda}{r} \]

The mean free path of gas molecules [19] is:

\[ \lambda = \frac{\mu}{p} \sqrt{\frac{\pi RT}{2M}} \]

The flow pattern can be divided into four types. Different persons have different limits for the definition of the viscous flow area, which researchers generally believed that the critical value of Knudsen number of viscous flow is 0.001[20], while in other papers is 0.01[21,22]. In this paper, the critical value is 0.001. Assume that the pore radius is 6.5nm, the pressure is from 200psi to 7000psi and the average free path is from 0.6nm to 8nm, so the Knudsen number is from 0.09 to 1.23. The flow pattern of studied is in the area of slip flow and transition flow.

<table>
<thead>
<tr>
<th>Knudsen number</th>
<th>flow pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Kn \leq 0.001 )</td>
<td>viscous flow</td>
</tr>
<tr>
<td>0.001 &lt; ( Kn &lt; 0.1 )</td>
<td>Slip flow</td>
</tr>
<tr>
<td>0.1 &lt; ( Kn &lt; 10 )</td>
<td>transition flow</td>
</tr>
<tr>
<td>( Kn \geq 10 )</td>
<td>molecular flow</td>
</tr>
</tbody>
</table>
3. Model of apparent permeability with slip factor and Knudsen diffusion

3.1 Model of absolute permeability with slip flow

Jones and Owens [23] studied 100 samples of tight gas reservoir, raising an experimental correlation of slip factor and absolute permeability. Sampath and Keighin provided a calculating equator of slip factor for the core samples (net confining pressure is 34.5Mpa, the diameter is 0.025 meters; the length is 0.05 meters) at the temperature of 80°C to 90°C. Florence et al put forward a relationship of nitrogen flow in porous media, which involve slippage effect in the formula of average free path and Knudsen number having nothing to do with pressure. In conclusion, the model of absolute permeability with slip factor can be seen in Tab.2.

Table.2 Model of absolute permeability with slip factor

<table>
<thead>
<tr>
<th>time</th>
<th>author</th>
<th>slip factor</th>
<th>significant factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>Jones and Owens</td>
<td>$b_k = 12.639(k_n)^{0.33}$</td>
<td>static slippage effect</td>
</tr>
<tr>
<td>1982</td>
<td>Sampath and Keighin</td>
<td>$b_k = 13.581(K_n^{0.53})$</td>
<td>static slippage effect</td>
</tr>
<tr>
<td>2007</td>
<td>Florence</td>
<td>$b_k = 43.345(K_n)^{-0.5}$</td>
<td>static slippage effect</td>
</tr>
</tbody>
</table>

3.2 Model of slip factor with Knudsen diffusion

Ertekin et al deduced the formula of Knudsen diffusion coefficient as shown in Tab.3. Michel et al established apparent permeability models referring to Beskok and Karniadakis. Michel et al took into consideration Knudsen diffusion, matrix porosity and tortuosity of porous media, average free path of gas molecules and pressure. The equation of equivalent matrix permeability could be seen in Tab.3 in SI.

Table.3 Slip factor model with Knudsen diffusion

<table>
<thead>
<tr>
<th>time</th>
<th>author</th>
<th>Knudsen diffusion</th>
<th>slip factor</th>
<th>significant factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>Ertekin</td>
<td>$D_k = \frac{31.54}{\sqrt{M}}(k_n)^{0.67}$</td>
<td>$b_k = \frac{D_k \mu c_\varepsilon p_{avg}}{\alpha_k k_n}$</td>
<td>Knudsen diffusion and dynamic slippage effect</td>
</tr>
<tr>
<td>2011</td>
<td>Michel</td>
<td>$D_k = \frac{2r (8RT)}{3(\pi M)^{0.5}}$</td>
<td>$b_k = \frac{3\pi D_m \mu}{16k_m}$</td>
<td>Knudsen diffusion and dynamic slippage effect</td>
</tr>
</tbody>
</table>

3.3 Models of apparent permeability within Knudsen number

Sakhaee-Pour and Bryant studied flow characteristics of different numbers according to Beskok and Karniadakis’s results and deduced flow equations of every region. They speculated that the relationship between apparent permeability and absolute permeability in transition flow is [24]:

$$k_a = k_n (0.8453 + 5.4576K_n + 0.1633K_n^2)$$

Here, $0.1 \leq K_n \leq 0.8$.

