# Motion Control of Intelligent Underwater Robot Using Integral LQG Control

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### Abstract

T Intelligent underwater robot has been drawing increasing interests in various areas such as ocean resource exploitation and etc. When the platform works, it often drifts from the working area because of environmental disturbances, and motion control is needed. In order to track the command input, the integral LQG control is used in the motion control design. The control loop is designed to also account for external environmental disturbances and sensor measurement noise. The control loop is accomplished using modern control theory which makes use of known intelligent underwater robot dynamics and assumptions associated with the environment and sensor measurement variances. Control loop performance results were obtained through testing on the generated simulation model. The results showed that the system can track the command accurately and has the capacity to reject environmental and noise disturbances.

### Keywords

### Motion Control, Underwater Robot, LQG.

### **1.** Introduction

Intelligent underwater robot is called AUV (Autonomous underwater vehicle) in general. AUV is the free swimming marine robot that requires little or no human intervention. One of the research goals is to develop an AUV capable to perform several challenging functions, such as maintaining a steady position for a particular task (station-keeping problem), following a prescribed trajectory in search of an object whose location has already been determined (tracking problem), and searching for missing or sought-after objects.

In fact, AUV motion control in the unstructured underwater environment, as an important component of AUV intelligent control system, is a research hotspot. The research is difficult because there are high nonlinearity and coupling among each freedom degree of AUV. Till now, many motion control algorithms have applied in AUV motion control such as PID, Fuzzy method, neural network, self-adaptive method. For this system a Linear Quadratic Gaussian (LQG) controller was selected which regulates a linear system perturbed by white (Gaussian) noise by minimizing a quadratic control cost function. The LQG is comprised of a Linear Quadratic Regulator (LQR) and a Linear Quadratic Estimator (LQE). The LQE is often referred to as an observer or Kalman Filter. In an LQG implementation it is common for some of the states to not be measured but rather estimated. One such concern is that the noise on the system must be Gaussian white noise. The other concern is that since this controller is of a linear basis, the system needs to be linearized around some point.

However, LQG as the feedback controller is a regulator, which is designed to make the state stay zero, can't follow the commands. The AUV autopilot design requirements is that the motion control system can accurately track the positon and angle instruction, and therefore the paper designs a AUV motion control system with integral LQG control. The requirement to minimize the variance of the fundamental output field is reflected in an LQG cost functional and the need for integral action is

included by modifying the control signals and the cost functional appropriately. Integral LQG control can not only make the system track the constant value commanding input, it can also reduce the influence of the constant environmental and noise disturbances.

#### 2. Dynamic Model of 6-DOF AUV

The six degrees-of-freedom nonlinear equations of motion of AUV are defined with respect to two coordinate systems as showed in Figure 1.



Figure 1. The coordinate systems for AUV

The vehicle coordinate system (o - xyz) has six velocity components of motion (surge, sway, heave, roll, pitch, and yaw). The velocity vector in the vehicle coordinate system is expressed as  $v = [u, v, w, p, q, r]^T$ . The global coordinate system  $(E - \xi\eta\zeta)$  is a fixed coordinate system. Translational and rotational movements in the global reference frame are represented by  $\eta = [x, y, z, \phi, \theta, \psi]^T$  that includes earth fixed positions and Euler angles.

The equations of motion for AUV without manipulators can be written as follows [1]:

$$M(v)\dot{v} + C_D(v)v + g(\eta) + d = \tau$$
  
$$\dot{\eta} = J(\eta)v$$
(1)

where  $M(v) \in \Re^{6\times 6}$  is a 6×6 inertia matrix as a sum of the rigid body inertia matrix and the hydrodynamic virtual inertia (added mass);  $C_D(v) \in \Re^{6\times 6}$  is a 6×6 Coriolis, centripetal and damping matrix;  $g(\eta) \in \Re^6$  is a 6×1 vector containing the restoring terms formed by the AUV's buoyancy and gravitational terms; d is a 6×1 disturbance vector representing the environmental forces and moments(e.g. current);  $\tau$  is a 6×1 vector including the control forces and moments;  $J(\eta)$  is a 6×6 velocity transformation matrix that transforms velocities of the vehicle-fixed to the earth-fixed reference frame.

