

A Permanent Magnet Synchronous Motor Position Estimation Algorithm Based on Improved MRAS

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Abstract

Permanent magnet synchronous motor is widely used in the field of motor. In the sensorless vector control system of permanent magnet synchronous motor, it is necessary to accurately detect the rotor position and speed. Motor parameters such as inductance, resistance and so on will change with the operation of the motor, will cause adverse effects on the detection of rotor position. In order to improve the accuracy of detection of rotor position, a model reference adaptive algorithm based on parameter identification is designed. Firstly, the parameters of the motor are identified online by Forgetting Factor Recursive Least-squares Algorithm, and then the result are fed back to the system. Finally, this algorithms was proved with simulink Simulation results show that this algorithms can overcome the adverse effect of the change of motor parameters and it has superior capability.

Keywords

permanent magnet synchronous motor; model reference adaptive system; forgetting factor recursive least-squares algorithm; sensorless; motor parameters.

1. Introduction

The growing popularity of permanent magnet materials and the constant progress of electronic technologies nowadays have led to the favorable application prospect of permanent magnet synchronous motors. With the continuous improvement of the performance of permanent magnet materials, PMSM control technology has become increasingly perfect and PMSM has been widely used in fields such as robot hands, CNC machine tools and aerospace^[1].

In the permanent magnet synchronous motor control system, a vital part is the estimation of the rotor position and speed. In the beginning of the development of permanent magnet synchronous motors, a general way for rotor position estimation is the use of position sensors. But in the subsequent application process, shortcomings of the position sensor gradually appear: 1 position sensors have high environmental requirements, and the application occasion is limited; 2 motor volume increases; 3 a wire port is increased, which makes the system more likely to be interfered, so the reliability is reduced; 4 the cost is increased^[2]. For these reasons, sensorless control came into being. Most commonly used methods for detecting the rotor position currently are the direct calculation method, high-frequency injection method^[3], various observation methods^[4-5], and model reference adaptive methods^[6]. In this paper, the model reference adaptive method is used, which contains three elements: a reference model, an adjustable model, and an adaptive mechanism. The main idea of the model reference adaptive method is: if the equation containing parameters to be estimated is called the adjustable model, the equation without unknown parameters is called the reference model, and both models have the same physical meaning of outputs, then the parameters of the adjustable model can be regulated through the adequate adaptive law of the output error when the two models work at the same time, thus reaching the purpose of control object output tracking reference model^[7]. However, because motor parameters (stator resistance, inductance) in the running process will vary with the motor running, the MRAS estimation for the rotor position will be influenced, which is likely to affect the performance of the entire system. Therefore, the forgetting factor recursive least squares method is used for online identification of motor parameters, and the identified values will return to MRAS, so as to improve the estimation accuracy of the rotor position and speed^[8].

2. The mathematical model of PMSM

PMSM belongs to the AC motor category, so the intuitive and simple mathematic model like DC motors cannot be established when analysis. In the PMSM study, the appropriate coordinate system is used, and the motor is abstracted to the DC motor for analysis, which is a common method applied nowadays. For a common PMSM coordinate system, it includes the stationary three-phase coordinate system, the stationary $\alpha\beta$ coordinate system, and the rotating dq coordinate system^[9]. To facilitate the follow-up study, a PMSM mathematical model is established based on the rotating dq coordinate system by this article.

To obtain a simplified PMSM mathematical model, the following assumptions have to be made^[10]:

- (1) Ignore the adverse effects of armature reaction and alveolar, and neglect the higher harmonics in the air gap;
- (2) Believe that the magnetic circuit is linear, and ignore the iron core saturation of the motor;
- (3) The conductivity of the permanent magnet is zero, and the hysteresis and eddy current losses are negligible;
- (4) Upon application of a symmetrical three-phase sinusoidal current in the stator winding, only the sinusoidal magnetic potential distribution will be generated in the air gap.

The voltage equation is:

$$\begin{cases} u_d = R_s i_d + \frac{d\psi_d}{dt} - \omega \psi_q \\ u_q = R_s i_q + \frac{d\psi_q}{dt} + \omega \psi_d \end{cases} \quad (1)$$

The current equation is:

$$\begin{cases} \frac{di_d}{dt} = -\frac{R_s}{L} i_d + \omega i_q + \frac{u_d}{L} \\ \frac{di_q}{dt} = -\frac{R_s}{L} i_q - \omega i_d - \frac{\psi_f}{L} \omega + \frac{u_q}{L} \end{cases} \quad (2)$$

The flux linkage equation is:

$$\begin{cases} \psi_q = L_q i_d + \psi_f \\ \psi_d = L_d i_q \end{cases} \quad (3)$$

Where, u_d and u_q are the voltage components on shaft d and shaft q , R_s is the stator resistance value, i_d and i_q are the current components on shaft d and shaft q , ω is the rotor angular velocity, ψ_d and ψ_q are the components of shaft d and shaft q of the stator flux linkage, L_d and L_q are the inductance components of shaft d and shaft q .

