A Neighborhood Wavelet Coefficient Image Denoising with Improved Threshold

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Abstract

Selection of threshold and threshold function is the key of wavelet-denoising. After wavelet decomposition and conversion, the low frequency part contains plenty of useful signals, while the high frequency part with noise distributed in the whole wavelet domain contains details of a few useful signals. Processing with fixed threshold and threshold function may cause loss of details of the useful signals of high frequency part. In this article, decomposition scale is introduced for threshold and combined with neighbor coefficient, different thresholds and threshold functions are used for processing of wavelet coefficient under various decomposition scales to retain the detail information under different scales. Simulation results show that the Peak Signal to Noise Ratio (PSNR) of the method proposed in this article is the maximum and Root Mean Square Error (RMSE) is the minimum, improving denoising effect.

Keywords

Threshold; Threshold function; neighbor coefficient; image denoising; Evaluation index.

1. Introduction

An image is often corrupted by noise during its acquisition, conversion, transmission and storage , which results in the degradation of image. Image denoising is used to remove the additive Gaussian noise while retaining image feature .

Wavelet transformation has good multi-scale and multi -resolution analytical characteristics, widely applied to image denoising. In 1994, Donoho et al. [1] proposed wavelet threshold denoising method based on wavelet transformation, and this method achieves the purpose of denoising through setting threshold within wavelet domain to reset the relatively small noise coefficients. The traditional threshold denoising method includes soft threshold method and hard threshold method[2], and the soft threshold processing method has constant deviation during denoising, thus giving rise to edge distortion. For hard threshold processing method, additional shock and Pseudo-Gibbs effect appear during denoising. Scholars at home and abroad make improvements for threshold function and threshold according to the above defects, putting forward eclectic non -smooth threshold method, logarithmic smooth threshold method, mould flat method etc. Wherein, literature [3] soft and hard threshold improvement eclectic method is able to effectively reduce the constant deviation between the estimated wavelet coefficient and wavelet coefficient, but has the defect of discontinuity of threshold function. As wavelet threshold denoising is processing of individual wavelet coefficient one by one, without consideration to influence on neighbor coefficient [4-6], in 2000, Cai and Silverman [7] proposed (NeighCoeff) wavelet coefficient denoising method, with better effect than traditional wavelet denoising, and verifying the correlation [8-11] between neighbor coefficient and current wavelet coefficient within the same scale. Literature [8] considers influence of decomposition scale in threshold setting, its test shows that the reconstructed image is smoother, performance index is promoted to some extent, but it is too smooth compared with original image, because of the excessive killing of wavelet coefficient during processing of detail information under different scales, causing that the reconstructed image still has relatively large error. Literature [10] adds size of neighboring window size to threshold function, compared with the original image, the image after denoising improves in denoising index, but has fine particles, because the wavelet coefficient has excessive retention during processing of detail information. Therefore, selection of threshold and threshold function of NeighCoeff denoising method is the key of image denoising.

2. Neighbor wavelet coefficient method

Model of image under noise pollution as shown in formula (1):

$$f(x,y) = s(x,y) + m(x,y) \tag{1}$$

Wherein, $f(x,y)$ is noise image, $s(x,y)$ is original image, $m(x,y)$ is gaussian white noise, subject to distribution $N(0,\sigma^2)$ As wavelet transformation is linear, after wavelet transformation of image, wavelet coefficient meet formula (2);

$$W_f = W_s + W_m \tag{2}$$

Wherein, W_f is coefficient of noise image after wavelet transformation, W_s is the coefficient of original image after wavelet transformation, and W_m is the coefficient of noise after wavelet transformation.

At the time of wavelet decomposition and transformation, useful signals are distributed in minority wavelet coefficients, amplitude of wavelet coefficient is relatively large, while noise signals are distributed in high frequency part, and wavelet coefficient is relatively small. Therefore, Donoho et al. put forward that purpose of wavelet threshold denoising method is selecting appropriate threshold to separate noise from useful signals, and then making quantization of wavelet coefficient after separation, the quantization rule is threshold function, then setting the coefficient with amplitude lower than threshold to be 0 and retaining (hard threshold processing) or shrinking (soft threshold processing) the coefficient with amplitude higher than threshold. Within certain neighbor, as certain correlation exists between wavelet coefficients, that is, the neighbor coefficient amplitude of wavelet coefficient with relatively large amplitude has a higher possibility, so, neighbor coefficient shall be given consideration during processing of current wavelet coefficient to avoid missetting of important coefficients.

Chen et al. [12]applied neighbor wavelet-denoising method proposed by Cai and Silverman to image denoising, called NeighShrink method, taking a neighboring window for each coefficient wjk of each sub-band, scale of each window may be 3X3,5X5,7X7, etc., Figure 1 gives an example of neighboring window, and window center is the wavelet coefficient of threshold to be selected.