The expansion equations of the motion for 6-DOF of AUV based on rigid-body dynamics are written as follows [2]:

$$\begin{cases} m \cdot [(\dot{u} - vr + wq) - x_G \cdot (q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X \\ m \cdot [(\dot{v} - wp + ur) - y_G \cdot (p^2 + r^2) + z_G(qr - \dot{p}) + x_G(pq + \dot{r})] = Y \\ m \cdot [(\dot{w} - up + vp) - z_G \cdot (q^2 + p^2) + x_G(pr - \dot{q}) + y_G(qr + \dot{p})] = Z \\ I_x \dot{p} + (I_z - I_y)qr + m \cdot [y_G(\dot{w} + pv - qu) + z_G(\dot{v} + ru - pw)] = K \\ I_y \dot{q} + (I_z - I_z)pr + m \cdot [z_G(\dot{u} + wq - vr) + x_G(\dot{w} + pv - uq)] = M \\ I_z \dot{r} + (I_y - I_y)qp + m \cdot [x_G(\dot{v} + ur - pw) + y_G(\dot{u} + qw - vr)] = N \end{cases}$$
(2)

where  $x_G, y_G, z_G$  is the center of gravity, *m* is the constant mass,  $I_x, I_y, I_z$  is the inertia matrix of AUV, X, Y, Z and K, M, N are vectors of external applied forces and moments, respectively.

The paper will focus on AUV motion maintaining a steady position for a particular task (station-keeping problem) in horizontal plane. Since this design is for a system which emphasizes zero movement the most logical point around which to linearize is zero velocity. In the damping of the system the linear components dominate over the quadratic. Using Taylor Series Expansion, the original system model equations of motion will be linearized with the strike through terms equating to zero. The resulting linearized equation

used for the LQG is shown as below:.

$$\dot{\eta} = v$$

$$\dot{v} = -M^{-1}Dv + M^{-1}\tau \tag{3}$$

where  $\eta = [x \ y \ \psi]^T$ ,  $v = [u \ v \ r]^T$ ,  $\tau = [X \ Y \ N]^T$ , M is a rigid-body and added mass inertia matrix; D is a linear viscous damping matrix.

It is important to note that the state space model shown in Equation 3 is one that does not incorporate noise. In the actual measurement process, there must be a measurement noise. In order to improve the robustness of the system, the system should be not sensitive to uncertainties brought about by the disturbance participated in the system.

For a more complete model of the plant state space, one must include the state disturbances (environmental) and output disturbances (sensor noise). The complete plant state space model is shown in Equation 4.

$$\dot{x} = Ax + Bu + B_w w$$

$$y = Cx + D_v v_n$$
(4)

where  $x = \begin{bmatrix} \eta \\ v \end{bmatrix}$ ,  $u = \tau$ ,  $y = \eta$ ,  $A = \begin{bmatrix} 0 & I \\ 0 & -M^{-1}D \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$ ,  $C = \begin{bmatrix} I & 0 \end{bmatrix}$ ,  $D = \begin{bmatrix} 0 \end{bmatrix}$ , w is state noise,  $v_n$  is

output noise,  $B_w$  is state noise gain,  $D_v$  is output noise gain.

#### 3. Integral LQG Control

The most common type of industrial controller is the proportional-integral-derivative controller. PID controller is a linear controller and calculates the e(t) (error) as the difference between c(t) (actual distances, Euler angle) and r(t) (target distance, Euler angle). If u(t) is the output from the controller, and e(t) is the error signal it receives, this control law has the form [4]

$$e(t) = r(t) - c(t) \tag{5}$$

$$u(t) = K_{P}e(t) + K_{I} \int_{0}^{t} e(t)dt + K_{D} \frac{de(t)}{dt}$$
(6)

where  $K_P$  is the proportional coefficient and  $K_I$  is the integral coefficient while  $K_D$  is the derivative coefficient. According to different parameters, the system responds differently.