3. Identification of motor parameters by forgetting factor recursive least squares

3.1 Forgetting factor recursive least squares

In the PMSM vector control study, the most commonly used method to identify the motor parameters is the recursive least squares method whose basic principle is to make use of corresponding rules to correct the previously identified data on the basis of the current data when the identification system is in operation, so as to reach the final purpose of reducing data errors^[11]. An advantage of the recursive least squares method is that it is simple and easy to be implemented.

By taking the self-regression model as an example, the differential equation of the system is:

$$y(k) = \varphi^T(k)\theta + \xi(k) \tag{4}$$

Where, $\xi(k)$ is the white noise; $\theta = [a_1, \dots, a_{n_1}, b_0, b_1, \dots, b_{n_2}]^T$ is the actual parameter vector; and $\varphi^T(k) = [-y(k-1), \dots, -y(k-n_1), u(k-d), \dots, u(k-d-n_2)]^T$ is the measurement vector.

Assume $\hat{\theta} = [a_1, \dots, a_{n_1}, \hat{b}_0, \hat{b}_1, \dots, \hat{b}_{n_2}]^T$ is the estimated value of the parameter vector, and the system error is set to be $e(k)$, the objective function will be:

$$J = \sum_{k=1}^j e^2(k) = \sum_{k=1}^j [y(k) - \varphi^T(k)\hat{\theta}]^2 \tag{5}$$

The purpose of the least square method is to calculate the value of $\hat{\theta}$ when the objective function J is minimized.

As can be seen, it needs to process numerous data by the system when using the least square method to identify parameters because each identified data has to be analyzed and received, so a large amount of memory is occupied. In this case, it is unrealistic to identify motor parameters accurately, so the recursive concept is introduced:

The new estimated value $\hat{\theta}(k)$ = the old estimated value $\hat{\theta}(k-1)$ + the corrected term

The recursive formula can be obtained according to the above equation:

$$\begin{cases} \hat{\theta}(k) = \hat{\theta}(k-1) + K(k)[y(k) - \varphi^T(k)\hat{\theta}(k-1)] \\ K(k) = \frac{P(k-1)\varphi(k)}{1 + \varphi^T(k)P(k-1)\varphi(k)} \\ P(k) = [I - K(k)\varphi^T(k)]P(k-1) \end{cases} \tag{6}$$

Where, $P(k) = (\Phi_k^T \Phi_k)^{-1}$, and $K(k) = P(k)\varphi(k)$.

With the recursive method, data is accumulated constantly, which will lead to the “data saturation”. To resolve this problem, the forgetting factor concept is introduced by this article.

The objective function is:

$$J = \sum_{k=1}^L \lambda^{L-k} [y(k) - \varphi^T(k)\hat{\theta}]^2 \tag{7}$$

Where, λ is the forgetting factor, $0 < \lambda < 1$.

As can be seen from formula (6), the forgetting factor recursive least square parameter estimation formula derivation result is shown below:

$$\begin{cases} \hat{\theta}(k) = \hat{\theta}(k-1) + K(k)[y(k) - \varphi^T(k)\hat{\theta}(k-1)] \\ K(k) = \frac{P(k-1)\varphi(k)}{\lambda + \varphi^T(k)P(k-1)\varphi(k)} \\ P(k) = \frac{1}{\lambda} [I - K(k)\varphi^T(k)]P(k-1) \end{cases} \tag{8}$$

Where, the initial value does not change, and the forgetting factor is usually: $0.9 < \lambda < 1$. When $\lambda = 1$, the forgetting factor recursive least square method will become the recursive least square method.

3.2 Online identification of PMSM parameters

According to formula (1) and (2), the motor state equation is:

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_d} & \frac{\omega L_q}{L_d} \\ \frac{\omega L_d}{L_q} & -\frac{R_s}{L_q} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{bmatrix} \begin{bmatrix} u_d \\ u_q - \omega \psi_f \end{bmatrix} \quad (9)$$

By dispersing $\frac{di_d}{dt}$ to $\frac{di_d}{dt} = \frac{i_d(k+1) - i_d(k)}{T}$, the following equations can be got:

$$i_d(k+1) = \left(1 - \frac{R_s T}{L_d}\right) i_d(k) + \frac{\omega L_q T}{L_d} i_q(k) + \frac{T}{L_d} u_d(k)$$

$$i_q(k+1) = -\frac{\omega L_d T}{L_q} i_d(k) + \left(1 - \frac{R_s T}{L_q}\right) i_q(k) + \frac{T}{L_q} (u_q - \omega \psi_f)$$

T is the identification interval.