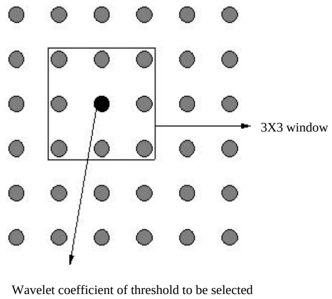

Fig 1 Example of neighboring window, window center is the coefficient of threshold to be selected.

Provide the coefficient in window with processing, as shown in formula (3)

$$S_{j,k}^2 = \sum w_{j,k}^2 \tag{3}$$

Wherein, $w_{j,k}$ is wavelet coefficient of j layer, $S_{j,k}^2$ is quadratic sum of coefficients in window. When processing wavelet coefficient in the center of window, taking neighbor wavelet coefficient into account, and then selecting proper threshold and threshold function for processing of current wavelet coefficient.

Threshold function selected by NeighCoeff is shown in formula (4).

$$\hat{w}_{j,k} = \begin{cases} w_{j,k}(1-\frac{T^2}{S_{j,k}^2}) & S_{j,k}^2 \geq T^2 \\ 0 & \text{else} \end{cases} \tag{4}$$

Wherein, T is threshold, and $T = \sigma\sqrt{2\lg(N)}$. *T is common threshold.*

The formula shows that T remains unchanged in whole scale space, T is determined as long as signals are determined. With increase of decomposition scale, wavelet coefficient of useful coefficient increases while noise decreases. Therefore, processing of wavelet coefficient is made on each layer with relevant threshold and threshold function, the above threshold and threshold function are constant in the whole scale, so “killing” or “excessive retention” appears, which affecting denoising performance to some extent.

3. Setting of threshold and selection of threshold function

3.1 Threshold improvement

Selection of threshold is the key of denoising of wavelet threshold, threshold is the limit value for separating the useful signals and noise, if too large threshold is selected, some useful signals may be set to be zero as noise, leading to excessive “killing”. If too small threshold is selected, some noise may be shrunk as useful signals, resulting in “excessive retention”. In the change process of wavelet, useful signals are mainly distributed in wavelet coefficients with relatively large amplitude, while noise is mainly distributed in lower layers, and with increase of number of decomposition layers, wavelet coefficient of noise decreases. Therefore, the high layer needs a relatively small threshold. Based on this, a new threshold is proposed, its expression is shown in formula (5):

$$T(j) = \sigma\sqrt{2\log(m\cdot n/2^{2j})} \tag{5}$$

Wherein, j is decomposition layer number, σ is noise RMSE, and $m\cdot n$ is image size. j with increase of number of decomposition layers, threshold decreases.

3.2 Selection of threshold function

Threshold function is the rule for processing of wavelet coefficient, the hard threshold method has shock during denoising; Soft threshold method has the problem that there is constant deviation between wavelet estimate coefficient of soft threshold function and wavelet coefficient. According to defects of the traditional method, a new threshold function is proposed.

$$\hat{w}_{j,k} = \begin{cases} w_{j,k}\beta & ,S_{j,k}^2 \geq T^2 \\ 0 & ,S_{j,k}^2 < T^2 \end{cases} \tag{6}$$

Wherein, β is shrinkage factor, the wavelet coefficient higher than threshold will be shrunk by β , and shrinkage factor is defined as:

$$\beta = 1 - \mu\frac{T^2}{S_{j,k}^2 * e^{k-1}} \tag{7}$$

Wherein, $0 \leq \mu \leq 1$, such as selecting μ=3/4, $k \geq 1$, integral, such as selecting $k=1$, j is the number of decomposition layer.

Formula (6) and (7) show that when making threshold processing, compare the quadratic sum within neighbor $S_{j,k}^2$ with threshold T^2, and only when $S_{j,k}^2$ is less than T^2, it is set to be zero. In other cases, wavelet coefficient will be shrunk properly according to shrinkage factor. Wherein, in formula (7), if $S_{j,k}^2$ is more than T^2 , and only if $S_{j,k}^2$ is larger ,gradually increase the value of k, and make the value of $T^2/(S_{j,k}^2 * e^{k-1})$ tend to be 0, and thus make $\hat{w}_{j,k}$ gradually tending to $w_{j,k}$ Additionally, with adjusting μ, it is able to reduce image blurring and detail loss caused by compression of detail wavelet coefficient.

The implementation steps of improved method are as follows:

(1)Perform multscale decomposition on the image corrupted by Gaussian noise. The 2-D wavelet transform on the noisy image is performed up to J th level to generate several subbands.

(2)Use the robust median to estimate the noise level σ that is given by:

$$\sigma^2 = [(median\,|W_{i,j}|)/0.06745]^2 \tag{8}$$

Where $W_{i,j}$ represent the wavelet coefficient matrix in the direction of diagonal after wavelet decomposition of image.

(3)For each subband (except the low pass residual), apply the proposed method to obtain the noiseless wavelet coefficients.

(4)Perform the inverse wavelet transform on the modified coefficients to obtain the denoised estimate image.