The design of the LQG control system was performed in two steps. First, we designed a linear quadratic regulator (LQR). Then, we designed a linear quadratic estimator (LQE or Kalman filter). Finally, we integrated both systems in a single control structure commonly know as a LQG controller. The objective of the first stage is to obtain a matrix K, the matrix that defines the controller, in such a way that itminimizes a cost function. Therefore, the selection of K is an optimization problem.

Optimization problem: Minimize the cost function

$$J_1 = \int \left( x^T Q x + u^T R u \right) dt \tag{7}$$

subject to

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \tag{8}$$

Here, Q and R are weight matrices that indicate which signal, x or u, has more importance in the optimization problem. The actual values of Q and R are not relevant. The ratio of their magnitudes is the factor that actually affects the priority in the optimization problem. A practical and simple method for selecting Q and R is to define Q as an identity matrix and change the value of R until one gets the expected results.

In next stage, we're going to consider noise in the measurements and disturbances in our model, i.e.

$$\dot{x} = Ax + Bu + w \tag{9}$$
$$y = Cx + Du + v$$

The objective of the LQE is to recover the states vector x using only information that is known, in other words, information from measurements y and the control signal u. Additionally, the LQE must minimize the estimation error  $e = x - \hat{x}$ . Here x is the unkown real vectors of states and  $\hat{x}$  is the estimation of that vector. Here again, we have an optimization problem.

Optimization problem: Minimize the cost function

$$\boldsymbol{J}_2 = \boldsymbol{E} \left[ \boldsymbol{e}^T \boldsymbol{e} \right] \tag{10}$$

Where  $E[ \cdot ]$  represent the mean squared error.

The LQG control system is built using both the linear quadratic regulator and the states estimator in a single feedback control loop.

According to the principle of separation, the feedback controller and Calman filter design in LQG method can be carried out respectively. Finally LQG optimal controller state equation is stated [3]:

$$\dot{x} = A\hat{x} + Bu + F(\hat{y} - y)$$
$$\hat{y} = C\hat{x}$$
(11)
$$u = K\hat{x}$$

K is the state feedback controller gain which stabilizes the closed loop system and minimizes the objective cost function. F is the observer gain such that it stabilizes the state estimation error and minimizes the cost function.

Adding an integral term to the control loop can be accomplished by adding another state to the state vector which maintains the integrated error value within the LQG controller. This change in the state vector requires a change in the state space model to account for the extra integral terms. The resulting LQG controller is the new observer based controller and is shown as:

$$\dot{e} = \overline{y} - y, \quad \overline{y} = \text{desired position}$$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & | \mathbf{0} \\ -C & | \mathbf{0} \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ \mathbf{0} \end{bmatrix} u + \begin{bmatrix} \mathbf{0} \\ I \end{bmatrix} \overline{y} \qquad (12)$$

The position output of the AUV is compared with the desired value. In order to minimizing the minimum deviation, energy consumption for the objective of the design of optimal control. The performance index is

$$J = \int_0^t \left[ x^T \mid e^T \right] \left[ \frac{Q \mid 0}{0 \mid Q_e} \right] \left[ \frac{x}{e} \right] + u^T R u dt$$
(13)

The LQG optimal controller is constructed from the solutioto a deterministic regulator problem and an optimal observe problem as follows <sup>[4][5]</sup>. The weighted matrix Q of the state variables is a symmetric positive semi definite matrix, which is a measure of the dynamic error of the system. The weighted R of the input variables is symmetric positive definite matrices. It not only can reflect the convergence rate of the state variables, but also can control the energy of the system. Because the system can be controlled can observed, so there is optimal control:

$$u(t) = -\left[K_x \mid K_e\right] \left[\frac{x}{e}\right] = -K_x x - K_e \int_0^t (\overline{y} - y) dt \qquad \left[K_e \mid K_x\right] = R^{-1} \left[\frac{0}{B}\right]^T P \qquad (14)$$

**P** is the only positive definite solution to the Riccati algebraic equation.

$$PA + A^T P - PBR^{-1}B^T P + Q = 0$$
<sup>(15)</sup>

The resulting LQG controller is the new observer based controller and is shown in Equation 10. One thing to note is that the Kalman filter (F) does not change in this implementation although the regulator gain (K) is altered and shown in Equation 10. The block diagram of the entire block diagram adding integral LQG controller is shown in Figure 2.



