Model for Evaluating Service Brand Extension with linguistic information

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Abstract

In this paper, we investigate the multiple attribute decision making problems for evaluating service Brand extension in which the attribute weights are usually correlative, attribute values take the form of linguistic variables. Firstly, some operational laws of linguistic variables are introduced. Then, an I-2TCOA operators-based approach is developed to solve the MADM problems for evaluating service Brand extension with linguistic variables. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords

Multiple attribute decision making; 2-tuple linguistic variables; Choquet Integral; induced 2-tuple linguistic choquet ordered averaging (I-2TCOA) operator; service Brand extension.

1. Introduction

The induced aggregation operators are an interesting research topic, which is receiving increasing attention [1-13]. Yager and Filev [2] developed induced ordered weighted averaging (IOWA) operators which take as their argument pairs, called OWA pairs, in which one component is used to induce an ordering over the second components which are exact numerical values and then aggregated. Xu[3-4] extended IOWA to linguistic environments and uncertain linguistic environments. Wei[5] developed Induced intuitionistic fuzzy ordered weighted averaging(I-IFOWA) operator. Xu and Da [6] proposed induced ordered weighted geometric averaging (IOWGA) operators, which have many desirable properties similar to those of the IOWA operators. Xu[7-8] extended IOWGA to linguistic environments and uncertain linguistic environments. Wei[9-10] extended IOWGA to intuitionistic fuzzy environments. Yager [11] developed an induced fuzzy integral aggregation operator, which extends the fuzzy integral aggregation operator by allowing the ordering operation to be based upon values other than those being aggregated. Chen and Chen [12] presented some FN-IOWA operators based on fuzzy numbers. Yager [13] suggests a number of applications of the IOWA aggregation operators, and extends the idea of order induced aggregation to the Choquet aggregation resulting in what he calls the induced Choquet ordered averaging (I-COA) operator. Tan and chen[14]defined two I-COA operators: the I-I-COA operator, which induces the ordering of the argument values based on the importance indexes of the information sources and the P-I-COA operator, which induces the ordering of the arguments based on the relative preference associated with each one of them.

In this paper, we investigate the multiple attribute decision making problems for evaluating service Brand extension in which the attribute weights are usually correlative, attribute values take the form of linguistic variables. Firstly, some operational laws of linguistic variables are introduced. Then, an I-2TCOA operators-based approach is developed to solve the MADM problems for evaluating service Brand extension with linguistic variables. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

2. Preliminaries

Let $S=\{s_i | i=1,2,\cdots,t\}$ be a linguistic term set with odd cardinality. Any label, s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics[15-22]:

(1) The set is ordered: $s_i > s_j$, if $i > j$; (2) Max operator: $\max(s_i, s_j) = s_i$, if $s_i \geq s_j$; (3) Min operator: $\min(s_i, s_j) = s_i$, if $s_i \leq s_j$. For example, S can be defined as

$$S=\{s_1 = extremely\ poor(EP), s_2 = very\ poor(VP), s_3 = poor(P), s_4 = medium(M),\\ s_5 = good(G), s_6 = very\ good(VG), s_7 = extremely\ good(EG)\}$$

Herrera and Martinez[15-17] developed the 2-tuple fuzzy linguistic representation model based on the concept of symbolic translation. It is used for representing the linguistic assessment information by means of a 2-tuple (s_i, α_i), where s_i is a linguistic label from predefined linguistic term set S and α_i is the value of symbolic translation, and $\alpha_i \in [-0.5, 0.5)$.

Definition 1[15-17]**.** Let β be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation, $\beta \in [1,t]$, being t the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [1,t]$ and $\alpha \in [-0.5, 0.5)$ then α is called a symbolic translation.

Definition 2[15-17]**.** Let $S=\{s_1, s_2, \cdots, s_t\}$ be a linguistic term set and $\beta \in [1,t]$ is a number value representing the aggregation result of linguistic symbolic. Then the function Δ used to obtain the 2-tuple linguistic information equivalent to β is defined as:

$$\Delta : [1,t] \rightarrow S \times [-0.5, 0.5) \tag{1}$$

$$\Delta(\beta) = \begin{cases} s_i, i = round(\beta) \\ \alpha = \beta - i, \alpha \in [-0.5, 0.5) \end{cases} \tag{2}$$

where round(.) is the usual round operation, s_i has the closest index label to β and α is the value of the symbolic translation.

Definition 3[15-17]**.** Let $S=\{s_1, s_2, \cdots, s_t\}$ be a linguistic term set and (s_i, α_i) be a 2-tuple. There is always a function Δ^{-1} can be defined, such that, from a 2-tuple (s_i, α_i) it return its equivalent numerical value $\beta \in [1,t] \subset R$, which is[8-13].

$$\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [1,t] \tag{3}$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \tag{4}$$

From Definitions 1 and 2, we can conclude that the conversion of a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation:

$$\Delta(s_i) = (s_i, 0) \tag{5}$$

Definition 4[15-17]**.** Let (s_k, a_k) and (s_l, a_l) be two 2-tuple, they should have the following properties.