4. Indexes of denoising evaluation

In order to evaluate the denoising effect of different methods, two indexes of denoising evaluation are selected[13]: peak signal-to-noise ratio (PSNR)、the mean squared error(RMSE).

4.1 Peak signal to noise ratio(PSNR)

$$PSNR = 10\log\frac{255^2 MN}{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(f(i,j)-\hat{f}(i,j)\right)} \tag{9}$$

Where M X N is the image size, $f(i,j)$ is the estimate of the image and $\hat{f}(i,j)$ is the original image without noise. In practical application, the larger PSNR is, the better the effect of denoising is getting.

4.2 Mean squared error (RMSE)

$$RMSE = \sqrt{\frac{1}{mn}\sum_{i,j}[f(i,j)-\hat{f}(i,j)]^2} \tag{10}$$

Where M X N is the image size, $f(i,j)$ is the estimate of the image and $\hat{f}(i,j)$ is the orginal image without noise. In practical application, the smaller RMSE is, the better the effect of denoising is getting.

5. Experiment simulation and analysis

In experiment, select db8 wavelet basis, number of decomposition layer is 3, window size is 3 X 3, with best effect [12]. In order to verify the effectiveness of denoising method proposed in this article, conduct denoising for Cameraman image (size 512 X 512) of gaussian noise with mean 0 and variance 25 with soft threshold method, hard threshold method, eclectic threshold method, IIDMWT method in literature [14] and the improved method proposed by this article respectively. Original image, noise image as well as obtained denoising image are shown in Fig 2.

(a) Gray scale Lena original scale (b) Noise image

 

(c) Hard threshold denoising (d) Soft threshold denoising

 

(e) Eclectic threshold denoising (f) NeighShrink denoising,

 

(g) Denoising with method in literature (h) Denoising with method of this article

Fig.2 the image de-noised with different wavelet methods

Fig.2(c) and 2(d) are effect pictures of denoising with traditional soft and hard threshold methods respectively, noise is removed to some extent, but the wave peak is not smooth and some noise is retained for soft threshold method. For hard threshold denoising method, due to discontinuity of threshold function, the denoising effect is poor. Eclectic method is proposed based on the above method, its denoising effect is improved so some extent comparing with traditional methods, but some noise is still retained in detail part as shown in Figure 2(e); The above is based on processing of wavelet coefficient one by one, ignoring the influence of neighbor coefficient. NeighShrink method introduces the concept of sliding window, giving full consideration to neighbor coefficient when processing current wavelet coefficient, but because of unreasonable selection of threshold, the detail feature of image is too smooth, and image denoising effect is poor as shown in Figure 2(f); Literature [14] is an improved method proposed based on NeighShrink method, but the proposed threshold function is constant in whole scale, excessive killing may appear during processing of wavelet coefficient, and loss of detail information of signals may be caused as shown in Figure 2(g). At last, the method put forward in this article is adopted as shown in Figure 2(h), more detail information is retained while removing noise, and the reconstructed signals are more complete.

In order to further verify the method put forward in this article, PSNR and RMSE are adopted for objective evaluation. The calculation results are shown in Table 1 and Table 2:

Table1 PSNR of different de-noising methods

σ	Soft threshold method	Hard threshold method	Eclectic method	NeighShrink	IIDMWT	Method of this article
10	28.79	30.64	30.65	33.53	33.65	34.17
20	26.90	27.99	28.00	30.43	30.76	31.06
30	25.98	26.67	26.69	28.52	28.96	29.14
40	25.39	25.78	25.79	27.27	27.56	27.68
50	24.94	25.10	25.11	26.22	26.45	26.55

Table 2 RMSE of different de-noising methods

σ	Soft threshold method	Hard threshold method	Eclectic method	NeighShrink	IIDMWT	Method of this article
10	9.27	7.49	7.48	5.37	5.07	4.99
20	11.52	10.17	10.15	7.67	7.22	7.13
30	12.81	11.83	11.81	9.56	8.98	8.90
40	13.71	13.10	13.09	11.04	10.61	10.49
50	14.45	14.18	14.16	12.46	12.11	11.99

Table 1 and Table 2 show that, under different noise intensities, compared with other methods, Peak Signal to Noise Ratio (PSNR) of the method proposed in this article is maximum and Root Mean Square Error (RMSE) is minimum, which indicates that the method of this article has better denoising effect, and combining with Table 2, we can see that the method of this article achieves relatively good unification in terms of denoising evaluation index and visual appearance.

6. Conclusion

According to the shortage of traditional wavelet-denoising method and NeighShrink method, and considering “excessive retention” and “excessive killing” of existing methods, improved threshold and threshold function based on neighbor coefficient are proposed for processing of noise image, aiming to add decomposition layer number restriction threshold and make improvement for threshold and threshold function, conducting processing of wavelet coefficient under different scales with different thresholds and threshold functions, retaining more detail information of wavelet coefficient under different scales while denoising and making the reconstructed image more approach the original image. Through processing of image polluted by gaussian noise, results show the method put forward in this article has better and more stable denoising effect than existing methods.

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