# **New Space Junk Removal Program**

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### Abstract

The increasing growth of space junk in the earth orbit has caused concern form various countries' national space agencies. Currently, there has been more than 4500 tons of space junk in the Earth's orbit. If those junks are not cleared, they will bring great threat to the security of space craft. Aiming at the lowest cost, by using space junk processors released by satellite carriers to remove space junk, we establish a satellite carrier switching orbit model based on the differential systems with Markovian Switching. More precisely, by observing whether the space junk within the each region range of satellite carrier scanning has processing value, we divide orbit switching strategy into two cases: one case is a differential system with Markovian Switching based on the probability transition matrix; the other case is the artificial switching system. To optimize the energy thrust, this paper designs the optimal index function for the optimum feedback controllers. Furthermore, we introduce some matrix to describe some stochastic failure of thruster, and give some appropriate sufficient conditions to ensure the stochastic stability of switching orbit system. Finally, we do the cost-benefit analysis, and further prove the feasibility of our technology.

## **Keywords**

satellite carrier; Markovian Switching; stochastic failure

## **1.** Introduction

Space junks are various man-made objects that do not have any purpose on orbit around the Earth running. With the expansion of the scope of human activities, the space is no longer restricted zones for humans, but also brought the space junk problems. As the number of space junks increasing annually, space junk problems began growing concern. Because of space junks running as orbit speed, if their collisions can severely damage functioning spacecraft, and even pose a threat to the safety of the astronauts during extravehicular activities.[1] Space junk is produced by artificial spread, the satellite and the rocket scrapped, equipment lost and anti-satellite weapons. [2] To avoid space junk with the spacecraft collision, reducing the probability of collision, cleaning up space junk problem is urgent.

### 2. Orbit Satellite Carrier Model Based on Markov Jump System

#### **2.1 Markovian Process**

Considering the random process  $\gamma(t)$ , and assuming  $t_0 < t_1 < \cdots < t_k$ , If its condition satisfies the probability density function

$$f\left(\gamma(t_{k+1})\right)\left|\gamma(t_{k}),\gamma(t_{k-1}),\cdots,\gamma(t_{0})=f\left(\gamma(t_{k+1})\right)\right|\gamma(t_{k})$$
(1)

It called  $\gamma(t)$  is Markov process. [4]

The above description of the definition, the Probability density function of  $\gamma(t_{k+1})$  only is related to  $\gamma(t_k)$ , and has nothing to do with  $\gamma(t_{k-1}), \dots, \gamma(t_0)$ , It indicates the Markov process sequence of no aftereffect, means that only affect the future now, and the past has no effect on the future, or the past affects the future only by today.

#### 2.2 Markovian Switching System

The more important one in the hybrid system is the linear switched systems or linear jump system.

$$x(t) = A(r(t))x(t) + B(r(t))u$$
(2)

Among  $r:[0,\infty) \rightarrow S = \{1, 2, \dots N\}$  represent switching signal. When the system is running, the system will jump from one mode to another in the switching signal. In general, the role of switching signal to the system will show some kind of rule, called switching rate. [5]

Here, r(t) switching is controlled by the combination of artificial control and system random.

Among them, the switching rate can be controlled, and the switching process can be controlled artificially. Random switching rate is defined as the switching mode when the system is in line with the corresponding conditions.

#### 2.3 Introduction of Random Failure Factors Thruster

Random failure of thruster has the characteristics of Markov the above relative motion equation can be rewritten in the form of the following random Markov hopping system

$$\begin{cases} \xi(t) = A\xi(t) + B(\gamma(t))u(t) \\ y(t) = C\xi(t) \end{cases}$$
(3)

Among them  $\{\gamma(t), t \ge 0\}$  is a stochastic Markov process that represents the change of system model. The stochastic process is taken from Finite countable set  $S = \{1, 2, \dots, N\}$ . S As the state space of the stochastic process, it represents the set of possible states of the system.  $B(\gamma(t))$  is known Input matrix set, which represents a variety of system actuator failures. System mode's Transition probability is:

$$\Pr\left\{\gamma(t+h) = j \middle| \gamma(t) = i\right\} = \begin{cases} \lambda_{ij}h + o(h), i \neq j\\ 1 + \lambda_{ij}h + o(h), i = j \end{cases}$$
(4)

