New method of Calculating Transmission Line Parameters Using PMU

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Abstract

Aiming at difficulty in measuring transmission line parameters, which changes with the system operating status, a new method for online estimating the parameters is proposed by using synchronized measurement data of the phasor measurement units (PMUs). This method derives the different estimating equations for medium length line and long transmission line respectively. For Π-type equivalent circuit of medium length lines, the method combines the two-port networks equation to estimate the parameters. For long lines, the method derives the parameters estimating equation based on the relationship between active power and reactive power. At last, by means of testing a 220kV, 300km line and two branches 2-5, 10-20 of IEEE-30 bus power system, the results from error curves show that the relative error is below 3% for the medium length line and no more than 1% for the long line. The result validates its correctness and also shows that this method has characteristics of simple calculation, accuracy and availability.

Keywords

Transmission line Parameters；Long line；Medium line；Phasor measurement unit(PMU)；Synchronized phasor

1. Introduction

In recent years, with the grid scale continuously expanding and the network structure becoming more and more complex, it has become an increasingly tough task to maintain safe and stable operation of the grid for the power stuff. In the calculation of relay protection and fault location, the main parameters (resistance, capacitance and inductance) of transmission line are needed as known parameters. The accuracy of these parameters has direct influence on the final result of relevant calculations, especially the accuracy of fault location. Accurate grid parameters are also the base of forming an accurate grid model for further power system analysis and calculation such as state estimation, power flow calculation, stability calculation, net loss analysis and so on. Therefore, it is of great significance for the safe, stable and reliable operation of the grid if the corresponding parameters of the current line operation can be calculated in real time [1].

Accurate line impedance parameters are of great significance on the following aspects in the power system: increasing the sensitivity of relay protection devices, in favor of accurate positioning and rapid recovery of system after accidents, improving transmission line model in system simulation such as state estimation calculation, determining parameter changes. When the transmission line parameters change in the case of capacitor connection in series, line extension and line replacement, and the existing data in the database are not updated synchronously in time, they can be discovered and modified in time.

In recent years with the rapid development of WAMS at home, great progress has been made on analysis research application which is based on PMU. There is a growing concern about how to get the actual grid parameters from the measured data of the grid among the scholars at home and abroad. An important way which is used to achieve power system wide area dynamic information in real time

is provided by Wide Area Measurement System (WAMS). The WAMS also helps develop ideas for each calculation of power system. The actual dynamic process of power grid can be truly monitored and reflected by using the measuring unit PMU. References [2-4] present a method to calculate the equivalent transmission line parameters using double-ended voltage phasor and current phasor, however, only single moment PMU data is involved in the calculation, and the error is not considered. Therefore, this requires further research . References [5-6] present a method of line parameters measurement which is based on synchronized phasor measurement technology. For Π-type equivalent circuit, the method writes the loop equations and solves equations, finally gets the line parameters. But the coefficient matrix is singular matrix in the process of derivation. Based on these researches, this paper proposes a new method of calculating parameters. This method measures by using the branch synchronized data which is provided by PMU, establishes relation equation between measurement data and line parameters, calculates to get the line parameter value and verifies the method of the theoretical derivation.

2. New method of Calculating Transmission Line Parameters

Impedance and admittance per unit length of transmission line parameters are as follows.

$$z_0 = r_0 + jwL_0 = r_0 + jx_0 \tag{1}$$

$$y_0 = g_0 + jwC_0 = g_0 + jb_0 \tag{2}$$

Among the parameters, r_0, x_0, g_0 and b_0 are resistance, reactance, conductance and susceptance.

In fact, when designing the circuit, corona loss is fully considered, always try to prevent corona from happening under normal weather conditions. The method increasing wire radius is used to prevent and reduce corona loss. Based on the previous work, the corona loss is generally neglected in electric power system calculation. We take g_0 approximately as zero.

In engineering calculation, we need to ensure the necessary accuracy as well as simplified calculation. Therefore, this paper uses different parameter calculation methods for medium lines and long lines.

