

Decision Making and Coordination Mechanism of Trilateral Supply Chain in Recycling Market

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Abstract

Effective recycling of industrial waste in the process of production has been paid more and more attention. This paper firstly proposes the Trilateral Supply Chain (TSC) model combining the horizontal with vertical supply chains, which aims to solve the recycling problem of industrial waste among Small & Middle Enterprises (SMEs), then analyses optimal centralized decision-making model and optimal dual master-slave model of decentralized decision-making in TSC, in the case where imbalance situation of supply and demand caused by uncertain multi-product and multi-market environment. Results show that: (1) the TSC model is feasible; (2) the centralized decision-making is superior to the decentralized decision-making; (3) revenue-cost sharing contract is an efficient policy to maximize returns of the system and realize “win-win” between the members in the supply chain.

Keywords

Recycling Market, Trilateral Supply Chain (TSC), Dual master-slave model, Revenue-cost sharing contract

1. Introduction

With the increasing resources shortage, serious environmental pollution, and intense global competition, the management of supply chain ushers in a new challenge. How to make supply chain and environment develop harmoniously is a serious problem for implementing the management of supply chain. Currently, study on resource recycling mainly focuses on the remanufacturing of closed-loop supply chain and the reverse supply chain. Savaskan R C [1,2] studies that a manufacturer and a retailer are how to make a decision in the remanufacturing of closed-loop supply chain, and the manufacturer is how to choose recycling channels by using the method in the game. Ruozhen Qiu [3] builds the Stackelberg model of closed-loop supply chain considering the manufacturer and the retailer are simultaneously responsible for product recycling channel. Zongsheng Huang [4] studies two kinds of recycling channels of remanufacturing closed-loop supply chain (CLSP) by using differential method, and builds dynamic loop supply chain (LSP) model. Mujing Wei [5] discusses the pricing decisions of supply chain in the fuzzy environment, and considers the effect of recycling on the loop supply chain. Research on the closed-loop supply chain mainly concentrates on the product recycling of the end of the supply chain [6-8] and optimal order coordination of the supply chain in the case of false failure returns [9]. However, few literatures refer to the related study about recycling of waste which is harmful to the environment in the process of production.

In order to make use of waste more effectively, Dan bin [10] puts forward the concept of Related Supply Chain (RSC) in 2006. Haiyan Li [11] proposes three operation modes of RSC: main and auxiliary integration; peer integration and parallel contract integration, and points that the cost of waste recycling is too high, so it's necessary to expand production scale. The output of waste is lower in some SEMs, and there is no advantage of recycling operation scale, so it is difficult to implement the management of related supply chain. To solve this problem, this paper puts forward the method based on the model of parallel contract of related supply chain, combining the horizontal with vertical supply chains, and establishes the Trilateral Supply Chain (TSC) model by introducing Third-party waste refiner. As is shown in Fig. 1

