Space Junk Optimal Control System Study

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Abstract

Space junk refers to what the human abandoned in the space in the space activity, namely the space trash. Therefore, how to clean up space junk, and how to reduce the hazards to the greatest extent, are two widely concerned problems by the international media. More precisely speaking, the problem is considered to be divided into the three stages of orbit determination, optimal control capture and orbit dynamic control, and all stages are modeled and simulated to validate the correctness of models. Taking space junk in low orbit as an example, this paper uses the motion law (including quality, size, location and other relevant data) of the space junk in low orbit (affected by air resistance) to establish the space junk orbit model. To minimize costs and risks, by changing the driving force, we establish the "Recycle Bin" orbit dynamic model, and give the shortest time performance index of the orbit control model. The Gauss pseudo spectral method is introduced to solve the optimal rendezvous points of the capture system, and the effectiveness of the method is verified.

Keywords

Space Junk; Simulation Experiment; Orbit Dynamic; Gauss Pseudo Spectral Method

1. Introduction

The number of the impact of the formation of space debris gradually increased because of human frequent space activities, the failure of the spacecraft, explosion, falling off. Space junk debris to a certain extent, has become the cause of the space shuttle in the future. It is a serious threat to the spacecraft that is about to fly into space. According to the relevant reports, it may occur a collision between space junk and spacecraft every 5-10 years. It is expected that it will take place once every two years.[1] Space junk will cause irreparable consequences even without a strong collision with the satellite. Therefore, the treatment of space debris has become a critical task.

2. The Establisnment of The Model

"Recycle Bin" the research on the issues of quickness, this article will transform it into a problem of the time, and made a Simulation picture for the coplanar orbit.

Before establish the orbit model of the "Recycle Bin", the author given the earth orbit dynamics model based on the spherical Perturbation [1]:

Among them: $J_2 = 1.08264\times10^{-3}$, Earth perturbation;

u : Geocentric gravitational constant;

R : Radius Earth;

$r = \sqrt{(x^2 + y^2 + z^2)}$: The geocentric distance of "Recycle Bin"

Based on the research, without considering the consumption of fuel, take the Orbit dynamics model [6-7] in to consideration, and set the state variable as: $x=\left[x,y,z,v_x,v_y,v_z\right]^T \in R^6$

$$\begin{cases} \dot{x}=v_x \\ \dot{y}=v_y \\ \dot{z}=v_z \\ \dot{v}_x=-\dfrac{ux}{r^3}\left[1-J_2\dfrac{3}{2}(\dfrac{R_e}{r})^2(5\dfrac{z^2}{r^2}-1)\right]+f_x \\ \dot{v}_y=-\dfrac{uy}{r^3}\left[1-J_2\dfrac{3}{2}(\dfrac{R_e}{r})^2(5\dfrac{z^2}{r^2}-1)\right]+f_y \\ \dot{v}_z=-\dfrac{uz}{r^3}\left[1-J_2\dfrac{3}{2}(\dfrac{R_e}{r})^2(5\dfrac{z^2}{r^2}-1)\right]+f_z \end{cases} \tag{1}$$

The initial state: $x_0=\left[r_0^T,v_0^T\right], x_f=\left[r_f^T,v_f^T\right]$

Covariate: $\lambda=\left[\lambda_x,\lambda_y,\lambda_z,\lambda_{vx},\lambda_{vy},\lambda_{vz}\right]^T \in R^6$ (λ:geocentric longitude)

According to the covariate, we can get the Hamiltonian function:

$$H(\lambda,r,f,t)=1+\lambda_x v_x+\lambda_y v_y+\lambda_z v_z+\lambda_{vx}\left[\frac{ux}{r^3}\left[1-J_2\frac{3}{2}(\frac{R_e}{r})^2(5\frac{z^2}{r^2}-1)\right]+f_x\right]$$
$$+\lambda_{vy}\left[\frac{uy}{r^3}\left[1-J_2\frac{3}{2}(\frac{R_e}{r})^2(5\frac{z^2}{r^2}-1)\right]+f_y\right]+\lambda_{vz}\left[\frac{uz}{r^3}\left[1-J_2\frac{3}{2}(\frac{R_e}{r})^2(5\frac{z^2}{r^2}-1)\right]+f_z\right]$$

The angular momentum based on Reference point O_s in the system： $H_s=\sum_{i=1}^{N} r_{si}\times m_i v_i$

Then, to derivative and summary the state variable, we can get this equation:
$$\dot{\lambda}_{vx}=-\frac{\partial H}{\partial x}=-\lambda_x$$
$$\dot{\lambda}_{vy}=-\frac{\partial H}{\partial y}=-\lambda_y$$
$$\dot{\lambda}_{vz}=-\frac{\partial H}{\partial z}=-\lambda_z$$

Through the observation of the state equation and the association equation, using the direct method [2], Applicant the Gauss Pseudo spectral Method-GPM [3-4] and will orbit model of the discrete optimal control problem to solve for NLP.

