Simulation and Analysis of Fundamental Current Extraction Algorithm based on Simulink
Zhongzheng Ren¹, Xiaoqin Bian²
¹The College of Electronic Science, Northeast Petroleum University, Daqing, P. R. China
²Nanjing power supply company of China State Grid Group Co., Ltd, Jiang’su 250200, China

Abstract
With large number of nonlinear loads connected to the grid, harmonic pollution has become an increasingly serious problem. This paper analyzes the theory of p-q method and ip-iq method based on instantaneous reactive power theory. Results of analysis reflect the difference of the two methods on accuracy of fundamental currents extraction with or without voltage distortion. Simulation models of p-q method and ip-iq method are established respectively on Simulink. Research results show that ip-iq method can accurately detect fundamental currents in both two cases. However, p-q method has significant error when there is voltage distortion. The experimental results validate the theoretical analysis.

Keywords
Nonlinear loads; harmonic problems; instantaneous reactive power; fundamental currents.

1. Introduction
In recent years, large number of nonlinear devices have been widely used in power system. The harmonic content in grid is rising, harmonic pollution has brought potential threat to both safe and stable operation of power system and normal operation of electrical equipment[1]. Therefore, how to accurately detect the harmonic and compensate, improve power quality exactly have been one of the most urgent problems.

At present, there are many methods of harmonic current detection: Analog filter detection, Fourier analysis, Instantaneous reactive power detection method and neural network, etc, among which Japanese scholar Akagi Thai’s "instantaneous reactive power theory" is the most widely used[2]. Detecting of harmonic current is equivalent to the extraction of current fundamental component. In order to analyze, the paper equals detecting of harmonic current to the extraction of current fundamental component for study, analyzes the theory of p-q method and ip-iq method based on instantaneous reactive power theory. Results of analysis reflect the difference of the two methods on accuracy of fundamental currents extraction with or without voltage distortion. In addition, the paper establishes fundamental component detecting circuit on Simulink Simulation Platform. Simulation results verify the theoretical analysis is correct.

2. Instantaneous reactive power detection principle
In 1983, Japanese scholar Akagi Thai proposed a three-phase instantaneous reactive power theory. Systematically expounded the concept of instantaneous real power and instantaneous reactive power theory breaking the traditional definition based on the average[3]. So instantaneous reactive power theory applied in harmonic current detection successfully. Instantaneous reactive power theory includes p-q method and ip-iq method. The following we will introduce the two methods respectively.

2.1 p-q method
In the three-phase circuit, First of all, convert instantaneous value of eₐ, eₕ, and eₖ-phase voltage to two orthogonal α-β coordinate system through 3S / 2R[4], let the converted voltage instantaneous value to eₐ, eₕ, SO:
Among them, the transformation matrix $C_{32}$ can be expressed as:

$$
C_{32} = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{pmatrix}
$$

Similarly, deal with three-phase instantaneous current $i_a, i_b, i_c$ with same transformation, we can get current $i_{\alpha}, i_{\beta}$ under two orthogonal $\alpha-\beta$ coordinate system.

By definition of three-phase instantaneous active and reactive power, its expression can get through the $pq$ transformation matrix $C_{pq}$\cite{5}, as formula (3):

$$
\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} e_{\alpha} & e_{\beta} \\ e_{\beta} & -e_{\alpha} \end{pmatrix} \begin{pmatrix} i_{\alpha} \\ i_{\beta} \end{pmatrix} = C_{pq} \begin{pmatrix} i_{\alpha} \\ i_{\beta} \end{pmatrix}
$$

After obtaining the instantaneous active and reactive power components, its fundamental component obtained through a low pass filter, deal the fundamental component with anti-transform of $C_{pq}$ and $C_{32}$. Finally, we can get fundamental current $i_{af}, i_{bf}, i_{cf}$ under initial a,b,c Coordinate System:

$$
\begin{pmatrix} i_{af} \\ i_{bf} \\ i_{cf} \end{pmatrix} = C_{32}^{-1} C_{pq}^{-1} \begin{pmatrix} p \\ q \end{pmatrix} = C_{23} \frac{C_{pq}}{e^2} \begin{pmatrix} p \\ q \end{pmatrix}
$$

Therefore, it can get detecting block diagram of p-q method, as shown in Figure 1.

