Beamforming Technology of MIMO System

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Abstract

Beamforming is a transmission technology to get full marks set gain in multi-input multi-output (MIMO) system, which maximizes the system-gain by adjusting the direction of sending signal in the channel vector space. This paper firstly simply expounds the technologies and model of MIMO system, then explains the beamforming technique and the limited feedback of beamforming, and lastly concludes the full texts.

Keywords

Beamforming, multi-input multi-output (MIMO), gain, communication.

1. The technologies of MIMO system

Beamforming is a transmission technology to get full marks set gain in multi-input multi-output (MIMO) system, which maximizes the system-gain by adjusting the direction of sending signal in the channel vector space. Meanwhile, in a multi-user system, beamforming adjusts sending signals of each user in the direction of the channel vector Spaces to make transmitting signals of users be orthogonal to each other, thereby eliminating the interference between users. Compared with other technologies of MIMO, beamforming can be applied not only to the point-to-point communication system in order to improve the reliability of information transmission, but also to multi-user communication system so that it eliminates the interference between users to approach channel capacity. Therefore beamforming is more suitable for application in wireless networks.

There are many kinds of technologies in MIMO system, such as the typical technologies: MIMO technology, diversity technology, multiplexing gain technology, and so on. MIMO technology installs multiple antennas in the sender and the receiver, using space diversity to obtain diversity gain and multiplexing gain, so that it can greatly enhance the transmission rate and stability of the system, while the beamforming is a kind of MIMO technology. Diversity gain and multiplexing gain is a pair of contradictions in MIMO system, the former can improve the reliability of information but reduce the rate of information, and yet the latter on the contrary can improve information rate, but reduces the information rate reliability. The literature [1], [2] and [4] have studied the diversity technology of MIMO system, and proved that under the condition of high SNR symbol error rate of the system declined in index law with the increase of SNR decline. The literature [5] and [6] have researched on the multiplex gain technique of the MIMO system, and demonstrated the linear relationship between the system capacity rate of the system and $\log SNR$ under the condition of high SNR. The literature [7] has pointed out the trade-off relationship between diversity gain and multiplexing gain.

2. The model of MIMO system

Assume that the number of antennas at the sender and the receiver is respectively M_T and M_R, and the channel state- $\mathbf{H}$ is the matrix of $M_R \times M_T$. The sending signals- $\mathbf{X}$ are the vector- $M_T \times 1$, and the receive signals are

$$\mathbf{Y} = \sqrt{P}\mathbf{HX} + \mathbf{N} \tag{1}$$

There into P is the sending power, the power constraint condition of sending signals is $Tr\left(\mathbf{XX}^H\right) \le 1$ and the complex Gaussian noise is $\mathbf{N} \sim CN\left(0, N_0\mathbf{I}\right)$.

The capacity of MIMO system is

$$C = \underset{\mathrm{Tr}(\mathbf{x}\mathbf{x}^H)\le 1}{\arg\max}\, E_{\mathbf{H}}\left\{\log\left(\det\left(\mathbf{I}+\frac{P}{N_0}\mathbf{H}^H\mathbf{H}\right)\right)\right\} \tag{2}$$

The literature [8] and [9] has calculated to obtain that the channel capacity was approximately

$$C = \min(M_T, M_R)\log SNR + o(1) \tag{3}$$

The definition of Multiplexing gain r and diversity gain d is separately

$$r = \lim_{SNR\to\infty}\frac{R(SNR)}{\log SNR} \tag{4}$$

$$d = -\lim_{SNR\to\infty}\frac{\log P_e(SNR)}{\log SNR} \tag{5}$$

Among it R is the information transmission rate, and P_e is information error probability. For any multiplexing gain r, the corresponding maximum diversity gain is $d(r)$. The literature [7] has given the trade-off relationship between diversity gain r and multiplexing gain d, when the length of information transmission time is $T \ge M_T + M_R - 1$, maximum diversity gain of the system is

$$d(r) = (M_T - r)(M_R - r) \qquad 0 \le r \le \min\{M_T, M_R\} \tag{6}$$

Therefore, when all the antennas are used to obtain diversity gain, the maximum diversity gain is $d = M_T M_R$ and the corresponding multiplexing gain is $r = 0$, namely the information transmission rate R is a fixed value; when all the antennas are used to obtain multiplexing gain, the maximum multiplexing gain is $r = \min\{M_T, M_R\}$ and the corresponding multiplexing gain is $d = 0$. Therefore, according to the channel state, it can adjust the system of diversity gain and multiplexing gain to meet the requirements of information transmission rate and reliability. When the channel state is good, more antennas can be applied to obtain multiplexing gain, in order to get higher information transmission rate; On the other hand, more antennas can be applied to acquire diversity gain, in order to gain high reliability of information transmission.