2.1 Medium Transmission Lines

In the calculation of power system steady state, Π-type circuit is equivalent to transmission lines, the model is shown in Fig.1
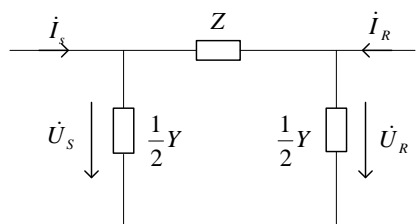

Fig.1 Π-type equivalent circuit

The two ends of branches are both equipped with PMUs, this measuring method can directly get voltage phasor of $\dot{U}_S$ and $\dot{U}_R$ and current phasor $\dot{I}_S$ and $\dot{I}_2$ of the two ends. According to Kirchhoff's law, there is the following relationship:

$$\dot{U}_R = \dot{U}_S - Z\left(\dot{I}_S - \frac{Y}{2}\dot{U}_1\right) \tag{3}$$

$$\dot{I}_R = -\dot{I}_S + \left(\frac{Y}{2}\dot{U}_1 + \frac{Y}{2}\dot{U}_2\right) \tag{4}$$

From expression (4) we can get the following calculation formula

$$Y = \frac{2\left(\dot{I}_S + \dot{I}_R\right)}{\dot{U}_S + \dot{U}_R} \tag{5}$$

Ps. in the above formula, the voltage phasor and current phasor of the two ends of the transmission lines must be synchronized phasor, PMU device measurement meets the conditions [9].

By substituting the admittance Y value calculated from expression (5) into the complex equation (3), we can get the following calculation formula.

$$Z = \frac{\left(\dot{U}_S - \dot{U}_R\right)\left(\dot{U}_S + \dot{U}_R\right)}{\dot{U}_R \dot{I}_S - \dot{U}_S \dot{I}_R} \tag{6}$$

Here we can get two real equations from formula $Z = R + jX$ according to the fact that the real parts equal and the imaginary parts equal correspondingly. Thereby we can get the resistance value R and the reactance value X after calculation. At this moment the line parameters can be solved already. In view of big error in single set of data, we do the following process with the combination of two-port networks equation [10].

The general equation of the known two-port networks is

$$\dot{U}_S = \dot{A}\dot{U}_R - \dot{B}\dot{I}_R \tag{7}$$

$$\dot{I}_S = \dot{C}\dot{U}_R - \dot{D}\dot{I}_R \tag{8}$$

We can get two groups of measurement data from PMU and write the following four equations:

$$U_S^1 = AU_R^1 - BI_R^1 \tag{9}$$

$$I_S^1 = CU_R^1 - DI_R^1 \tag{10}$$

$$U_S^2 = AU_R^2 - BI_R^2 \tag{11}$$

$$I_S^2 = CU_R^2 - DI_R^2 \tag{12}$$

With the unknown A, B, C and D, we can obtain the following formulas through the above four equations

$$A = \frac{I_R^1 U_S^2 + I_R^2 U_S^1}{U_R^1 I_R^2 - U_R^2 I_R^1} \tag{13}$$

$$B = \frac{U_S^1 U_R^2 - U_R^1 U_S^2}{U_R^1 I_R^2 - U_R^2 I_R^1} \tag{14}$$

$$C = \frac{I_S^1 U_R^2 - I_S^2 U_R^1}{U_R^1 I_R^2 - U_R^2 I_R^1} \tag{15}$$

$$D = \frac{I_S^1 U_R^2 - I_S^2 U_R^1}{U_R^1 I_R^2 - U_R^2 I_R^1} \tag{16}$$

Both line parameters and A, B, C, D have relationship with the two-end voltage phasor and current phasor. The relationship between them is derived:

$$A = 1 + 0.5YZ \tag{17}$$

$$C = Y\left(1 + 0.25YZ\right) \tag{18}$$

According to the two groups of measurement data, we can obtain the value of parameter A, B, C and D. Then we can obtain the value of the line parameters by formula (17) and (18). With the help of the median A, B, C and D, we take two-step calculation to obtain the parameter value. Reference [11] adopts Π-type equivalent circuit model to calculate 500kV long lines. Theoretically the equivalent model is applicable to the lines which are not longer than 300km. Therefore this paper adopts this method for medium lines and the following method for the long lines.

2.2 Long Transmission Lines

With the development of the Wide Area Measurement System (WAMS), many important busbars of the power grid have been equipped with high-precision PMU measurement device. According to the voltage phasor and current phasor of the two ends of the transmission lines, the existing calculation method derives the spread impedance and attenuation coefficient expression according to the long

line equation, thereby obtaining the resistance value and reactance value. The hyperbolic function is involved in the calculation, a little more complex [11-12].