(1)If $k < l$ then (s_k, a_k) is smaller than (s_l, a_l); (2) If $k = l$ then if $a_k = a_l$, then (s_k, a_k), (s_l, a_l) represents the same information; (b) if $a_k < a_l$ then (s_k, a_k) is smaller than (s_l, a_l); (c) if $a_k > a_l$ then (s_k, a_k) is bigger than (s_l, a_l).

Definition 5[15-17]**.** A 2-tuple negation operator.

$$neg(s_i, \alpha) = \Delta\left(t + 1 - \left(\Delta^{-1}(s_i, \alpha)\right)\right) \tag{6}$$

Where t is the cardinality of S, $S = \{s_1, s_2, \cdots, s_t\}$.

3. Induced 2-tuple linguistic choquet ordered averaging (I-2TCOA) operator

However, the above aggregation operators with linguistic information is based on the assumption that the attribute of decision makers are independent, which is characterized by an independence axiom[23], that is, these operators are based on the implicit assumption that attributes of decision makers are independent of one another; their effects are viewed as additive. For real decision making problems, there is always some degree of inter-dependent characteristics between attributes. Usually, there is interaction among attributes of decision makers. However, this assumption is too strong to match decision behaviours in the real world. The independence axiom generally can not be satisfied. Thus, it is necessary to consider this issue.

Let $m(x_j)(j = 1, 2, \cdots, n)$ be the weight of the elements $x_j \in X$ $(j = 1, 2, \cdots, n)$, where m is a fuzzy measure, defined as follows:

Definition 6 [23]. A fuzzy measure m on the set X is a set function $m: \theta(x) \to [0,1]$ satisfying the following axioms:

(1) $\mu(\phi) = 0$, $m(X) = 1$;

(2) $A \subseteq B$ implies $m(A) \le m(B)$, for all $A, B \subseteq X$;

(3) $m(A \cup B) = m(A) + m(B) + \rho m(A) m(B)$, for all $A, B \subseteq X$ and $A \cap B = \phi$, where $\rho \in (-1, \infty)$.

Especially, if $\rho = 0$, then the condition (3) reduces to the axiom of additive measure:

$m(A \cup B) = m(A) + m(B)$, for all $A, B \subseteq X$ and $A \cap B = \phi$.

If all the elements in X are independent, and we have

$$m(A) = \sum_{x_j \in A} m(\{x_j\}), \text{ for all } A \subseteq X.$$

Definition 7[24]. Let f be a positive real-valued function $f: X \to R^+$ and m be a fuzzy measure on X. The discrete Choquet integral of f with respective to m is defined by

$$C_m(f) = \sum_{j=1}^{n} f_{\sigma(j)} \left[m\left(A_{\sigma(j)}\right) - m\left(A_{\sigma(j-1)}\right) \right] \tag{7}$$

where $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, such that $f_{\sigma(j-1)} \ge f_{\sigma(j)}$ for all $j = 2, \cdots, n$, $A_{\sigma(k)} = \{x_{\sigma(j)} \mid j \le k\}$, for $k \ge 1$, and $A_{\sigma(0)} = \phi$.

It is seen that the discrete Choquet integral is a linear expression up to a reordering of the elements.

Definition 8[25-26]. Let f be a positive real-valued function on X and m be a fuzzy measure on X. The induced Choquet ordered averaging operator of dimension n is a function I-COA $:(R^+ \times R^+) \to R^+$, which is defined to aggregate the set of second argument of a list of 2-tuples $(\langle u_1, f_1\rangle, \langle u_2, f_2\rangle, \cdots, \langle u_n, f_n\rangle)$ according to the following expression:

$$\text{I-COA}_m(\langle u_1, f_1\rangle, \langle u_2, f_2\rangle, \cdots, \langle u_n, f_n\rangle) = \sum_{j=1}^{n} f_{\sigma(j)}\left[m(A_{\sigma(j)}) - m(A_{\sigma(j-1)})\right] \tag{8}$$

where $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, such that $u_{\sigma(i-1)} \geq u_{\sigma(i)}$ for all $j = 2, \cdots, n$, i.e., $\langle u_{\sigma(j)}, f_{\sigma(j)}\rangle$ is the 2-tuple with $u_{\sigma(j)}$ the jth largest value in the set $(u_1, u_2, \cdots, u_n)$, $A_{\sigma(k)} = \{x_{\sigma(j)} \mid j \leq k\}$, for $k \geq 1$, and $A_{\sigma(0)} = \phi$.