![Figure 1 Detecting block diagram of p-q method](image)

**2.2 ip-iq method**

The main difference of this method and p-q method is harmonic current detected starting with instantaneous active and reactive current without collecting three-phase voltage signal to coordinate transformation\cite{6}, and as long as the acquisition of a phase voltage fundamental positive sequence component, it can accurately detect the three-phase harmonic currents. Its block diagram is shown in Figure 2.

![Figure 2 Detecting block diagram of ip-iq method](image)

This method need Phase-Locked loop and sine, cosine generator to generate positive cosine signal $\sin \omega t$ , $\cos \omega t$ which is same phase with a-phase voltage fundamental positive sequence component.

Firstly, similar to the p-q method, transform three-phase current to $i_{a}, i_{b}$ under two orthogonal $\alpha-\beta$ coordinate system, then get active and reactive current $i_{p}, i_{q}$ by C Transformation\cite{7} matrix, as shown in Formula (5):
\[
\begin{bmatrix}
    i_p \\
    i_q
\end{bmatrix} = C
\begin{bmatrix}
    i_p \\
    i_q
\end{bmatrix} =
\begin{bmatrix}
    \sin \omega t & -\cos \omega t \\
    -\cos \omega t & -\sin \omega t
\end{bmatrix}
\begin{bmatrix}
    i_p \\
    i_q
\end{bmatrix} \tag{5}
\]

Since the three-phase three-wire circuit does not contain the current zero-sequence component, and thus in the calculation only consider positive, negative sequence component. Get active and reactive current DC component \( I_p, I_q \) though low-pass filter.

\[
\begin{bmatrix}
    I_p^- \\
    I_q^-
\end{bmatrix} = \begin{bmatrix}
    \sqrt{3} I_{11} \cos \phi_{11} \\
    -\sqrt{3} I_{11} \sin \phi_{11}
\end{bmatrix} \tag{6}
\]

In the formula, \( I_{11} \) is the amplitude of the current fundamental positive sequence component, \( \phi_{11} \) is phase angle. The Formula (6) shows that the DC component of active and reactive current, \( I_p^-, I_q^- \), are related to current fundamental positive sequence component. The original fundamental negative sequence component has been filtered after a series of coordinate transformation. It shows that even in the case of the current asymmetry fundamental positive sequence current component will still be extracted, as shown in Formula (7):

\[
\begin{bmatrix}
    I_{a(f)} \\
    I_{b(f)} \\
    I_{c(f)}
\end{bmatrix} = C_m^{-1} C_2 \begin{bmatrix}
    I_p^- \\
    I_q^-
\end{bmatrix} = \begin{bmatrix}
    \sqrt{2} I_{11} \sin(\omega t + \phi_{1f}) \\
    \sqrt{2} I_{11} \sin(\omega t + \phi_{2f} - \frac{2\pi}{3}) \\
    \sqrt{2} I_{11} \sin(\omega t + \phi_{3f} + \frac{2\pi}{3})
\end{bmatrix} \tag{7}
\]

As long as detect the unit sine and cosine signals which is same phase with a-phase voltage fundamental positive sequence component accurately\(^8\), it still can detect fundamental positive sequence component accurately by ip-iq method, thus it can obtain summation of current harmonics and fundamental negative sequence current.

