Estimation in Partially linear Varying-coefficient Errors-in-Variable with stochastic linear restrictions

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Abstract

In this paper we study the estimator of the Partially linear Varying-coefficient Errors -in-Variable model with stochastic linear restrictions. We present a mixed Profile least squares estimator when the covariates in the linear part are measured with additive error and some additional stochastic linear restrictions on the parametric component are available.

Keywords

Partially additive model; Measurement error; Stochastic linear restrictions.

1. Introduction

Consider the following partially linear Varying-coefficient Errors-in-Variable model:

$$\begin{cases} Y = X'\beta + Z'\alpha(U) + \varepsilon \\ V = X + \eta \end{cases} \tag{1}$$

where Y denotes the response, Z, X, U denote random regressors. And we also assume that U is univariate. $\beta = (\beta_1, \ldots, \beta_p)'$ denotes the unknown parameters; $\alpha(\bullet) = (\alpha_1(\cdot), \ldots, \alpha_q(\cdot))'$ denotes a q-dimensional vector of unknown coefficient functions, ε denotes the random errors with $E[\varepsilon | X, Z, U] = 0$; η denotes the measurement error with mean 0 and covariance matrix $\sum_\eta$ and is independent of (Y, X, Z, U). In this paper we suppose that $\sum_\eta$ is known, When $\sum_\eta$ is unknown, we may estimate it by repeatedly measuring V (Liang et al. [2]).

As a extension of partially linear model and varying-coefficient model, partially linear Varying-coefficient Errors-in-Variable model has big application value. In many applications, there often exists covariate measurement errors. Such as, it has been well documented in the literature that covariates such as blood pressure Carroll et al. [1]. You and Chen [6] construct a Profile least squares estimator and also discuss the properties of this estimator. In some cases, some prior information bout some prior information about the regression coefficients is available from outside sample sources. Use of such information may improve upon the efficiency of the estimator. This information may contain linear restrictions or stochastic linear restrictions, see Liu [3-4]. Wei [7] construct a restricted Profile least squares estimator when some additional linear restrictions are assumed to hold.

To the best of our knowledge, there is no paper to study the estimator of (1) when some additional stochastic linear restrictions are suppose to hold. In this paper we will consider how to estimate (1) under some stochastic linear restrictions [5]. Consider the following stochastic linear restrictions

$$b = A\beta + \upsilon \tag{2}$$

where A denotes a $k \times p$ matrix of known constants with *rank(A)=*, and b is a known vector, υ denotes random error with mean 0and covariance matrix Ω, where Ω is a known positive definite matrix.

The rest of the paper is organized as follows, the new estimator of model (1) with (2) is given in Section 2 and some conclusion remarks are given in Section 3.

2. Mixed Profile least squares estimator

2.1 Modified Profile least squares estimator

Suppose $\{Y_i, X_i, Z_i, U_i\}_{i=1}^{n}$ be an independent identically distributed (iid) random sample which comes from model (1), and satisfy

$$Y_i = X_i'\beta + Z_i'\alpha(U_i) + \varepsilon_i \tag{3}$$

If the parametric component β is known, then model(3) may be written as

$$Y_i - X_i'\beta = \alpha_1(U_i)Z_{i1} + \cdots + \alpha_q(U_i)Z_{iq} + \varepsilon_i \tag{4}$$

it is easy to know that (4) is a varying coefficient model. Wei (2012) use the following method to present least squares estimator. For $\alpha_j(\bullet)$, using the Taylor method, we have:

$$\alpha_j(u) = \alpha_j(u_0) + \alpha_j'(u_0)(u-u_0),\ j=1,...,q \tag{5}$$

where $\alpha_j'(u) = \partial\alpha_j(u)/\partial u$, then we can minimize (6) to derive $\{(\alpha_j(u_0), \alpha_j'(u_0))\}$

$$\sum_{i=1}^{n}\left[(Y_i - X_i'\beta) + \sum_{j=1}^{q}\{\alpha_j(u_0) + \alpha_j'(u_0)(U_i-u_0)\}Z_{ij}\right]^2 K_h(U_i-u_0) \tag{6}$$

where K denotes the kernel function, h denotes a bandwidth, and $K_h(\bullet) = K(\bullet/h)/h$, then by (6) we obtain

$$\left[\hat{\alpha}_1(u_0),...,\hat{\alpha}_q(u_0),\hat{\alpha}_1'(u_0),...,\hat{\alpha}_q'(u_0)\right]' = \{D_{u_0}'W_{u_0}D_{u_0}\}^{-1}D_{u_0}'W_{u_0}(Y - X\beta) \tag{7}$$

where

$$X = \begin{bmatrix} X_1' \\ \vdots \\ X_n' \end{bmatrix}, D_{u_0} = \begin{bmatrix} Z_1' & (U_1-u_0)Z_1' \\ \vdots & \\ Z_n' & (U_n-u_0)Z_n' \end{bmatrix}$$

$Y = [Y_1,...,Y_n]', W_{u_0} = diag\{K_h(U_1-u_0),...,K_h(U_n-u_0)\}$

Substituting $\{(\alpha_j(u_0), \alpha_j'(u_0))\}$ in (3), we may obtain a linear regression model

$$Y_i - \hat{Y}_i = (X_i - \hat{X}_i)'\beta + \varepsilon_i \tag{8}$$

where $\hat{Y} = [\hat{Y}_1,...,\hat{Y}_n]' = SY, \hat{X} = [\hat{X}_1,...,\hat{X}_n]' = SX$, and

$$S = \begin{bmatrix} (Z_1'\ 0)\{D_{u_1}'W_{u_1}D_{u_1}\}^{-1}D_{u_1}'W_{u_1} \\ \vdots \\ (Z_n'\ 0)\{D_{u_n}'W_{u_n}D_{u_n}\}^{-1}D_{u_n}'W_{u_n} \end{bmatrix}$$

