A new smoothing kernel function of SPH

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Abstract

In the traditional SPH method, pressure instability will cause the agglomeration of particles and even collapse in the progress of calculation. In this paper, a new quartic spline smoothing function was putted forward, which can eliminate the pressure instability with high accuracy and smoothness. And an example were presented to verify the applicability and accuracy of the function. The results show that the pressure instability can be resolved effectively by this smoothing function, and the smoothing function could be used more widely.

Keywords

pressure instability; smoothing function; Smoothed Particle hydrodynamics; SPH

1. Introduction

Stress instability is always a important defect and restricted the development of Smoothed Particle hydrodynamics method (SPH). When the particles are in compress or tensile stress state, the motion of the particle becomes unstable, and leads to particles agglomerated or scattered or even lead to calculation collapse[1].

According to the theory of stability analysis, Swegle pointed out that the sufficient condition for the unstable growth is $W''\sigma>0$ [2] (Where W'' is the second derivative of the smoothing function, and σ is stress state). By using the method of dispersion equations, Morris studied the stability of different discrete format of SPH and demonstrated that there are no smoothing function can both eliminate the compress and tensile instability [3].However, for the fluid problems which do not existed tensile state, we could construct a smoothing function of which the second derivative is nonnegative. In this paper, in order to solve the pressure instability problem, the basic properties of smoothing function is discussed, and the defect of conventional smoothing function is analyzed. Then, in the full consideration of the smoothness, precision, stability, a new quartic spline kernel was established. Finally, through two typical numerical examples, the practicability of the improved smoothing function is verified.

2. Basic theory of SPH

2.1 Basic properties of smoothing function

For any meshless method, a key problem is to approximate the discrete points by function. For SPH method, the progress is realized by smoothing function, and can be expressed by the following equation:

$$< f(x') >= \int_{\Omega} f(x')W(x-x',h)dx' \tag{1}$$

Where $W(x-x',h)$ is the smoothing function, it not only determines the form of function approximation, and also definite the support domain, the consistency and accuracy of approximation. Therefore, smoothing function is of great important in SPH method. Usually, the more smooth the smoothing function is (included the one and two order derivative), the fewer it will be affected by the irregular distribution of particles.

2.2 Control equations in the form of SPH

Under the Lagrangian description, control equations of flow can be written as partial differential equations as follow:

$$\frac{d\rho}{dt} = -\rho \frac{\partial v^{\beta}}{\partial x^{\beta}} \tag{2}$$

$$\frac{dv^{\alpha}}{dt} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^{\beta}} \tag{3}$$

The above differential equations are the famous Nervier-Stokes control equations. The formulas (4), (5) are mass conservation equation, momentum conservation equation respectively, and its corresponding SPH expression can be written as follow:

$$\frac{d\rho_i}{dt} = \rho_i \sum_{j=1}^{N} m_j v_{ij}^{\beta} \bullet \frac{\partial W_{ij}}{\partial x_i^{\beta}} \tag{4}$$

$$\frac{dv_{ij}^{\alpha}}{dt} = \sum_{j=1}^{N} m_j \left(\frac{\sigma_{ij}^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_{ij}^{\alpha\beta}}{\rho_j^2}\right)\frac{\partial W_{ij}}{\partial x_i^{\beta}} \tag{5}$$

Where ρ indicates density, m represents mass, v denotes velocity, e represents energy, σ are the total stress tensor of particles.

3. A new quartic spline smoothing function

According to the basic properties of the smoothing function described above, combined with the $W'' \geq 0$ condition and the central peak condition, a new quartic spline kernel (in the following text, we use NQS kernel to represent the function) was constructed in this paper to solve the pressure instability problem:

$$W(R,h) = \alpha_d \begin{cases} (3-R)^4 - \frac{1}{2}(2-R)^4 - 7(1-R)^4 & 0 < R \leq 1 \\ (3-R)^4 - \frac{1}{2}(2-R)^4 & 1 < R \leq 2 \\ (3-R)^4 & 2 < R \leq 3 \end{cases} \tag{6}$$

Where $R = |x - x'|/h$, and the values of α_d are $\frac{1}{88h}$, $\frac{1}{46\pi h^2}$, and $\frac{105}{8464\pi h^3}$ corresponding to first-, second-, and three-dimensional spaces respectively. The NQS smoothing function and its derivatives can be seen as follows:

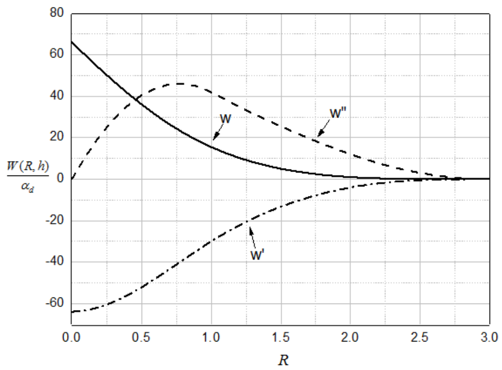

Figure 1 The NQS kernel and its one- and two-order derivative

In addition to meet the basic properties of smoothing function, the NQS smoothing function also has the following characteristics:

1. The two order derivative of the NQS smoothing function is constantly greater than 0, so pressure instability problem can be eliminated when using the NQS kernel, and especially suitable for fluid mechanics problems which does not exist tensile stress.
2. The one- and two-order derivatives of the NQS kernel are more smooth than traditional kernals, so it will be fewer affected by the irregular distribution of particles.
3. Central peak was considered when constructed the function, so the approximate solution will be more accurate by using NQS kernel.