The mapping function:

$$\tau=-1+\frac{2(t-t_0)}{t_f-t_0} \tag{2}$$

At first, we will turn $t\in\left[t_0,t_f\right]$ to the scale between $\left[-1,1\right]$, make $t_0=0$ and simplify the function: $\tau=-1+\dfrac{2t}{t_f}$

Then in interval by N LG discrimination points, (3), (4),(5) for the orbit model of power constraints and control constraints at the end of the early, as shown below:

At the end of the early constraint conditions:

$$\begin{cases} x(t_f) = x(t_0) + \dfrac{t_f}{2}\sum_{k=1}^{N} w_k v_x(k) \\ y(t_f) = y(t_0) + \dfrac{t_f}{2}\sum_{k=1}^{N} w_k v_y(k) \\ z(t_f) = z(t_0) + \dfrac{t_f}{2}\sum_{k=1}^{N} w_k v_z(k) \end{cases} \tag{3}$$

$$\begin{cases} v_x(t_f) = v_x(t_0) + \dfrac{t_f}{2}\sum_{k=1}^{N} w_k \left\{ -\dfrac{ux(k)}{r(k)^3}\left[1 - J_2\dfrac{3}{2}(\dfrac{R_e}{r(k)})^2(5\dfrac{z(k)^2}{r(k)^2} - 1)\right] + f_x(k) \right\} \\ v_y(t_f) = v_y(t_0) + \dfrac{t_f}{2}\sum_{k=1}^{N} w_k \left\{ -\dfrac{uy(k)}{r(k)^3}\left[1 - J_2\dfrac{3}{2}(\dfrac{R_e}{r(k)})^2(5\dfrac{z(k)^2}{r(k)^2} - 1)\right] + f_y(k) \right\} \\ v_z(t_f) = v_z(t_0) + \dfrac{t_f}{2}\sum_{k=1}^{N} w_k \left\{ -\dfrac{uz(k)}{r(k)^3}\left[1 - J_2\dfrac{3}{2}(\dfrac{R_e}{r(k)})^2(5\dfrac{z(k)^2}{r(k)^2} - 1)\right] + f_z(k) \right\} \end{cases} \tag{4}$$

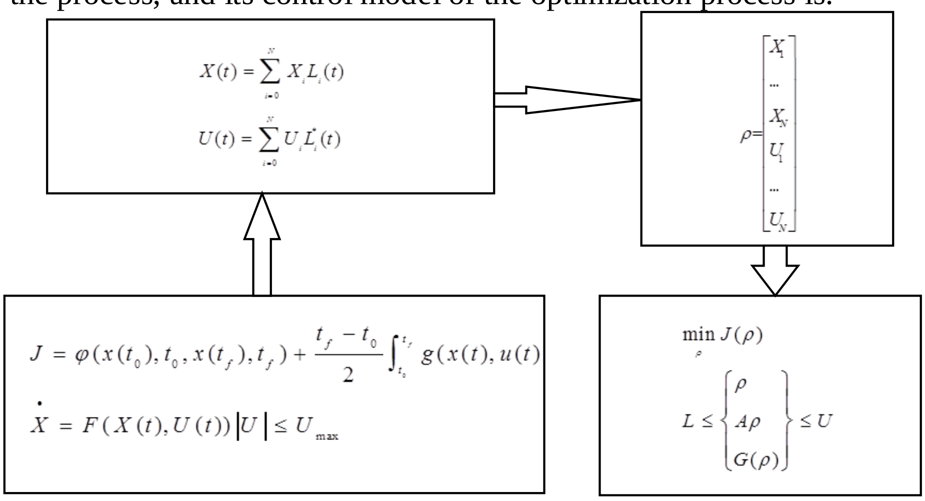
Among them: ω_k Stand for the Gauss integral weight.

The condition of the constraint power:

$$\begin{aligned} -0.04 \le f_x(k) \le 0.04 \\ -0.04 \le f_y(k) \le 0.04 \\ -0.04 \le f_z(k) \le 0.04 \end{aligned} \tag{5}$$

Select its shortest time performance indicators of the model: $J = t_f$

Then simplify the process, and its control model of the optimization process is:



3. Simulations of Statistics

The results of Simulation is based on the MATLAB,using relevant software packages for solving NLP, first selected LG point, this article take 50, the Simulation experiment.

