The Stability analysis of dynamic model of unilateral fish which cannot survive independently

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Abstract

By improving the classical Lotka-Volterra model, a reasonable dynamic model is established for the unilateral fish which cannot survive independently. We combine the established model with the stability theory of differential equations to obtain the equilibrium point of the dynamic model about fish mutualism , and analyze the locally stability of the equilibrium point.At last, we give the explanation of the biological process from the angle of the change of fish school.

Keywords

global asymptotical stability; dynamic model; the unilateral fish cannot survive independently

1. Introduction

Malthus applies differential equations to the prediction of population growth, and he accurately predict the population change for decades. Verhulst who is a Holland biologists build the Logistic model. And this model accurately predicted the change of population quantity at that time. Based on this conjecture, we can build a similar differential equation model to study the change of the number of shoal of fish.

The relationship between species of fish are a common competitive relationship mutually beneficial symbiotic relationship, symbiotic relationship and the relationship of cooperation and competition. In the study of marine fish, we found that the survival of fish A depends on the survival of fish B, and fish B can survive independently. For this relationship that one side can not survive independently. Is there any corresponding differential equation model worth studying?

In 1940s, Lotka and Volterra set up the theoretical basis of interspecific competition, and they propose the Lotka-Volterra model. The Lotka-Volterra model is the most common one among the study groups. In the ecological system of fish school, study on the competition pattern of the prey and the competition model of the non predatory fish school, we find that the latter is more universal. In the non predator relationship, the researchers have done a lot of research on the competition between the populations. We are lack of research on the dynamic model of fish which can't live independently. In this paper, we build the stability analysis of dynamic model of unilateral fish which cannot survive independently to explore the quantity of shoal of fish in the sea.

2. Model

We take into account a phenomenon that the B fish can survive alone, while fish A can not live alone without B. In the face of this phenomenon, we set up a model of Lotka-Volterra cooperation which belongs to this type. It is efficient to predict the development trend of this kind of fish by establishing the dynamic model. By improving the classical Lotka-Volterra model, a reasonable dynamic model is established for the unilateral fish which cannot survive independently.[3][4]

Assuming that A and B are two schools of fish, they are living in the natural environment of the sea. Without the effects of human beings. Meanwhile we do not consider the effect of self feeding on the number of fish.And when they are in the ocean, the number of fishes obey the law of Logistic.

Let $y_1(t)$ and $y_2(t)$ are as the density of fish A, B in turn.Let r_1 and r_2 are as their growth rates. Let y_1^m and y_2^m are as the maximum environmental capacities of the ocean for their individual growth.

For fish A, when the fish B survive in the same area, because fish a can't live fish when they leave B, then fish B will promote the growth of fish A. Fish B have a positive impact on the growth of fish A, we need to add condition for the logistic model which cannot live independently. Finally we get a similar Lotka-Volterra model.

We establish a differential dynamic equation

$$\frac{dy_1}{dt} = r_1 y_1(-1 - \frac{y_1}{y_1^m} + \frac{b_1 y_2}{y_2^m}) = f_1(y_1, y_2) \tag{1}$$

Among them: b_1 refers that the amount of food which unit quantity of the School of fish B provide to the fish A is b_1 times than the amount of food which unit quantity of the School of fish A provide to the fish B . $-1-y_1/y_1^m$ refers that the fish B can survive alone, while fish A can not live alone without fish B.

The fish B is consistent with the classical Lotka-Volterra model. We establish a differential dynamic equation.

$$\frac{dy_2}{dt} = r_2 y_2(1 - \frac{y_2}{y_2^m} + \frac{b_2 y_1}{y_1^m}) = f_2(y_1, y_2) \tag{2}$$

Similarly, among them: b_2 refers that the amount of food which unit quantity of the School of fish A provide to the fish B is b_2 times than the amount of food which unit quantity of the School of fish B provide to the fish A .

3. Stability analysis

We get the Lotka-Volterra model (3).

$$\begin{cases} \dfrac{dy_1}{dt} = r_1 y_1(-1 - \dfrac{y_1}{y_1^m} + \dfrac{b_1 y_2}{y_2^m}) \\ \dfrac{dy_2}{dt} = r_2 y_2(1 - \dfrac{y_2}{y_2^m} + \dfrac{b_2 y_1}{y_1^m}) \end{cases} \tag{3}$$

Equation of the model (4)

$$f_1 = 0; f_2 = 0 \tag{4}$$

System (1) admits three nonnegative equilibrium point:

$$A_1(0,0), A_2(0, y_2^m), A_3(y_1^m(b_1-1)/(1-b_1b_2), y_2^m(1-b_2)/(1-b_1b_2))$$

For the positive equilibrium, we discuss the stability of their respective respectively.