### 2.3 Comparison of the grid voltage distortion

Under actual conditions, the grid voltage is prone to distortion. Therefore, it is of great significance to detect results of two methods’ comparison with voltage distortion. Three-phase voltage distortion is:

\[
\begin{align*}
    e_a &= \sum_{n=1}^{\infty} \sqrt{2} U_a \sin(n \omega t + \theta_a) \\
    e_b &= \sum_{n=1}^{\infty} \sqrt{2} U_a \sin(n \omega t - 2\pi / 3 + \theta_a) \\
    e_c &= \sum_{n=1}^{\infty} \sqrt{2} U_a \sin(n \omega t + 2\pi / 3 + \theta_a)
\end{align*} \tag{8}
\]

### 3. Verification of model simulation based on Simulink

#### 3.1 Simulation model

(1) Main circuit model

In order to verify the detection accuracy of the two methods, firstly, a harmonic generation circuit, the main circuit model, is built. The three-phase voltage source used in this paper is three phase programmable voltage source with phase voltage of 380V and frequency of 50Hz. The load is three-phase uncontrolled rectifier bridge, filter capacitor is 3300\( \mu \)F, the load resistor is 10\( \Omega \). The model is shown in Figure 3.
(2) Detection model of p-q method

From the above analysis we can know, detection model of p-q method mainly contain three-phase to two-phase conversion, two-phase to three-phase conversion, p-q arithmetic module and its inverse module and low-pass filter module. Its modeling approach is similar. For the sake of simplicity, lists p-q inverse arithmetic module only, as shown in Figure 4.

Figure 4 Inverse module of p-q method

Design of low pass filter is fdatool filter design tool of simulink. At present, the most commonly used low-pass filter is FIR filter and IIR filter. IIR filter uses less order than FIR filter in the same requirements, so its application is more extensive \(^{[10]}\).

In IIR filter, ButterWorth low pass filter has better response characteristics. In the simulation model of this paper, in order to eliminate the influence of two methods test results’ comparison, it both adopts ButterWorth low pass filter \(^{[11]}\) which cut-off frequency is 25Hz.

In summary, integrated simulation model of p-q method can be obtained, as shown in Figure 5.

Figure 5 Integrated Simulation Model of p-q method

(3) ip-iq method

The main difference of ip-iq method and p-q method is: ip-iq method does not need to participate in the three-phase voltage coordinate transformation. A-phase voltage is required to generate unit sinusoidal component and unit cosine component through PLL phase-locked loop only. Its ip-iq operation module is the main module which is different from the p-q method, as shown in Figure 6.

Figure 6 Transformation module of ip-iq method
Its inverse module is similar, not repeat them here. In addition, ip-iq method also need to use a phase locked loop circuit to generate input function \( \sin \omega t \) and \( \cos \omega t \) of ip-iq transformation module, as shown in Figure 7.

![Phase locked loop circuit](image)

Figure 7. Phase locked loop circuit

Therefore, the overall simulation model is shown in Figure 8.

![Overall simulation model of ip-iq method](image)

Figure 8. Overall simulation model of ip-iq method

### 3.2 Analysis of simulation result

(1) Three phase power supply voltage without distortion

Simulation of standard sine wave voltage is the first, then do FFT analysis of fundamental current component obtained by the two methods respectively by FFT analysis in powergui. The results shown in Figure 9, Figure 10, respectively.

![FFT analysis of p-q method](image)

Figure 9. FFT analysis of p-q method

![FFT analysis of ip-iq method](image)

Figure 10. FFT analysis of ip-iq method

The comparison shows, in the case of the power supply voltage without distortion, fundamental current Harmonic content obtained by the two methods were 1.56%, 1.38% respectively. There is no obvious distortion, it also shows that ip-iq method is more accurate to extract the fundamental current component under the same conditions.

### 4. Conclusion

This paper based on instantaneous reactive power theory and the Simulink simulation platform. Simulation models of p-q method and ip-iq method are established respectively on Simulink. Simulate in condition of voltage with or without distortion. Research results show that both of the two methods have good accuracy and can meet the engineering requirements under the condition of voltage without distortion. However, in case of voltage distortion, there are harmonic component mainly of 3 times and 5 times in the extracted fundamental current component of p-q
method. Ip-iq method can still maintain good detection accuracy verifying the correctness of theoretical analysis.

References