Thus, for long lines, it is proposed to establish the contact of the variables of two ends and substitute the known quantity to solve parameters according to the active measurement equation and the reactive measurement equation, under the condition that the voltage phasor and the current phasor of the two ends and other measurement data which is provided by PMU are known [13].

Figure 2 is a Π-type equivalent circuit, measuring its power and voltage at the two ends, the PMU provides the measured value of the angle difference between node i and node j, making an analysis of the two-end active and reactive power measurement equation of the transmission line:

$$P_{ij} = V_i^2 g - V_i V_j (g \cos \theta_{ij} + b \sin \theta_{ij}) \tag{19}$$

$$P_{ji} = V_j^2 g - V_i V_j (g \cos \theta_{ij} - b \sin \theta_{ij}) \tag{20}$$

$$Q_{ij} = -V_i^2 (b + y) + V_i V_j (b \cos \theta_{ij} - g \sin \theta_{ij}) \tag{21}$$

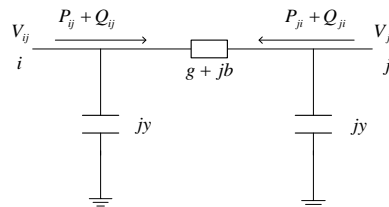$$Q_{ji} = -V_j^2 (b + y) + V_i V_j (b \cos \theta_{ij} + g \sin \theta_{ij}) \tag{22}$$

Fig.2 power equivalent circuit line

In the formula: the line admittance value is $y = g + jb = \dfrac{1}{r + jx}$, r is line resistance, x is line reactance,

g is line conductance, b is line susceptance and θ_{ij} is the two-end voltage phasor angle difference of the branch.

The four power equations above are only influenced by the three parameters of resistance, reactance and capacitance besides the state variables. The state variables are obtained according to the devices on both sides. Therefore, from the above relationship we can obtain the calculation method of the following three parameters. Add the active equation (19) and expression (20) together and eliminate other unknown quantities, we obtain

$$P_{ij} + P_{ji} = V_i^2 g + V_j^2 g - 2V_i V_j g \cos \theta_{ij} \tag{23}$$

$$g = \frac{P_{ij} + P_{ji}}{V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij}} \tag{24}$$

From expression (7) we can derive the following expression of susceptance b.

$$b = \frac{V_i^2 g - g V_i V_j \cos \theta_{ij} - P_{ij}}{V_i V_j \sin \theta_{ij}} \tag{25}$$

Substitute the conductance value from expression (24) into expression (25) and we can obtain the computation result.

From expression (21), we can obtain the admittance expressions

$$y = \frac{V_i V_j (b \cos \theta_{ij} - g \sin \theta_{ij}) - b V_i^2 - Q_{ij}}{V_i^2} \tag{26}$$

Substitute the value obtained from expression (24) and expression (25) into expression (26), we can

obtain admittance calculation formula. And we know the formula $r + jx = \dfrac{1}{g + jb}$, thus, the line

impedance and admittance have been obtained.

Synchronized phasor measurement unit PMU can provide voltage amplitude and phase angle measurement value with high precision, the measurement error is no more than 5%. Therefore, in the parameter calculation, the voltage measurement value can be used as a known quantity. Substitute the measurement value of the branch active power and reactive power into expression (24), (25) and (26), then we can obtain line impedance and admittance value.

3. Simulation and Verification

3.1 Medium Line Example of Verification

The medium lines verify and calculate the 220kV and 300km system single phase. Using the given length of the lines and the voltage and current phasor value measured by PMU, compare the line parameters obtained according to formula with the given line parameters.

The known parameters: $r_1 = 0.25673\,\Omega/\text{km}$, $x_1 = 0.10623\,\Omega/\text{km}, b_1 = 4.3261\times10^{-7}\,\text{S/km}$. According to expression (5) and expression (6), do six groups of verification. The original data is shown in the following Table 1:

Table 1 original data

	$\dot{U}_1$	$\dot{i}_1$	$\dot{U}_2$	$\dot{i}_2$
1	$200\angle 0°$	$0.1225\angle 2.6526°$	$210\angle 0°$	$0.1154\angle -0.2839°$
2	$200\angle 0°$	$3.2464\angle -1.2911°$	$210\angle -30°$	$3.2646\angle 1.8532°$
3	$200\angle 0°$	$0.1167\angle -0.2944°$	$190\angle 0°$	$0.1264\angle 2.6543°$
4	$200\angle 0°$	$3.0878\angle -1.2324°$	$190\angle -30°$	$3.1059\angle 1.9113°$
5	$200\angle 0°$	$3.2084\angle -1.2740°$	$205\angle -30°$	$3.2266\angle 1.8701°$
6	$200\angle -30°$	$0.1082\angle -0.2740°$	$205\angle -30°$	$0.1164\angle 2.4543°$