Civan studied the flow of shale reservoirs by Beskok and Karniadakis’s way. The equation of apparent permeability based on Knudsen number is:

$$k_a = k_n f(K_n)$$

Here, $f(K_n)$ is the function of Knudsen number, tenuity factor and slip factor.

$$f(K_n) = (1 + \alpha_2 K_n)(1 + \frac{4K_n}{1 - bK_n})$$

The dimensionless tenuity factor $\alpha_2$ is:
\[ \alpha_2 = \alpha_0 \left( \frac{K_n^B}{A + K_n^B} \right) \]

In the above equation, the down limit of \( \alpha_2 \) is zero, which corresponds to slip flow; when \( K_n \) tends to be infinite, the up limit of \( \alpha_2 \) is \( \alpha_0 \), which corresponds to free molecular flow and \( A \) and \( B \) are curve fitting coefficients. Civan fetch a specific set of data (\( \alpha_0=1.358 \), \( A=0.178 \), \( B=0.4348 \)) for transition flow in tight sandstone. Civan came forward an approximate value of the above equation according to Florence’s results. When \( Kn \) was less than or equal to 1, the dynamic slip factor is [25] (viscosity is a function of pressure):

\[ b_k = \frac{2790 \mu}{\sqrt{M}} \left( \frac{k_v}{\phi} \right)^{-0.5} \]

3.4 Models of apparent permeability within Knudsen diffusion and slip effect

Javadpour et al. studied Knudsen diffusion and slip effect of nanopores in the shale reservoirs, and the formula of apparent permeability is:

\[ k_a = \frac{D_k \mu M}{RT \rho} + F k_D \]

Here, \( D_k \) is Knudsen diffusion coefficient; \( \mu \) is gas viscosity; \( M \) is the molecular mass of gas molecules; \( R \) is gas constant; \( T \) is temperature; \( \rho \) is gas average density; \( F \) is slip factor; \( k_0 \) is absolute permeability; The equation of slip factor is [26]:

\[ F = 1 + \left( \frac{8\pi RT}{M} \right)^{0.5} \frac{\mu}{p r} \left( \frac{2}{f} - 1 \right) \]

In the above formula, \( p_{avg} \) is average pressure; \( r \) is nanopore radius; \( f \) is the percentage of collision on the pore wall; the apparent permeability item that Javadpour introduced can revised to the absolute permeability as follows[27]:

\[ k_a = c_g D_k \mu + F k_D \]

4. Calculation and result analysis

4.1 Parameter value

In this paper, apparent permeability along with pressure and pore radius is calculated according to the all above different researchers. Gas compression coefficient is calculated by the BWR relationship Matter raised. Gas viscosity is computed with Kobayashi and Burrow’s method. The tortuosity value that worked out is 65. The pore radius is calculated from the below equation:

\[ r = 2.665 \left( \frac{k_m}{100 \phi_m} \right)^{0.45} \]

Assume the matrix permeability is 12.5md and porosity is 0.15, the calculation result of pore radius is 6.5nm. Single-phase methane gas is selected in the calculation. Shale reservoir physical parameter and the formula constants are: \( M=0.018 \text{kg/mol} \), \( R=8.314 \text{J/(mol·K)} \), \( f=0.8 \), \( A=0.178 \), \( B=0.4348 \), \( b=-1 \).