3. The beamforming technique

Beamforming is a kind of MIMO transmission technology to gain full mark set, after which weights sending signals to be sent out through all the antennas. Assuming that the weighted vector of the transmitting end is v, the weighted vector for the receiving end is u, in order to satisfy the power constraint- $\|v\| = \|u\| = 1$, then the equation of the system can be written as

$$z = \sqrt{P}u^H\mathbf{H}vx + u^H\mathbf{N} \tag{7}$$

and Receiving signal-to-noise ratio(SNR) of the destination nodes is

$$rD = \frac{P\|u^H\mathbf{H}v\|^2}{N_0} \tag{8}$$

In order to maximize the receiving SNR, destination nodes use the maximum ratio combining (MRC), namely $u = \frac{\mathbf{H}v}{\|\mathbf{H}v\|}$, and Thus, rD can be rewritten as

$$rD = \frac{P\|\mathbf{H}v\|^2}{N_0} \tag{9}$$

And the optimal weighted vector of the sending end is

$$v_{opt} = \arg\max_{\|v\|=1} \|\mathbf{H}v\| \tag{10}$$

When the transmitting end knows the channel state information- $\mathbf{H}$, the optimal solution of (10) is the corresponding eigenvector with the maximum eigenvalues of matrix- $\mathbf{H}$. Assuming that the maximum eigenvalues of matrix- $\mathbf{H}$ is $\lambda_{\max}$, the channel capacity of beamforming system is

$$C = E\left\{\log\left(1+\lambda_{\max}^2 \frac{P}{N_0}\right)\right\} \tag{11}$$

When the number of antennas at the sender is $M_R = 1$, the receiving SNR can be written as

$$rD = \frac{M_T P}{N_0} \times \frac{\|\mathbf{H}\|^2}{M_T} \tag{12}$$

From the above equation, it can be seen that energy gain of beamforming linearly increases with the number of transmitting antennas- M_T, and diversity gain is M_T.

4. The limited feedback of beamforming

In the beamforming system, the sender need to obtain vector information of beamforming. In the time division duplex (TDD) system the sender, using the reciprocity of the forward and backward channel, can calculate to acquire beamforming vector, while in the frequency division duplex (FDD) there does not exist the reciprocity of the forward and backward channel, so the sender can't directly measure the beamforming vector. In the FDD system, the method is usually adopted that channel state information or beamforming vector information is fed back to the sender through the limited bandwidth of the feedback channel. The limited feedback of beamforming firstly offline designs the vector code book of beamforming- $\{w_1, w_2, \cdots, w_{2^R}\}$, and then based on channel state information in the code book the receiver chooses the optimal vector, the label of the optimal vector is fed back to the sender. After the sender receives the feedback information, it selects the corresponding beamforming vector to weight sending signals from the code book as shown in Fig.1.

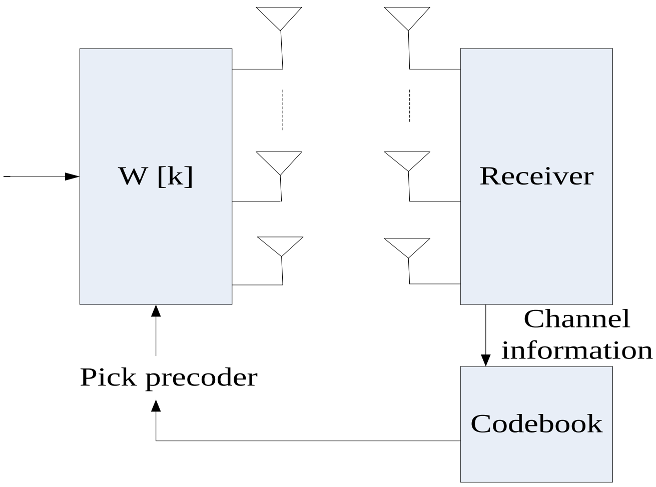

Fig.1 The distributed beamforming of the system

The receiver selects the optimal vector of beamforming from the vector code book of beamforming, which can be seen as a kind of coding scheme, namely

$$Q_w : C^{M_R \times M_T} \to \{w_1, w_2, \cdots, w_{2^R}\} \tag{13}$$

The goal of coding scheme is to maximize the receiving SNR, thus the mapping function from the channel state- $\mathbf{H}$ to feedback on the label is

$$Q_w(\mathbf{H}) = \arg\max_{1\le i\le 2^B} \|\mathbf{H}w_i\|^2 \tag{14}$$

The key of limited feedback beamforming system is how to design a vector code book of beamforming, the design methods of existing code book mainly include the vector quantize(VQ), Grassmannian line packing and Random vector quantize(RVQ), etc.

The main idea of the design method of vector quantization is derived from the damage source coding that regards the beamforming vector as a random vector source, the scheme of the optimum quantization minimizes distortion function under the condition of fixed feedback information. Assume that the quantizatin of beamforming vector- w is $\hat{w} \in \{w_1, w_2, \cdots, w_{2^R}\}$, and the distortion function is defined as $E_w\left\{d\left(w, \hat{w}\right)\right\}$, there into

$$d\left(w, \hat{w}\right) = \|\mathbf{H}w\|^2 - \left\|\mathbf{H}\,\hat{w}\right\|^2 \tag{15}$$

That is to say, the distortion function is the loss of the receive SNR caused by vector quantization. Using Lloyd method to design vector quantization code book to minimize distortion function is able to get the optimal beamforming code book, but this method is difficult to gain theoretical analysis results.

5. Conclusion

In a word, beamforming is a transmission technology to get full marks set gain in MIMO system and there is still a long way to research on more other techniques.

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