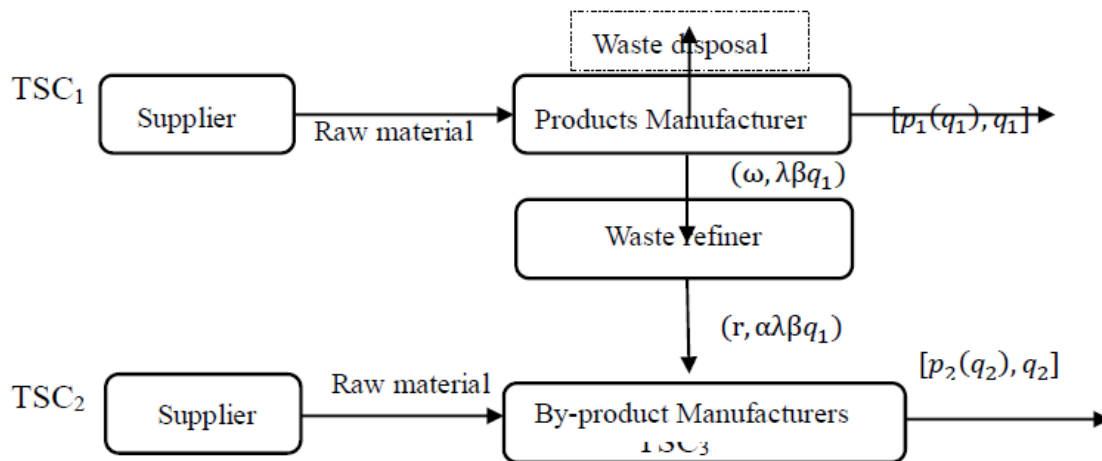


Fig.1 TSC model

2. Hypothesis and Description of Model

- (1) Suppose that main-product manufacturer, waste refiner and by-product manufacturer are perfect information.
- (2) Suppose that the price of raw material of main-product manufacturer is c_1 , the demand of raw material per unit is μ , the price of main product is p_1 ; meanwhile, main products produce waste during production, the output rate is β , the cost of waste disposal is h , main-product manufacturer can also sell the waste to the waste refiner at the price of ω ; when ω is negative, the main-product suppliers need to pay part charge to the waste refiner for dealing with the waste.
- (3) Suppose that waste refiner sells the waste after processing to by-product manufacturer at the price of r , purchases rate of waste refiner from manufacturer is λ , the cost of waste disposal is $\lambda^2 c_0$, Where c_0 represents the investment scale which waste refiner deals with waste, $\alpha(0 < \alpha < 1)$ represents refining rate of waste, the output of waste refiner is $\alpha\beta\lambda q_1$.
- (4) Suppose that by-product can absolutely be produced by raw material, the cost per unit is c_2 ; can also be produced by raw material refined by waste refiner, the cost per unit is r , where $r \leq c_2$; ξ represents the demand of waste per unit by-product, and the purchase amount of by-product manufacturers from waste refiner is ξq_2 .
- (5) Suppose that market demand for the main product and by-product is linear, and the demand curve is $p_i(q_i) = \frac{a_i - q_i}{b_i}$, $i = 1, 2$, where 1 represents main-product, 2 represents by-product, a_i represents market scale, b_i represents sensitive coefficient of price, where both of them are parameter which is great than zero.
- (6) Suppose that all of the decision makers among TSC are rational, that is to say, they can produce just when the profit is greater than zero.

According to the hypothesis above, we can get three profit functions of supply chain. The profit function of main-product manufacturer can be given as follow:

$$\pi_1 = [p_1(q_1) - \mu c_1]q_1 + \omega\lambda\beta q_1 - h(1 - \lambda)\beta q_1;$$

The profit function of by-product manufacturer can be given as follow:

$$\pi_2 = p_2(q_2)q_2 - r \min(\xi q_2, \alpha\lambda\beta q_1) - c_2 \max(\xi q_2 - \alpha\lambda\beta q_1, 0);$$

The profit function of waste refiner can be given as follow:

$$\pi_3 = r \min(\xi q_2, \alpha\lambda\beta q_1) - \omega\lambda\beta q_1 - \lambda^2 c_0$$

3. Decision-making model of two supply chains in traditional mode

In traditional mode, main-product manufacturer deals with waste at the cost of h , and by-product manufacturer uses raw material to produce, they belong to different markets, so two supply chains

carry out optimal decision-making model respectively, their profit functions can be shown in Eq. (1) and Eq. (2):

$$\max \pi_{01} = [p_1(q_1) - \mu c_1]q_1 - h\beta q_1 \tag{1}$$

$$\max \pi_{02} = p_2(q_2)q_2 - c_2\xi q_2 \tag{2}$$

Let the first derivative of q_1 in Eq. (1) and q_2 in Eq. (2) be zero respectively, we can get these:

$$q_{01}^* = \frac{A}{2}, \pi_{01}^* = \frac{A^2}{4b_1},$$

$$q_{02}^* = \frac{a_2 - b_2 c_2 \xi}{2}, \pi_{02}^* = \frac{(a_2 - b_2 c_2 \xi)^2}{4b_2}$$