4.2 Ratio analysis of apparent and absolute permeability

Ratio of apparent and absolute permeability of different models under different pressures is worked out as Fig.1 shown. Simulation results show: the lower the pressure is, the ratio is higher; when the pressure value is very high, the apparent permeability is approaching to absolute permeability. The ratio increases quickly with pressure decreasing when the pressure is less than 800psi. To a lesser degree, it can increase a few orders of magnitude; to a more degree, it can increase over twenty orders of magnitudes.
Fig. 1 Diagram of the ratio of apparent permeability to Darcy permeability with pressure. Value added of ratio is minimum in the Klinkenberg and Jones-Owens’s forecasting model. Their models assume a static slippage factor (based on study of tight gas samples), which might not hold very well for shale where matrix pore size is much smaller than tight gas reservoirs. Michel et al. took Knudsen diffusion into account, and the results fall in between those by Jones and Ertekin. Ertekin et al. who included a dynamic slippage factor which changes with pressure, predicts a greater change in apparent permeability with pressure as compared to static slippage models. Sakhaee-Pour and Bryant used a relationship between apparent permeability and Knudsen number and the results were higher than the above models. Javadpour’s model predicts much more permeability change than other models since it considers both Knudsen diffusion and slippage. The prediction using the Florence et al. theoretical model falls above those from other models—we believe the reason is the multiplying factor 43.345 which amplifies the effect. Also notable is Fathi et al.’s prediction, which cuts across the other models.

Figure 2 shows the variation in the ratio of apparent permeability to matrix Darcy permeability with respect to pore radius. As can be seen, the average pressure is 1000psi and the pore radius is in a more practical range, 1nm to 100nm. The smaller the pore radius is, the ratio of apparent permeability to Darcy permeability is higher. When the pore radius is 1nm, the ratio of every model reached a maximum. The ratio changed at 1.5 when the pore radius increased to 100nm, where apparent permeability was basically the same as Darcy permeability.

Fig. 2 Diagram of the ratio of apparent permeability to Darcy permeability with pore radius.
4.3 Slippage analysis of different models

Figure 3 shows how the ratio of apparent permeability to matrix Darcy permeability varies with pressure. The value range of pressure is from 200psi to 8000psi and the pore radius is 6.5nm. As can be seen, models given by Jones-Owens, Sampat-Keighin and Florence predict the static slippage factor remains unchanged with pressure. As pressure increases, the slippage factor predicted by Ertekin increase firstly and then decrease. Michel et al. took Knudsen diffusion into account and the results of slippage factor simulation continuously increases with pressure boosting. The slippage factor given by Klinkenberg increases linearly with pressure boosting.

![Fig.3 Diagram of slippage factor with pressure](image)

Figure 4 shows the variation in the ratio of apparent permeability to matrix Darcy permeability with respect to pore radius. As can be seen, the larger the pore radius is, the slippage factor is smaller; the smaller the pore radius is, the slippage factor is larger. The slippage factor got by Florence is highest and the factor by Jones-Owens is lowest for the pore radius in a range, 1nm to 1000nm.

![Fig.4 Diagram of slippage factor with pore radius (0.1 to 1000nm) under the pressure 1000psi](image)
5. Conclusion

The ratio of apparent permeability to matrix Darcy permeability varies with pressure and makes important effects. The simulation results show: the higher the pressure is, the ratio is higher; for higher pressure, apparent permeability is closer to matrix Darcy permeability. Under the same pressure, the value by only taking static slippage into account is less than by Knudsen diffusion; the value by only considering Knudsen diffusion is less than by dynamic slippage effect; the value by dynamic slippage effect is less than by dynamic slippage effect and Knudsen diffusion.

The pore radius has an important influence on the ratio of apparent permeability to matrix Darcy permeability. The smaller the pore radius is, the ratio is higher. When the pore radius is in a range, a few nanometers to a few tens of nanometers, the apparent permeability is several or dozens of times of Darcy permeability.

The theoretical basis of Javadpour’s model is reliable, which takes Knudsen diffusion and gas molecules slippage flow into account. The ratio of apparent permeability to Darcy permeability is the second largest from calculation analysis and the simulation result is conform to theory and accurate.

The pore radius has an important influence on slippage factor. The slippage factor decreases with the pore radius becoming larger; the smaller the pore radius is, the slippage factor is higher. The Florence’s simulation result is highest and the Jones-Owens’s is lowest in a range of pore radius, 1nm to 10nm.

References