In the following, we shall introduce induced 2-tuple linguistic choquet ordered averaging (I-2TCOA) operator based on the Definition 4, I-COA operator.

Definition 9[27]. Let $X(x_1, x_2, \cdots, x_n)$ be a finite set, m be a fuzzy measure on X, and $(\langle u_1, (r_1, a_1)\rangle, \langle u_2, (r_2, a_2)\rangle, \cdots, \langle u_n, (r_n, a_n)\rangle)$ be a set of 2-tuple, An I-2TCOA operator of dimension n is a mapping I-2TCOA: $S^n \to S$, furthermore,

$$\begin{aligned} &\text{I-2TCOA}(\langle u_1, (r_1, a_1)\rangle, \langle u_2, (r_2, a_2)\rangle, \cdots, \langle u_n, (r_n, a_n)\rangle) \\ &= \Delta\left(\sum_{j=1}^{n}\left(m(A_{\sigma(j)}) - m(A_{\sigma(j-1)})\right)\Delta^{-1}(r_{\sigma(j)}, a_{\sigma(j)})\right) \end{aligned} \tag{9}$$

where u_j in 2-tuple $\langle u_j, (r_j, a_j)\rangle$ is referred to as the order-inducing variable and (r_j, a_j) as the argument variable, $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, such that $u_{\sigma(j-1)} \geq u_{\sigma(j)}$ for all $j = 2, \cdots, n$, $A_{(i)} = \{x_{(1)}, x_{(2)}, \cdots, x_{(i)}\}$ when $i \geq 1$ and $A_{\sigma(0)} = \phi$.

4. An approach to multiple attribute decision making with linguistic information

In this section, we shall utilize the developed operators to multiple attribute decision making. For a multiple attribute decision making problems with linguistic information, let $A = \{A_1, A_2, \cdots, A_m\}$ be a discrete set of alternatives, $G = \{G_1, G_2, \cdots, G_n\}$ be the set of attributes, whose weight vector is $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$, with $\omega_j \geq 0$, $j = 1, 2, \cdots, n$, $\sum_{j=1}^{n}\omega_j = 1$. Suppose that $R = (r_{ij})_{m\times n}$ is the multiple attribute decision making matrix, where $r_{ij} \in S$ is an attribute value, which take the form of linguistic variable, given by the decision maker for the alternative $A_i \in A$ with respect to the attribute $G_j \in G$.

In what follows, we shall apply the I-2TCOA operator to solve the MADM problems in which both the attribute weights are usually correlative, attribute values take the form of linguistic variables.

Step 1. Transforming linguistic decision matrix $R = (r_{ij})_{m\times n}$ into 2-tuple linguistic decision matrix $\tilde{R} = (r_{ij}, 0)_{m\times n}$

Step 2. We utilize the decision information given in matrix H, and the I-2TCOA operator

$$\tilde{r}_i = \text{I-2TCOA}_\lambda\left(\left\langle u_{ij},\left(r_{ij},a_{ij}\right)\right\rangle,\left\langle u_{ij},\left(r_{ij},a_{ij}\right)\right\rangle,\cdots,\left\langle u_{ij},\left(r_{ij},a_{ij}\right)\right\rangle\right)$$
$$=\Delta\left(\sum_{j=1}^{n}\left(m\left(A_{\sigma(j)}\right)-m\left(A_{\sigma(j-1)}\right)\right)\Delta^{-1}\left(r_{\sigma(ij)},a_{\sigma(ij)}\right)\right)$$

to derive the overall preference values $\tilde{r}_i\left(i=1,2,\cdots,m\right)$ of the alternative A_i.

Step 3. Rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ in accordance with the overall 2-tuple linguistic preference values $\tilde{r}_i\left(i=1,2,\cdots,m\right)$ of the alternative A_i. The larger the overall preference values $\tilde{r}_i$, the better the alternative A_i will be.

5. Numerical example

The term―brand‖came from America. In the early20th century,"brand" has been moreapplied to sales. Since1930s,"brand" has began been widely used in academia, media andmarketing.In recent years, with the more incentive market competition and rising advertisingcosts, the introduction of new products has been under lots of risk. Between1970and1980,when companies introducing new products, there is only20%got success. The 30%-35% failure was due to not being accepted by consumers and the high cost of initial marketintroduction. In Western countries, brand extension has been a generic way to introduce newproducts. According to statistics, between1977and1984,in Western countries, there isapproximately40%between every120-175new brands being imported to supermarketachieved through brand extension. Thus, the brand extension has been a general trend.As you can see, in the field of physical product, Haier company constantly develop newproducts, Toyota company constantly develop a series of cars in different price, and so on. Inlate1970s, Tuber systematically presented the theoretical issues of the brand extension firstly.In the next eras, scholars have done a lot of research around the brand extension and Aakerand Keller made the greatest impact. However, you can find a lot of scholars have been doingresearch about the physical product brand extension. There is few researchs about servicebrands. With the development of society, the proportion of tertiary industry has accounted forlarger and larger. Service brands, such as hotels, are also engaged in extending and theyrequire theoretical guidance. In this section, we present an empirical case study of evaluating the service brand extension with 2-tuple linguistic information. The service brand extension quality of five possible enterprises $A_i\left(i=1,2,3,4,5\right)$ is evaluated. Assume that an expert group newly identified an investment with service brand extension quality, and in order to maximize the expected profit, we need to determine the service brand extension quality of the five enterprises so as to choose the optimal one. The expert group must take a decision according to the following four attributes: (1)G_1 is the brand asset value; (2)G_2 is the relation between core brand and extensive product; (3)G_3 is the internal environment factors; (4)G_4 is the external environment factors. The five possible enterprises $A_i\left(i=1,2,\cdots,5\right)$ are to be evaluated using the linguistic term set