So the profit of two supply chains is $\pi_0^{d*} = \pi_{01}^* + \pi_{02}^* = \frac{A^2}{4b_1} + \frac{(a_2 - b_2 c_2 \xi)^2}{4b_2}$

4. The model analysis of imbalance situation between supply and demand

Traditional supply chain just involves single product market, and faces the problem of competition and cooperation of single product market. Presented here is three complete supply chains, and every supply chain is equivalent, different product belong to different market. Due to the uncertainty of market demand, when TSC₁ and TSC₂ meet the demand of market, it's difficult to ensure the supply-demand balance of the waste in related supply chain. Therefore, the author thinks that the supply and demand of waste should be in two states: one is the supply exceeding the demand, another is the demand exceeding the supply.

4.1 The supply exceeding the demand in the waste market.

When the demand of main-product is greater, while the demand of by-product is smaller, supply of waste will be oversupplied. In the TSC model, the supply chain between main-product and by-product is effectively combined by waste and refiner, waste refiner is in charge of refining the waste produced during production by main-product manufacturer, and then form a horizontal supply chain taking waste as raw material and extract as product.

When $\xi q_2 > \alpha\beta\lambda q_1$, that is to say, the demand exceeds the supply in the extract market.

(1) Centralized decision-making model

Firstly, the centralized decision-making model should be built with the goal of profit maximization in the whole RSC. In the model, main-product, by-product and waste purchase rate determine the optimal production of three markets together.

$$\max_{c_0\lambda_1^c} \pi_1^c = [p_{12}^c(q_{12}^c) - c_2\xi]q_{12}^c + [p_{11}^c(q_{11}^c) - \mu c_1 + c_2\alpha\beta\lambda_1^c + h\beta(1 - \lambda_1^c)]q_{11}^c - c_0\lambda_1^c \tag{3}$$

Make the first derivative of q_{11}^c , q_{12}^c and λ_1^c is zero respectively, there

$$\begin{cases} q_{11}^{c*} = \frac{A + b_1 B \lambda_1^{c*}}{2} \\ q_{12}^{c*} = \frac{a_2 - b_2 c_2 \xi}{2} \\ \lambda_1^{c*} = \frac{B q_{11}^{c*}}{2c_0} \end{cases} \tag{4}$$

Where $A = a_1 - b_1 c_1 \mu - b_1 \beta h$, $B = (c_2 \alpha + h)\beta$.

Substitute q_{11}^{c*} , q_{12}^{c*} into Eq. (3), then calculate the first derivative and second derivative of λ_1^{c*} . We can get conclusion below.

$$\lambda_1^{c*} = \frac{AB}{4c_0 - b_1 B^2}, q_{11}^{c*} = \frac{2Ac_0}{4c_0 - b_1 B^2}$$

Substitute λ_1^{c*} , q_{11}^{c*} , q_{12}^{c*} into Eq. (3), we can get the maximum benefit when the three supply chains are centralized decision-making.

$$\pi_1^{c*}(q_{11}^{c*}, q_{12}^{c*}, \lambda_1^{c*}) = \frac{A^2 c_0}{b_1 (4c_0 - b_1 B^2)} + \frac{(a_2 - b_2 c_2 \xi)^2}{4b_2}$$

(2) Dual master-slave model of decentralized decision-making

When $\xi q_2 > \alpha\beta\lambda q_1$, extract refined by waste refiner cannot meet the need of by-product manufacturer. And then, the horizontal supply chain can be the main supply chain among three supply chains, waste refiner is the leader of horizontal market, by-product manufacturer and main-product manufacturer are followers of horizontal market. Due to oversupply of waste, main-product manufacturer is also the follower of by-product manufacturer, dual master-slave model emerges, so the problem of decision making converted the function below:

$$\max \pi_{13} = \alpha\beta r_1^d \lambda_1^d q_{11}^d - \beta w_1^d \lambda_1^d q_{11}^d - c_0 (\lambda_1^d)^2 \quad (5)$$

s.t.