4. Numerical examples

In this section, a numerical examples were simulated to verify the effectiveness of the NQS kernel. The case is a hydrostatic pressure test, and the purpose is to verity the effectiveness of NQS kernel of eliminate the pressure instability problem.

A certain amount of fluid is placed in a quadrate tub. The width and depth of the fluid is 1m and 0.5m respectively. The density and gravity acceleration is $\rho = 10^3 kg/m^3$ and $g = 9.8 m/s^2$ respectively. In order to observe the pressure instability phenomenon, the initial smooth length was set as 4/3 times as the distance between the particles (Where the second derivation of qunitic spline smoothing function is zero). The time step is $2\times10^{-5}s$, 10000 time steps was calculated.

Four points were selected to monitor the pressure of the fluid, and the coordinate of the points is (0.5, 0.1), (0.5, 0.2), (0.5, 0.3), (0.5, 0.4) respectively. The results of particle distributions after 10000 time steps were shown at figure 9, 11, 13, and the pressure changes of the monitor points were shown at figure 3 and 5, which use qunitic spline kernel and NQS kernel respectively.

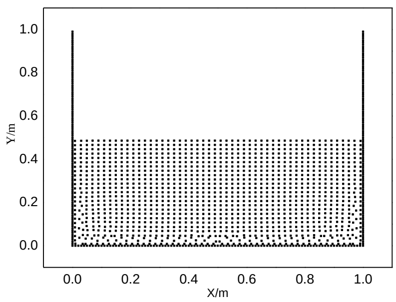

Figure 2 Particle distributions when using qunitic kernel

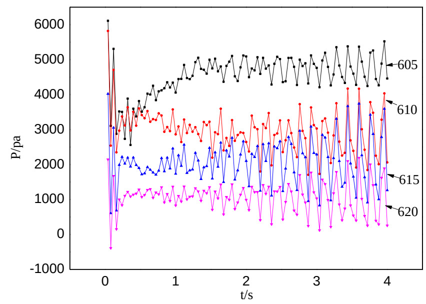

Figure3 Pressure of monitored points when using qunitic kernel

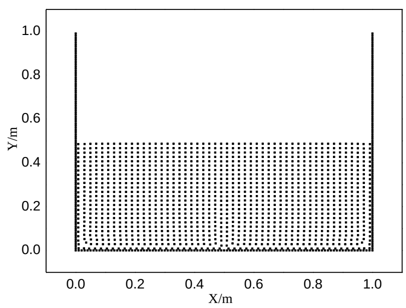

Figure 4 Particle distributions of NQS kernel

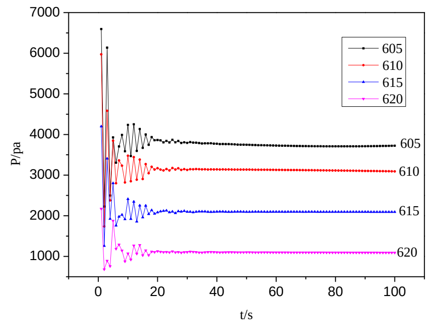

Figure5 Pressure of monitored points of NQS kernel

As shown in the figure 2and 3, when using the qunitic spline kernel, the particles near the bottom and side wall had obvious trend of aggregation (figure 2), significant pressure fluctuations can be seen in each monitor points, and had a trend of increase (figure 3).

When using NQS kernel, the distribution of the particles is basically stable (figure4). After a short time of pressure fluctuation, the pressure became stable in figure 5. Proved that the NQS can removed the pressure instability.

5. Conclusions

In This article, the reason of the stress instability was discussed, and a new quartic spline smoothing function was represented to solve pressure instability. A typical example was discussed and proved that, compared to the traditional smoothing function, the NQS kernel can not only solve the pressure instability, but also has higher accuracy and stability.

Reference

[1] G.R. Liu and M.B. Liu. Smoothed Particle Hydrodynamics: A meshfree particle method. Singapore: World Science (2003)

[2] Swegle J.W. and Hicks D. L. and Attaway S. W. Smoothed particle hydrodynamics stability analysis, Journal of Computational Physics, 116(1):123 (1995)

[3] Morris. J. P. Analysis of smoothed particle hydrodynamics with applications, Ph. D. thesis, Monash University. (1996)