$$S=\{s_1=extremely\ poor,s_2=very\ poor,s_3=poor,s_4=medium,$$
$$s_5=good,s_6=very\ good,s_7=extremely\ good\}$$

by the decision makers under the above four attributes, and construct, respectively, the decision matrices as follows $R=\left(r_{ij}\right)_{5\times4}$. The linguistic decision matrix is shown in Table 1.

Table 1 Decision matrix *R*

	G1	G2	G3	G4
A1	VP	M	EG	VP
A2	M	VP	P	VP
A3	VP	VP	P	M
A4	G	G	VP	EP

A5	VG	EG	VG	G

Then, we utilize the proposed procedure to get the most desirable enterprise(s).

Step 1. Suppose the fuzzy measure of attribute of $G_j\left(j=1,2,\cdots,n\right)$ and attribute sets of G as follows:

$$\mu\left(G_1\right)=0.30, \mu\left(G_2\right)=0.35, \mu\left(G_3\right)=0.30, \mu\left(G_4\right)=0.22, \mu\left(G_1,G_2\right)=0.70,$$
$$\mu\left(G_1,G_3\right)=0.60, \mu\left(G_1,G_4\right)=0.55, \mu\left(G_2,G_3\right)=0.50, \mu\left(G_2,G_4\right)=0.45,$$
$$\mu\left(G_3,G_4\right)=0.40, \mu\left(G_1,G_2,G_3\right)=0.82, \mu\left(G_1,G_2,G_4\right)=0.87,$$
$$\mu\left(G_1,G_3,G_4\right)=0.75, \mu\left(G_2,G_3,G_4\right)=0.60, \mu\left(G_1,G_2,G_3,G_4\right)=1.00$$

Step 2. The experts use order-inducing variables to represent the complex attitudinal character involving the opinion of different members of the board of directors[28]. The results are shown in Table 2.

Table 2. Inducing variables

	G1	G2	G3	G4
A1	23	16	26	27
A2	24	18	21	16
A3	18	16	15	12
A4	14	23	22	25
A5	22	15	20	16

Step 3. Transforming linguistic decision matrix $R=\left(r_{ij}\right)_{m\times n}$ into 2-tuple linguistic decision matrix $\tilde{R}=\left(r_{ij},0\right)_{m\times n}$. We utilize the decision information given in matrix $\tilde{R}=\left(r_{ij},0\right)_{m\times n}$, and the I-2TCOA operator to obtain the overall preference values $\tilde{r}_i$ of the enterprises $A_i\left(i=1,2,3,4,5\right)$. The results are shown in Table 3.

Table 3. The overall preference values for the alternative by utilizing the IG-2TCOA operator

	A1	A2	A3	A4	A5
I-2TCOA	(VP, -0,32)	(VP, -0,38)	(VP, -0,46)	(EP, 0,49)	(VP, -0,35)

Step 4. Rank all the enterprises $A_i\left(i=1,2,3,4,5\right)$ in accordance with the overall preference values $\tilde{r}_i$ of the enterprises $A_i\left(i=1,2,3,4,5\right)$. The ordering of the enterprises is shown in Table 4. Note that > means "preferred to". As we can see from Table 4, the investor can properly select the desirable enterprise according to his interest and the actual needs. Therefore, the best enterprises is A_4.

Table 4 . Ordering of the alternative by utilizing the I-2TCOA operator

	Ordering
I-2TCOA	A4 > A3 > A2 > A1 > A5

6. Conclusion

In this paper, we investigate the multiple attribute decision making problems for evaluating service Brand extension in which the attribute weights are usually correlative, attribute values take the form of linguistic variables. Firstly, some operational laws of linguistic variables are introduced. Then, an I-2TCOA operators-based approach is developed to solve the MADM problems for evaluating service Brand extension with linguistic variables. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness. In the future, we shall continue working in the extension and application of the developed operators to other domains.

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