$$\begin{cases} \max \pi_{12} = p_{12}^d (q_{12}^d) q_{12}^d - \alpha\beta r_1^d \lambda_1^d q_{11}^d - c_2 (\xi q_{12}^d - \alpha\beta \lambda_1^d q_{11}^d) \\ \max \pi_{11} = [p_{11}^d (q_{11}^d) - \mu c_1] q_{11}^d + \beta w_1^d \lambda_1^d q_{11}^d - h\beta (1 - \lambda_1^d) q_{11}^d \end{cases} \quad (6)$$

Eq. (5) and Eq. (6) are dual master-slave models overlapped by two Stackelberg decision-making model. Purchase price of waste is w_1^d , purchase rate is λ_1^d , and sale price of extract is r_1^d , which is determined by waste refiner, the market follower. By-product manufacturers regulate the production and maximize the benefits; main-product manufacturers determine their own optimal decision-making based on the first two supply chains. Then the optimal response strategies will be feedback to by-product manufacturer and waste refiner, waste refiner will also adjust the initial decision according to the optimal output q_{11}^d of main-product manufacturer and the optimal output q_{12}^d of by-product manufacturer, to make optimal waste purchase price w_1^d , purchase rate λ_1^d , extract sale price r_1^d .

Make the first derivative of q_{11}^d and q_{12}^d in Eq. (6) be zero, we can get conclusions as follows after computing (calculation process is omitted).

$$\begin{cases} q_{11}^{d*} = \frac{2c_0 A}{8c_0 - b_1 B^2} \\ q_{12}^{d*} = \frac{a_2 - b_2 c_2 \xi}{2} \\ \pi_{11}^* = \frac{4A^2 c_0^2}{b_1 (8c_0 - b_1 B^2)^2} \\ \pi_{12}^* = \frac{(a_2 - b_2 c_2 \xi)^2}{4b_2} \\ \pi_{13}^* = \frac{A^2 c_0}{b_1 (8c_0 - b_1 B^2)} \end{cases}$$

$$\pi_1^{d*} = \pi_{11}^* + \pi_{12}^* + \pi_{13}^* = \frac{4A^2 c_0^2}{b_1 (8c_0 - b_1 B^2)^2} + \frac{(a_2 - b_2 c_2 \xi)^2}{4b_2} + \frac{A^2 c_0}{b_1 (8c_0 - b_1 B^2)}$$

When $\xi q_2 < \alpha\beta\lambda q_1$, that is to say, the supply exceeds the demand in the extract market.

When supply exceeds demand of extracts, by-product manufacturer can take full advantage of the extract to produce, instead of purchasing raw materials with high price; the main-product manufacturer must deal with the excess waste at the cost of h .

Centralized decision-making model

When waste and extract are both oversupplied, centralized decision-making model will change into the function below:

$$\max \pi_2^c = [p_{21}^c (q_{21}^c) - \mu c_1] q_{21}^c - h\beta (1 - \lambda_2^c) q_{21}^c + p_{22}^c (q_{22}^c) q_{22}^c - c_0 (\lambda_2^c)^2$$

Dual master-slave model of decentralized decision-making

When $\xi q_2 < \alpha\beta\lambda q_1$, the output of extract exceeds the demand of by-product manufacturer, at this time, by-product manufacturer will become follower among TSC market. Then the problem turns into function below:

$$\max \pi_{22} = [p_{22}^c (q_{22}^c) - \xi r_2^d] q_{22}^c$$

$$\begin{cases} \max \pi_{23} = \xi r_2^d q_{22}^c - \beta \omega_2^d \lambda_2^d q_{21}^c - c_0 (\lambda_2^d)^2 \\ \max \pi_{21} = [p_{21}^c(q_{21}^c) - \mu c_1] q_{21}^c + \beta \omega_2^d \lambda_2^d q_{21}^c - h\beta(1 - \lambda_2^d) q_{21}^c \end{cases} \text{ s.t.}$$

4.2 The demand exceeding the supply in the waste market

In this case, whatever extracts that by-product manufacturer purchases from waste refiner can't meet the need of production, that is to say $\xi q_{32}^c < \alpha \beta \lambda_3^c q_{31}^c$.