Among them h > 0,  $\lim_{h \to 0^+} \frac{o(h)}{h} = 0$ , Diversion Probability Matrix  $\Lambda$  expressed as

$$\Lambda = \begin{bmatrix} \lambda_1 & \cdots & \lambda_{1N} \\ \vdots & \ddots & \vdots \\ \lambda_{N1} & \cdots & \lambda_{NN} \end{bmatrix}$$
(5)

Among them,  $\lambda_{ij}$  represents the transition from modality i to modality j (Transfer rate), and when  $i \neq j$ , that  $\lambda_{ij} \ge 0$ ,  $\lambda_{ij} = -\sum_{j=1, i \neq j}^{N} \lambda_{ij}$ .

Transtion-probablity Matrix Settings

To optimize the satellite carrier for tracking large garbage removal process, and enable satellite carriers to complete satellite carrier with a large space junk (relative hover Relative hover) work under the premise of saving fuel, we adopt the artificial provisions combination method which make transition probability matrix Transfer Probability Matrix and random transition probability matrix Random transition probability matrix combination. Specifically, the following:

The Satellite carrier finds mass greater than 500kg of space junk within the range, [3] which is a radius of 60km itself as the center of the space. Each track is set at a 15km, then there are a total of 9

tracks, which make this space divide into 10 regions, starting from near the center of the earth where the label  $1, 2, \dots, 10$ . Since the outside space of the track 1 and 9 will be more than the detection range of the satellite carrier, we just consider the 1 to 9 zone. And signing every district has  $(n \ge 0)$  space junk of meeting the requirements. Then we determine the parameter *m* for the quality of space junk, *l* for distance between the spacecraft and space junk, *s* for Radar cross section.

First, we define  $Q_i$  for the  $i(1 \le i \le 8, i \in N)$  area space junk's processing value, then

$$Q_i = \frac{ms}{l} \tag{6}$$

Then, Satellite carriers within the scan area of space junk the total value  $Q_m$ 

$$Q_m = \sum_{0}^{n} \frac{m_n s_n}{l_n} (n \ge 0) \tag{7}$$

Making the  $j(1 \le j \le 8, j \in N)$  track of satellite area locating currently

Finally, we determine whether space junk should be dealt with in this area. The different results of discrimination as well as corresponding programs measures below:

(1) If there is space junk of compliance with the requirements in 1 to 10 regions, Let

$$\lambda_{ij} = Q_i / Q_m \tag{8}$$

 $\lambda_{ij}, i(1 \le i \le 8, i \in N)$  Corresponding weights of space garbage value in the *i* orbit which was measured in the *j* orbit. Then the transition probability matrix:

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{18} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{28} \\ \vdots & \vdots & \cdots & \vdots \\ \lambda_{101} & \lambda_{102} & \cdots & \lambda_{88} \end{bmatrix}$$
(9)

When space junk of an orbit are cleaned up, the spacecraft evaluated changing orbit behavior to determine the next changing orbit behavior.



Fig. 1 Transfer system

(2) If there is no compliance with the requirements of space junk in the area of 1 to 8. Then we set spacecraft countermeasure behavior was only 3 kinds, that is flying to the 1 orbit, the original orbit continues to fly and fly to the 9 orbit, and transfer Probability Matrix of the  $[1/3 \quad 1/3]$ , meaning

for the spacecraft to fly to the 1 orbit, the original orbit continues to fly with a probability of 9 tracks each for 1/3. We fly to the 1 or 9 track process does not detect and transfer, because if the spacecraft orbit to orbit approaching again after monitoring and will monitor to track 1 or 9 tracks outside the region are in line with the requirements of space junk to spacecraft will choose to fly to orbit 1 or 9 tracks.

Therefore, there are three countermeasures of optimal behavior.

In order to facilitate understanding, we have drawn into a flow chart based on the above approach, as shown in Fig. 1:

### 3. Cost Benefit Analysis

In this project, we use the design of a satellite carrier with a plurality of space garbage processors to eliminate the space junk.