By the ordinary least squares estimator method we have

$$\tilde{\beta}_n = \arg\min\left[(\bar{Y} - \bar{V}\beta)'(\bar{Y} - \bar{V}\beta)\right] = (\bar{V}'\bar{V})^{-1}\bar{V}'\bar{Y} \tag{9}$$

where $\bar{Y} = Y - \hat{Y}, \bar{V} = V - \hat{V}, \hat{V} = \left(\hat{V}_1, \ldots, \hat{V}_n\right)' = SV, V = \left(V_1, \ldots, V_n\right)'$.

However, there exists measurement error, X_i cannot be exactly observed. If one ignores the measurement error and replaces X_i by V_i in (9), one can see that there resulting estimator is inconsistent. By the correction for attenuation technique, the modified profile least squares estimator of β was denoted by

$$F_1(\beta) = \left(\bar{Y} - \bar{V}\beta\right)' \left(\bar{Y} - \bar{V}\beta\right) - n\beta' \Sigma_\eta \beta \tag{11}$$

Then solve (11), we can obtain that modified profile least squares estimator of β

$$\hat{\beta}_n = \left(\bar{V}'\bar{V} - n\Sigma_\eta\right)^{-1} \bar{V}'\bar{Y} \tag{12}$$

The asymptotic normality of the estimator (12) has been studied by You and Chen (2006).

2.2 Mixed Profile least squares estimator

In this subsection, we construct the mixed profile least squares estimator.

You and Chen [6] present a modified profile least squares estimator, however stochastic linear restrictions (2) were not satisfied. In order to deal with this problem, we construct a function

$$F_2(\beta) = \left(\bar{Y} - \bar{V}\beta\right)' \left(\bar{Y} - \bar{V}\beta\right) - n\beta' \Sigma_\eta \beta + \left(b - A\beta\right)' \Omega^{-1} \left(b - A\beta\right) \tag{13}$$

By differentiating $F_2(\beta)$ with respect to β, we obtain the following equations:

$$\frac{\partial F_2(\beta)}{\partial \beta} = -2\bar{V}'\bar{Y} - 2n\Sigma_\eta \beta + 2\bar{V}'\bar{V}\beta - 2A'\Omega^{-1}\left(b - A\beta\right) = 0 \tag{14}$$

Solving (14), we obtain

$$\hat{\beta}_r = \left(\bar{V}'\bar{V} - n\Sigma_\eta + A'\Omega^{-1}A\right)^{-1} \left(\bar{V}'\bar{Y} + A'\Omega^{-1}b\right) \tag{15}$$

By Rao et al. (2008), this estimator can also be written as

$$\hat{\beta}_r = \hat{\beta}_n - \left(\bar{V}'\bar{V} - n\Sigma_\eta\right)^{-1} A' \left[\Omega + A\left(\bar{V}'\bar{V} - n\Sigma_\eta\right)^{-1} A'\right]^{-1} \left(A\hat{\beta}_n - b\right) \tag{16}$$

Then we can get the estimator of $Z'\alpha(U)$.

By (16), if $\Omega = 0$, then (16) becomes

$$\hat{\beta}_r = \hat{\beta}_n - \left(\bar{V}'\bar{V} - n\Sigma_\eta\right)^{-1} A' \left[A\left(\bar{V}'\bar{V} - n\Sigma_\eta\right)^{-1} A'\right]^{-1} \left(A\hat{\beta}_n - b\right)$$

Wei (2012) has presented this paper, we can see that our estimator contains Wei's work.

3. Conclusions

In this paper, we consider the parameter estimator in Partially linear Varying-coefficient Errors-in-Variable model with stochastic linear restrictions. We present a mixed profile least squares estimator. However we did not discuss the properties of the new estimator, in future study we will study the asymptotic normality of the new estimator.

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References

[1] Carroll, R.J., Ruppert, D., Stefanski, L.A., Crainiceanu, C.M., 2006. Measurement Error in Nonlinear Models, second ed. Chapman and Hall, New York.
[2] Liang, H., Hardle, W., Carroll, R.J., 1999. Estimation in a semiparametric partially linear errors-in-variables model. The Annals of Statistics 27, 1519–1935.
[3] Liu, C. H., Yang, H., Wu, J. B., 2013. On the weighted mixed almost unbiased ridge estimator in stochastic restricted linear regression, Journal of Applied Mathematics, Article ID 902715, 10 pages
[4] Liu, C. H., Jiang, H.N., Shi, X. H., Liu, D. L., 2014. Two Kinds of Weighted Biased Estimators in Stochastic Restricted Regression Model, Journal of Applied Mathematics, Article ID 314875, 10 pages
[5] Rao, C.R., Toutenburg, H., Shalabh, Heumann, C., 2008. Linear Models: Least Squares and Alternatives. Springer, Berlin.
[6] You, J.H., Chen, G.M., 2006. Estimation of a semiparametric varying-coefficient partially linear errors-in-variables model. Journal of Multivariate Analysis 97, 324–341.
[7] Wei, C.H., 2012. Statistical inference for restricted partially linear varying coefficient errors-in-variables models. Journal of Statistical Planning and Inference, 142, 2464-2472.