Centralized decision-making model

When adopting the centralized decision-making mechanism, the profit function of three related supply chain system can be expressed as follows:

$$\max \pi_3^c = [p_{31}^c(q_{31}^c) - \mu c_1] q_{31}^c - h\beta(1 - \lambda_3^c) q_{31}^c + p_{32}^c(q_{32}^c) q_{32}^c - c_0 (\lambda_3^c)^2 - c_2 (\xi q_{32}^c - \alpha \beta \lambda_3^c q_{31}^c)$$

Dual master-slave model of decentralized decision-making

The price of extract is lower than raw material, therefore, by-product manufacturer prefers to purchase raw material made by waste refiner. At this time, main-product manufacturer is leader of the market in the horizontal supply chain, and has an impact on the price and output of RSC. So the profit function of three related supply chain systems can be expressed as follows:

$$\max \pi_{31} = [p_{31}^c(q_{31}^c) - \mu c_1] q_{31}^d + \beta \omega_3^d \lambda_3^d q_{31}^d - h\beta(1 - \lambda_3^d) q_{31}^d$$

$$\begin{cases} \max \pi_{33} = \alpha \beta r_3^d \lambda_3^d q_{31}^d - \beta \omega_3^d \lambda_3^d q_{31}^c - c_0 (\lambda_3^d)^2 \\ \max \pi_{32} = p_{32}^d(q_{32}^d) q_{32}^d - \alpha \beta r_3^d \lambda_3^d q_{31}^d - c(\xi q_{32}^d - \alpha \beta \lambda_3^d q_{31}^d) \end{cases} \text{ s.t.}$$

According to the total profit of three kinds of models, summaries can be made.

a. Supply and demand of waste is inconsistent, and demand exceeds supply in the extract market, if $\frac{b_1 B^2}{4} < c_0 < \frac{b_1 B^2}{2}$, so TSC mode of decentralized decision-making is superior to the traditional mode of two independent supply chains; only if $c_0 > \frac{b_1 B^2}{4}$, TSC mode of centralized decision-making is constantly superior to the traditional mode of two independent supply chains and TSC mode of decentralized decision-making.

b. The supply exceeds the demand in the waste and extracts market, if $4c_0 > b_1 h^2 \beta^2$, TSC mode of centralized decision-making is constantly superior to the traditional mode of two independent supply chains and TSC mode of decentralized decision-making.

5. Revenue-cost sharing contract of TSC

Analysis above shows that TSC model of centralized decision-making model is optimal. However, every supply chain is an independent economic body, and they make optimal decision for their own interests, so it's difficult to realize the common decision of TSC system. Therefore, it's necessary to establish a suitable coordination mechanism to maximize respective interests and the whole system interests.

Literature [12] uses an improved coordination mechanism of revenue-cost sharing contract, and makes supply chain system get effective coordination. Based on above, this paper coordinates revenue-cost sharing of TSC. Main-product supply chain, by-product supply chain and horizontal supply chain share profit with proportion of ψ_1 , ψ_2 and ψ_3 respectively.

$$\psi_1 + \psi_2 + \psi_3 = 1, 0 < \psi_i < 1, i \in \{1,2,3\}.$$

Due to space limitations, this paper takes the first case as an example. The profit functions of main-product supply chain, by-product supply chain, horizontal supply chain and supply chain can be expressed below.