1. The number of major cost for x1 each satellite launch scientific research costx2, cost of space garbage processor  $x_2$ , independent space garbage processor costx3, cost of satellite carrier and foundation launch costsx4, and increase a space garbage processor launch costs increasex5, the other costsx6,

Among them, the satellite carrier cost

$$X_1 = x_1 + nx_2 + x_3 + x_4 + nx_5 + x_6$$

If used in space junk processing functions of a single spacecraft costs

$$X_2 = x_1 + nx_2 + nx_4 + x_6$$

Is the satellite carrier and single spacecraft scheme in this paper are compared, and the cost can decrease

$$\Delta X = n(x_2 - x_2) - x_3 + (n - 1)x_4 - nx_5 \tag{10}$$

2.For the problem of the number of space garbage processor, the more we know space garbage processor, relative cost is lower, but must be satisfied

$$M \leq m_1 + m_2 + nm_3$$

In the formula, M is the maximum load of the rocket,  $m_1$  is the fuel quality,  $m_2$  is the quality of the satellite carrier,  $m_3$  is the quality of space garbage processor. And space garbage processor for folding of main components of cable and rope nets, not only can save space, and quality is lighter, provides the more load may, therefore, this scheme is superior to other solutions.

3.In this scenario, orbit satellite carrier process, consider the random failure of thruster, make their own system safe and stable operation for a long time.

4. This scheme mainly for larger and threatening the first satellite and other space junk. Because, given the large space debris to spread out more small garbage, select a larger space junk to eliminate will get higher relative gains.

5.Combined with literature [6] study respectively the bare cable length and bare cable diameter for the tension, the effect of the result is shown in Fig. 2.

Showed that the longer the cable length, diameter, the greater the exposed, the greater the tension, but bare cable length and the diameter of the increase will be a big system, launch costs increase, at the same time, the bare cable length increase will reduce the security system, increase the system risk. To weigh the bare cable length and the diameter of the selection, this article will bare cable tensile

and bare cable quality ratio is defined as the efficiency, bare cable parameters on its influence is shown in Fig. 3





Fig. 3 Bare cable parameters on its influence

Can be seen from the diagram, pull efficiency increases with bare cable length increases, and decreases with the bare cable diameter. Considering the cost and effect factors determine the length of 15 km bare cable diameter of 0.5 mm. According to bare cable length to determine the most nearly every time hovering orbit satellite carrier.

As a result, this scheme has a certain economic attraction.

## 4. Conclusions

According to the satellite carrier transfer process, we use satellite carrier Markov hopping system based on orbit model. In the model, in order to determine the transtion-probablity matrix  $\Lambda$ , transtionprobablity matrix and the use of a combination of the method of random transtion-probablity matrix; In order to achieve the energy consumption in the process of orbit satellite carrier, thrust optimization, we set the index function optimization optimum feedback controller; considering propulsion system stochastic breakdowns, We introduce the matrix group  $\{F_i\}$  describes the thruster fault conditions,

to ensure the stability of emergency transfer system. Finally, we present a cost-benefit analysis of the program, it also proves the feasibility of the space junk processing method, whereby this program has great commercial value.

## Acknowledgements

Natural Science Foundation.

# References

- [1]Liu Huan, Zhong Yong. Design of fly-around obit for space debris capturing[J]. Journal of Deep Space Exploration , 2015, 2(4): 376-378.
- [2]Yuuki Ishige, Satomi Kawamoto, Seishiro Kibe. Study on electro dynamic tether system for space debris removal[J], Acta Astronautica, 2004, 55: 917-929.
- [3]GanLiang et al.Satellite communication system[M].WuHang:Wuhan University press, 2001,12.
- [4]Gajic Z, Losada R. Monotonicity of algebraic Lyapunov iterations for optimal control of jump parameter linear systems[J]. Systems & Control Letters, 2000, 41(3):175-181.
- [5]Cui Zhenmao.Model reference control method for networked control systems based on jumping system theory [D]. Harbin: Harbin Institute of Technology,2015.
- [6]Su Hang, Cao Xibin, Zhang Shijie et al. Performance analysis for bare anode electro dynamic tether deorbit system[C] Yantai, Shandong, China, Research Center of Satellite Technology HIT,2015:398-402