For coplanar orbit, this paper selected the data shown in the following table;

Table1 the parameters of the recycle Bin (coplanar orbital maneuver)

"Recycle Bin" position	orbital semi major axis	Eccentricity	orbital dip angle	the Ascending Node	argument of periapsis	Trueanomaly
Before the transfer orbit parameters	$8000km$	0	28.5°	100°	0°	
After the transfer orbit parameters	$9000km$	0	28.5°	100°	0°	170°

By Simulation experiment, we can get the results shown in the following picture:

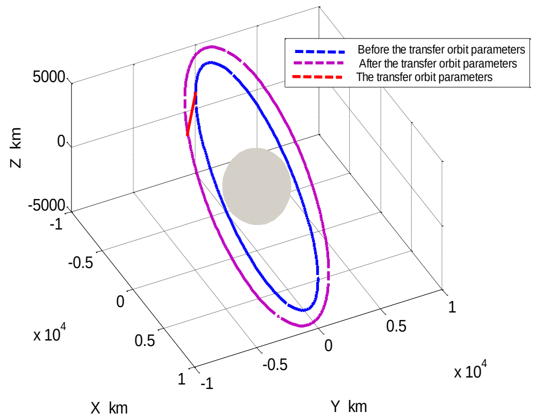


Figure1 Optimization locus of to"Recycle Bin".

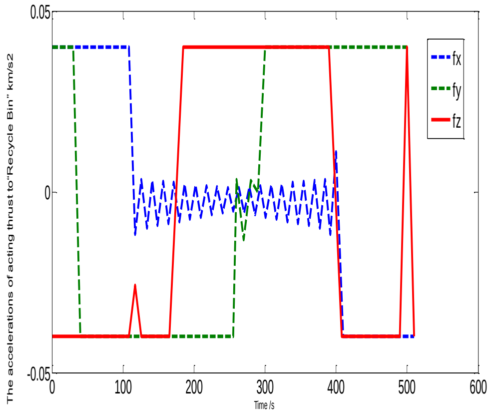


Figure2 Accelerations error of acting thrust "Recycle Bin"the transfer orbit parameters.

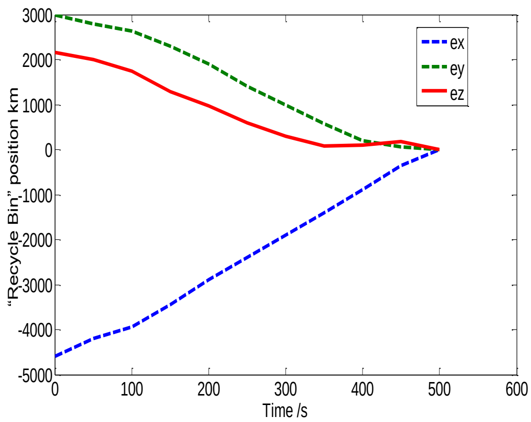


Figure3 "Recycle Bin" position error

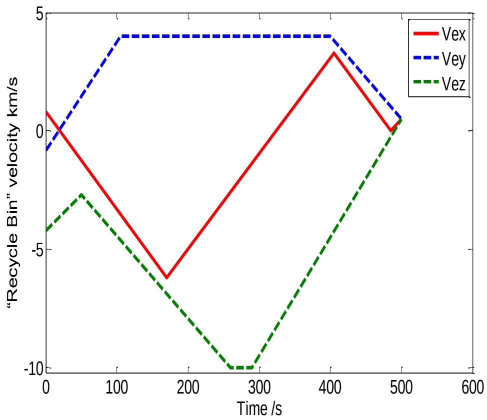


Figure4 "Recycle Bin" velocity error

By Using Gauss, the fastest time of the coplane orbit is: $t_f = 511.4595s$

Thought the result of the simulation, it comes true the optimum time in the recycle bin control system, the control system model is based on the formula that earth centered inertia system, and consider the perturbation that have major influence to recycle bin orbit. So the formula is universal.

4. Tethered Capture Systems

Establishing the inertia coordinate system, supposing that mass is far outweigh the catcher's mass in recycle bin, and establishing the orbital coordinate system that as the recycle bin is center.

E_2, E_3 make periodic motion, Track parameter is orbital dip angle: i, the Ascending Node Ω,The Angular distance of near location: M true anomaly θ.The tether is simplified as Elastic rod, regardless of quality, whose rigidity is EA. and its Side interior angle is ϑ, Side exterior angle ϕ ,and the present length is l .