$$\begin{aligned} \pi_1 &= \psi_1 [p_1(q_1)q_1 + p_2(q_2)q_2 - c_1 \mu q_1 - c_2 (\xi q_2 - \alpha \beta \lambda q_1) - h\beta q_1 - \lambda^2 c_0] + \beta \omega \lambda q_1 + h\beta \lambda q_1 \\ \pi_2 &= \psi_2 [p_1(q_1)q_1 + p_2(q_2)q_2 - c_1 \mu q_1 - c_2 (\xi q_2 - \alpha \beta \lambda q_1) - h\beta q_1 - \lambda^2 c_0] - \alpha \beta r \lambda q_1 \end{aligned}$$

$$\pi_3 = \psi_3 [p_1(q_1)q_1 + p_2(q_2)q_2 - c_1\mu q_1 - c_2(\xi q_2 - \alpha\beta\lambda q_1) - h\beta q_1 - \lambda^2 c_0] + \alpha\beta r\lambda q_1 - \beta\omega\lambda q_1$$

$$\pi = p_1(q_1)q_1 + p_2(q_2)q_2 - c_1\mu q_1 - c_2(\xi q_2 - \alpha\beta\lambda q_1) - h\beta q_1 - \lambda^2 c_0 + h\beta\lambda q_1$$

Make the first derivative of λ and q_1 in the function π_1, π_3 be zero, there

$$\begin{cases} \lambda^* = \frac{(\psi_3 c_2 \alpha + r\alpha - \omega)\beta q_1}{2\psi_3 c_0} \\ q_1^* = \frac{\psi_1 A + [\psi_1(c_1\alpha + c_2\alpha) + \omega + h]b_1\beta\lambda}{2\psi_1} \end{cases}$$

It can be calculated by computing above

$$\pi_1 = \psi_1 \pi_1^{c*}, \pi_2 = \psi_2 \pi_1^{c*}, \pi_3 = \psi_3 \pi_1^{c*}, \pi = \pi_1^{c*}$$

Therefore, we can get some conclusions according to analysis above.

(1) Revenue-cost sharing contract can make the profit of TSC system get optimal, main-product supply chain, by-product supply chain and horizontal supply chain share profit with proportion of ψ_1, ψ_2 and ψ_3 respectively.

(2) ψ_1, ψ_2 and ψ_3 reflect the ability of bargain of every supply chain, meanwhile, the profit respectively after coordinating must be greater than the one decentralized decision-making.

6. Case analysis

In this section, a numerical example will be used to validate the conclusion is correct. Assume that related parameters of a certain TSC are shown as follows:

$$p_1 = \frac{20000 - q_1}{300}, p_2 = \frac{1050 - q_2}{30}, c_1 = 30, c_2 = 10, \mu = 0.95, \beta = 0.2, h = 120,$$

$\alpha = 0.7, c_0 = 80000, \xi = 0.7, \psi_1 = 0.45, \psi_2 = 0.25, \psi_3 = 0.3$. Results are shown in Table 1

Table 1 Optimal decision-making and profit comparison of every mode

	traditional mode	TSC mode		
		decentralized mode	centralized mode	After coordination
q_1	2125	1523.12	5377.54	5377.54
q_2	840	420	420	420
λ	—	0.24	0.85	0.85
ω	—	-202.97	—	-66
r	—	10	—	-42.86
π_1	15052.08	7732.98	—	19786.89
π_2	5880	5880	—	10992.72
π_3	—	10788.77	—	13191.26
π_T	20932.08	24401.75	43970.87	43970.87

As can be seen from Table 1, the purchase rate of waste improves significantly in the TSC mode after coordinate. The transfer price of waste is controlled in an acceptable range for main-product manufacturers, which is lower than the price that main-product manufacturers deal with the waste by themselves. The profit of every supply chain in TSC has been greatly increased, system profit make the optimal profit possible, and also illustrates that model above is feasible in practice.

7. Conclusion

This paper proposes the Trilateral Supply Chain (TSC) model, which is aimed to solve the recycling problem of industrial waste during production. Then analyses centralized decision-making model and dual master-slave model of decentralized decision-making, and shows that the centralized decision-

making is superior to the decentralized decision-making. Finally a revenue-cost model is used to coordinate the profit of TSC, the interests of all parties achieve the best finally, and this also opens up a new channel for deeper research of supply chain management.

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