Figure 2. Complete Control System

### 4. Simulation Results

In this section, the newly proposed control scheme was numerically evaluated on a simulation example of an AUV with some parameters as follows [6]: Mass: m = 800kg; Length: L=2.5m; Diameter : D=1.2m; Volume: V= 2.5m<sup>3</sup>; Water density: pwater =1000kg / m<sup>3</sup>; AUV hull density:  $\rho_{AUV}=320$ kg / m<sup>3</sup>;

#### 4.1 LQG control with no current disturbances

The next step in the paper is to minimize the cost functions for an expected whitenoise disturbance. Since this is a simulation it is reasonable to have knowledge of the level of disturbance forces applied to the AUV and therefore set up the controller accordingly. For this simulation, the maximum expected value of the disturbances was on the order of  $10^2$ . Therefore the magnitude of  $B_w$  was set to a magnitude of  $10^2$ . Tuned LQG control loop response of AUV horizontal plane motion is shown in Figure 3. From the figure it is apparent that this control loop is capable of maneuvering the AUV back to the zero position. In this simulation there are no environmental disturbances or sensor noise and this is controlling the linear system it was specifically designed for.

We use a step function as reference signal, in the operating point, to evaluate the performance of the control system. We add Gaussian noise to the output position of the AUV noise in the measurements. In Fig. 3, the estimator (Kalman filter) achieves perfect filtering and delivers anadequate signal to be used by the regulator. In Fig. 3, the control signal varies between the saturation limits and this variation is smooth, i.e. the control signal is not affected by noise in the measurements of the output position.



Figure 3. LQG Control response with whitenoise

#### 4.2 Integral LQG control with current disturbance

With the satisfactory performance, the next test in the progression was to begin adding disturbances. The disturbance added was an ocean current. The ocean current disturbance is a current applied at 45 degrees off broadside of the AUV. This invoked the most force components for both X and Y as well as the largest Yaw moment. After applying the ocean current, the control loop was able to stabilize the AUV but it came to a stability point which was not the desired location (Figure 4). After some further tuning, which failed to resolve the issue, it became apparent that the controller would require an integral term to make up for this constant error value.

Adding an integral term to the control loop can be accomplished by adding another state to the state vector (Equation 12) which maintains the integrated error value within the LQG controller. Performing the same ocean current test as with the non-integral LQG implementation results in significant improvements. Figure 4 demonstrates how the controller reacts to the current pushing it off the reference point. The AUV response appears to be practical. Now that the control loop is successfully performing DP on an AUV in a current.



Figure 4. LQG Control response with whitenoise

### 5. Summary

The paper adopts integral action as the standard LQG technique isnot able to deal with the type of disturbance present in our application. Based on the mathematical model of the system, the integral LQG controller is designed. At a suitable operating point, the system was linearized and an optimal state feedback control law was determined from a given cost functional. The cost functional was chosen to reflect our goal of minimizing the variance of the amplitude quadrature of the fundamental output field. Simulation results obtained show the effectiveness of the controller in cancelling out noise and that the control system performance becomes limited only by noises. It is easy for engineering implementation. The introduction of Kalman filter make the system has a certain amount of resistance ability to noise interference and measure noise. Using this model, we designed a LQG control system joining a linear quadratic regulator and a linear quadratic estimator (a Kalman filter). Then, we tested the LQG control system under ideal conditions to evaluate its performance. In order to reject constant perturbations, an integral action was added to the previous LQG design to get the integral LQG controller. The main contribution of this work was the detailed design of a control system, starting from identification and ending with algorithms for a controller and an estimator for a AUV with using simplified theoretical models.

# Acknowledgements

This work is financially supported by the fund of YiChang Science and Technology Bureau (A13-302a-11) and the doctoral scientific research foundation of China Three Gorges University (KJ2012B016).

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