For the model (3), we obtain the derivative (5).[2][5]

$$\begin{cases} \dfrac{\partial f_1}{\partial y_1} = c_{11} = -r_1 - 2r_1y_1/y_1^m + r_1b_1y_2/y_2^m, \\ \dfrac{\partial f_1}{\partial y_2} = c_{12} = r_1b_1y_1/y_2^m, \\ \dfrac{\partial f_2}{\partial y_1} = c_{21} = r_2b_2y_2/y_1^m, \\ \dfrac{\partial f_2}{\partial y_2} = c_{22} = r_2 - 2r_2y_2/y_2^m + r_2b_2y_1/y_1^m. \end{cases} \tag{5}$$

For equilibrium point $A_1(0,0)$, From (5) we know that

$$c_{11} = r_1(b_1 - 1); c_{12} = 0; c_{21} = y_2^m r_2 b_2 / y_1^m; c_{22} = -r_2$$

Characteristic root equation

$$\begin{vmatrix} c_{11}-\lambda & c_{12} \\ c_{21} & c_{22}-\lambda \end{vmatrix} = 0 \text{ ,we have } \begin{vmatrix} r_1-\lambda & 0 \\ 0 & r_2-\lambda \end{vmatrix} = 0 ,$$

The solution of the equation is $\lambda_1 = -r_1 > 0; \lambda_2 = r_2 > 0$ .We get equilibrium point A_1 is unstable.

For equilibrium point A_2 , From (5) we know that

$$a_{11} = -r_1; a_{12} = r_1 b_1 y_1^m / y_2^m; a_{21} = 0; a_{22} = r_2(1+b_2)$$

The solution of the corresponding eigenvalue equation is

$$\lambda_1 = -r_2 < 0,\ \lambda_2 = r_1(1+b_1)$$

If $b_1 < -1$, we get the equilibrium point A_2 is a stable node. If $b_1 > -1$, we get equilibrium point A_2 is a unstable saddle point .

For equilibrium point A_3, From (5) we know that

$$a_{11} = r_1(1-b_1)/(1-b_1 b_2),$$
$$a_{12} = r_1 b_1 y_1^m (b_1 - 1)/[y_2^m (1-b_1 b_2)],$$
$$a_{21} = r_2 b_2 y_2^m (1-b_2)/[y_1^m (1-b_1 b_2)],$$
$$a_{22} = r_2(b_2 - 1)/(1-b_1 b_2).$$

And we calculate the Δ.

$$\Delta = a_{11}a_{22} - a_{12}a_{21} = r_1 r_2 (1-b_1)(b_2-1)/(1-b_1 b_2)$$

If $b_1 < 1$, $b_2 > 1$, $\Delta < 0$, we get equilibrium point A_3 a is a saddle point .

If $b_1 > 1$, $b_2 < 1$, $\Delta > 0$, we get equilibrium point A_3 is not a saddle point .

we calculate the ω.

$$\omega = a_{11} + a_{22} = r_1(1-b_1)/(1-b_1 b_2) + r_2(b_2-1)/(1-b_1 b_2)$$

If $\omega < 0$, we get equilibrium point A_3 is a stable node or focus.

If $\omega^2 - 4\Delta > 0$, we get equilibrium point A_3 is a stable node.

If $\omega^2 - 4\Delta < 0$, we get equilibrium point A_3 is a focus.

4. Conclusion

1.We get equilibrium point A_1 is unstable. System is stable .

2.If $b_1 < -1$, we get the equilibrium point A_2 is a stable node. If $b_1 > -1$, we get equilibrium point A_2 is a unstable saddle point .Two kinds of fish resources for their growth, fish A depends on the survival of fish B. So the two kinds of fish to promote the growth of two kinds of fish, the existence of each other is conducive to their growth.

3. If $b_1 < 1$, $b_2 > 1$, $\Delta < 0$, we get equilibrium point A_3 a is a saddle point .The fish A will eventually stable, the fish B will eventually stable.

4. If $b_1 > 1$, $b_2 < 1$, $\Delta > 0$, we get equilibrium point A_3 is not a saddle point .If $\omega < 0$, we get equilibrium point A_3 is a stable node or focus. If $\omega^2 - 4\Delta > 0$, we get equilibrium point A_3 is a stable node. If $\omega^2 - 4\Delta < 0$, we get equilibrium point A_3 is a focus.

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