Substitute the six groups of original data into expression (5) and expression (6) respectively to calculate and obtain this line parameter calculation values and check calculation accuracy, write them in table 2

Table 2 Line parameters calculated results and error

Parameter NO.	r_1		x_1		b_1	
	Value	Error/%	Value	Error /%	Value	Error /%
1	0.25665	0.03	0.1065	0.25	4.3298×10^{-7}	0.09
2	0.25119	2.2	0.10646	0.22	4.3273×10^{-7}	0.03
3	0.25138	2.1	0.10648	0.24	4.3287×10^{-7}	0.06
4	0.25118	2.16	0.10645	0.21	4.3284×10^{-7}	0.05
5	0.2513	2.12	0.10558	0.61	4.3134×10^{-7}	0.29
6	0.25138	2.1	0.10648	0.24	4.3287×10^{-7}	0.06
Initial Value	0.25673		0.10623		4.3261×10^{-7}	

According to the data in table 2, draw error curve as is shown in the following Fig.3:
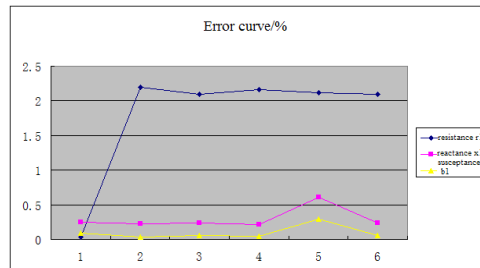
Fig.3 lines error curve

By comparing the above six groups of parameter original values with calculation values, in the error curve we can intuitively see that the calculation accuracy of this calculation method is higher. The resistance calculation error can reach as low as 0.03%, and the maximum error 2.2% can also meet the requirements of other power system calculation, the inductance calculation error is below 1%, the susceptance calculation error reaches below 0.1%. When the Matlab program calculates, the method takes shorter time, which means rapid high-precision calculation and better meeting the requirement for real-time operation of the grid.

3.2 Long Line Example of Verification

For long line parameter calculation, according to the method proposed in this paper, take the standard IEEE 14 bus test system which is shown in Figure 4 as an example to do the testing, examine the accuracy of this method. Choose the two branches whose head end and terminal end node numbers are 2-5 and 10-20 as estimation object. When testing, suppose that both the two ends of branch 2-5 and the two ends of branch 10-20 are equipped with PMU devices, as is shown in the figure, and the node voltage amplitude and line head terminal power flow are provided. The true values of the example are taken from references [14].
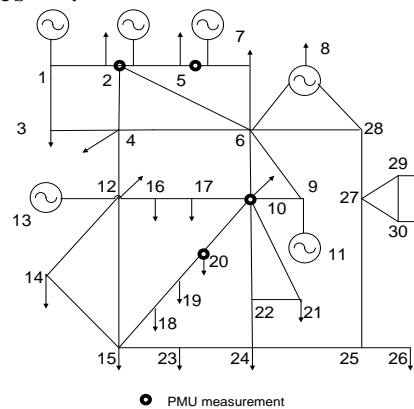

Fig.4 IEEE 30 Schematic diagram of test system

When obtaining the sample data, add the error which is less than 2% to active power and voltage amplitude value of head and terminal ends of the line respectively, add the random relative error which is not more than 4% to the reactive power of head and terminal ends of the line.

For the obtained more than 400 groups of sample data, use the method proposed in this paper to do the parameter estimation of transmission line, use expression (24) to (26) to calculate. Table 3 presents the parameter estimation results of line 2-5 and line 10-20. The results are shown in Table 3.

Figure 5 to figure 7 presents parameter calculation result cartogram of line 2-5, wherein the abscissa represents the parameter calculation results, the ordinate represents the frequency that the calculation results appear in this value interval. From figure 5 to figure 7 we can see that the parameters mostly spread close to the true value range. This illustrates that not only the parameter estimation accuracy is high, but also the data which is provided by PMU is accurate with high stability.