Write control input u_1 as two order derivative strain on time, That is $u_1=\varepsilon''$, besides Elastic tether's strain is $\varepsilon=(l-l_0)/l_0$.

Impose $x=\{\vartheta,\phi,\xi,\varepsilon,\vartheta',\phi',\xi',\varepsilon',\}\in R^n$, $\xi=\dfrac{l_0}{L_f}$ is non-dimensional length of Tether. L_f is the unreformed tether length being released in all.

Non dimensional state equation which satisfies the system [5] is :

$$\begin{cases} x_1'=x_5, x_2'=x_6, x_3'=x_7, x_4'=x_8 \\ x_5'=-2(x_5-1)\left(\dfrac{x_7}{x_3}-x_6\tan x_2+\dfrac{x_8}{1+x_4}\right)-\dfrac{3}{2}\sin 2x \\ x_6'=-2x_6\left(\dfrac{x_7}{x_3}+\dfrac{x_8}{1+x_4}\right)-\dfrac{1}{2}\sin 2x_2\left[(x_5-1)^2+3\cos^2 x_1\right]+\dfrac{u_2}{x_3(1+x_4)} \\ x_7'=-\dfrac{2x_7x_8+x_3u_1}{1+x_4}+x_3\left[(x_5-1)^2\cos^2 x_2+x_6^2+3\cos^2 x_1\cos^2 x_2-1\right]-\dfrac{EAx_4}{m_2\Omega^2 L_f(1+x_4)} \\ x_8'=u_1 \end{cases}$$

Among them :the " ' " means the derivatives of true anomaly; $\Omega=\sqrt{\mu/R_s^3}$

μ is the gravitational constant, and $\mu=3.986005\times10^5\ km^3/s^2$;

R_s is "Recycle Bin" the distance of Centroid to the center of the earth;

$u_2=F/m_2\Omega^2L_f$ is the control force of dimensional surface;

$T=EA\varepsilon$ is the tether tension;

State equation which contains the control variable says:

$$X'=f\left(x(v),u(v)\right) \tag{6}$$

Among them, $u(v)=\{u_1,u_2\}\in R^m$ is Control vector.

5. Confirm the Junction Point

Tethered capture system and captured garbage have different orbits, when the capture device in the end of rope catches the garbage that is the junction point [6].

For General, major axis is a_m, orbital eccentricity is , in orbital coordinate system, medium particle vector and velocity vector is follow.

$$r_m=-\frac{a_m(1-e_m^2)}{1+e_m\cos v_m}e_k\quad v_m=\sqrt{\frac{\mu}{a_m(1-e_m^2)}}e_m\sin v_m e_i-\sqrt{\frac{\mu}{a_m(1-e_m^2)}}(1+e_m\cos v_m)e_k \tag{7}$$

Above mention, m is form recycle bin. ($e_s=0$)

In orbital coordinate system, the capture device in the end of rope, its position vector:

$r = l\sin\theta\cos\phi e_i - l\sin\phi e_j + l\cos\theta\cos\phi e_k$

In the coordinate system, (7) it can be:

$$\Lambda = \frac{\sqrt{(r_{rel}\bullet e_i)^2 + (r_{rel}\bullet e_j)^2 + (r_{rel}\bullet e_k)^2}}{L_f}$$

$$\theta = a\tan 2\left(r_{rel}\bullet e_i r_{rel}\bullet e_k\right)$$

$$\phi = -\sin^{-1}\left(\frac{r_{rel}\bullet e_j}{\Lambda L_f}\right)$$

In this formula, Λ is the rope which is out of shape dimensionless length.

According to junction condition, the speed of catcher and garbage is the same, according to the hypothesis; the speed of orbit angular velocity in the system is that is the speed of orbit angular velocity in the recycle bin. When they are junction, the relative radial speed of the catcher and the garbage is zero, that is, Based on the actual deformation of a tether cannot be unlimited, thus the rope dimensionless length is , above all and the hypothesis junction, are zero, thus finding the accessible junction point.

6. Conclusions

Taking space junk in low orbit as an example, this paper uses the motion law (including quality, size, location and other relevant data) of the space junk in low orbit (affected by air resistance) to establish the space junk orbit model.

To minimize costs and risks, by changing the driving force, we establish the "Recycle Bin" orbit dynamic model, and give the shortest time performance index of the orbit control model. The Gauss pseudo spectral method is introduced to solve the optimal rendezvous points of the capture system, and the effectiveness of the method is verified.

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