Table 3 below:

Table 3 Estimation results for parameters datas

target line	type	mean value	standard deviation	95% confidence interval	true value	relative error /%
	y	0.0208	0.0118	[0.0198,0.0218]	0.0209	0.5
2-5	g	1.1515	0.2486	[1.1296,1.1733]	1.1360	1.4
	b	-4.7745	0.5157	[-4.8189,-4.7292]	-4.7725	0.04
	y	0	0.0094	$[-0.9424,0.7168]_{\times 10^{-3}}$	0	—
10-20	g	1.7351	0.4397	[1.6964,1.7737]	1.7848	2.8
	b	-3.9369	0.6289	[-3.9922,-3.8817]	-3.9854	1.2

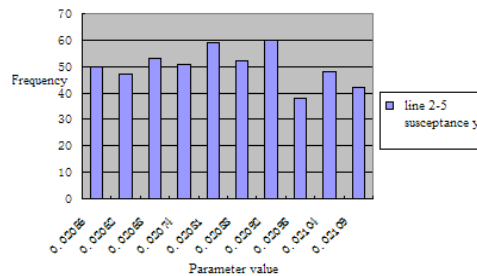The following figure 5 to figure 7:

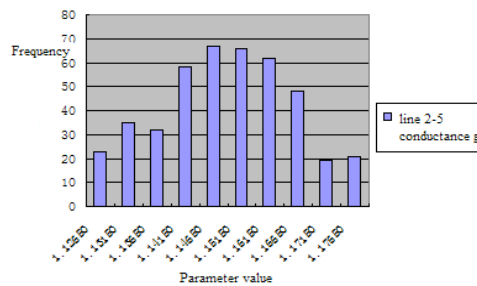Fig.5 line 2-5 parameters susceptance results cartogram

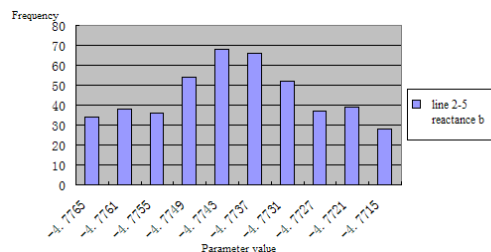Fig.6 line 2-5 parameters conductance results cartogram

Fig.7 line 2-5 parameters reactance results cartogram

If the sample data is reduced to 50 groups, in table 4, the parameter estimation results of line 2-5 are shown again.

Compare table 3 with table 4 and make an analysis, we find that the more the quantity of samples is, the higher the accuracy of estimation results for parameters is. And there is not too much difference

between the consuming time of computer operation, therefore, if possible, we can obtain multiple sets of measurement data for parameter estimation in different time periods as much as possible.

Table 4 Estimation results for 500 groups parameters data

target line		mean value	standard deviation	95% confidence interval	true value	relative error /%
2-5	y	0.0214	0.0111	[0.0182,0.0245]	0.0209	2.4
	g	1.0987	0.2690	[1.0223,1.1752]	1.1360	3.3
	b	-4.7493	0.4876	[-4.8879,-4.6107]	-4.7725	0.5

The above selects two branches for calculation, in order to realize the observability of the whole power grid, it is necessary to site and install the PMU in the actual electric network. Then we can use this method to calculate for each branch, on the basis of error analysis, it can be seen that this method meets the requirements of the actual power grid safe operation analysis, it is simple for Matlab programming, it takes shorter time to input the measurement data and calculate, the real-time effect is good.

4. Conclusion

This paper presents the methods of parameter calculation for medium lines and long lines according to the relation between the line double-side variables. The calculation process is simple, the parameters of the medium lines calculated by this method can be used in parameter calculation directly. Considering the large transmission capacity and much electric equipment of long lines, use multiple groups of sample data for estimating calculation and take the average value as the final result. After verification calculation and error analysis, it can be concluded that the above calculation method is feasible. When the power system is operating, the PMU can provide various kinds of electrical quantities with high precision. Although there is slight error, it still can meet the requirements of the electrical calculation. This paper is different from the previous method that using line equation to derive parameters, but establishes relationship between the parameters of both ends of the branch in the power grid according to PMU measurement dada. Thus the unknown parameters are obtained and the calculation is simplified. This paper increases the calculation speed while improving the accuracy of calculation and meets the rate requests for real-time calculation of the power networks.

Acknowledgement

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