The least square form of the motor model is:

$$\begin{bmatrix} i_d(k+1) & i_q(k+1) \end{bmatrix} = \begin{bmatrix} i_d(k) & i_q(k) & u_d(k) & u_q(k) - \omega \psi_f \end{bmatrix} \begin{bmatrix} 1 - \frac{R_s T}{L_d} & -\frac{\omega L_d T}{L_q} \\ \frac{\omega L_q T}{L_d} & 1 - \frac{R_s T}{L_q} \\ \frac{T}{L_d} & 0 \\ 0 & \frac{T}{L_q} \end{bmatrix} \quad (10)$$

As can be seen from formula (9), the only unknown quantity is the permanent magnet flux linkage ψ_f which only exists in i_q rather than i_d . Since the $i_d = 0$ vector control strategy is adopted by this article, the above formula can be simplified to:

$$i_d(k+1) = \begin{bmatrix} i_d(k) & i_q(k) \omega(k) & u_d(k) \end{bmatrix} \begin{bmatrix} 1 - \frac{TR_s}{L} \\ T \\ \frac{T}{L} \end{bmatrix} \quad (11)$$

Thus, the motor stator resistance and inductance can be identified by making use of the forgetting factor recursive least square method. By making $a(k) = 1 - \frac{T(k)R_s(k)}{L(k)}$ and $b(k) = \frac{T(k)}{L(k)}$, then:

$$\begin{bmatrix} a(k) \\ T(k) \\ b(k) \end{bmatrix} = \begin{bmatrix} a(k-1) \\ T(k-1) \\ b(k-1) \end{bmatrix} + K(k) \begin{bmatrix} y(k) - \varphi^T(k) \begin{bmatrix} a(k-1) \\ T(k-1) \\ b(k-1) \end{bmatrix} \end{bmatrix} \quad (12)$$

Where, $y(k) = i_d(k+1)$, $\varphi(k) = \begin{bmatrix} i_d \\ i_q(k) \omega(k) \\ u_d(k) \end{bmatrix}$,

$K(k) = \frac{P(k-1)\varphi(k)}{\lambda + \varphi^T(k)P(k-1)\varphi(k)}$, $P(k) = \frac{1}{\lambda} [I - K(k)\varphi^T(k)]P(k-1)$, $P(0) = \alpha I$, and the data range of

α is $10^4 \sim 10^{10}$. The value of a and b can be obtained by making use of the above formula, and R_s and L can be calculated by making use of $a(k)$ and $b(k)$, thus identifying the motor parameters.

4. The model reference adaptive algorithm design

As one of the commonly used algorithms for PMSM position estimation, the advantage of MRAS is that its response speed is fast and it is easy to be realized. The basic principle of MRAS is to construct an adequate parameter model and adjustable model, construct an appropriate adaptive law by taking advantage of the error of two model output physical quantities, and adjust the parameters of the adjustable model continuously, thus reaching the effect of tracking the reference model by the adjustable model to reduce errors [7]. Its basic principle is shown in the figure below.

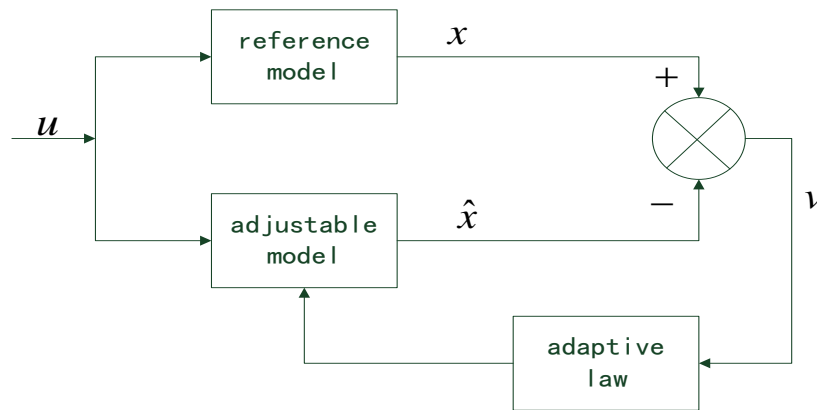


Figure 1 The basic principle diagram of MRAS

4.1 The reference model and adjustable model of MRAS

The parallel structure is adopted. According to the formula (2), the motor stator current model is taken as the adjustable model, and the motor itself as the reference model. Formula (2) can be transformed into:

$$\frac{d}{dt} \begin{bmatrix} i_d + \psi_f \\ i_q \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L} & \omega \\ -\omega & -\frac{R_s}{L} \end{bmatrix} \begin{bmatrix} i_d + \psi_f \\ i_q \end{bmatrix} + \frac{1}{L} \begin{bmatrix} u_d + \frac{R_s \psi_f}{L} \\ u_q \end{bmatrix} \quad (13)$$

By making $i'_d = i_d + \frac{\psi_f}{L}, i'_q = i_q, u'_d = u_d + \frac{R_s \psi_f}{L}, u'_q = u_q$, formula (13) can be simplified to:

$$\frac{di'}{dt} = Ai' + Bu' \quad (14)$$

Where, $i' = \begin{bmatrix} i'_d \\ i'_q \end{bmatrix}, u' = \begin{bmatrix} u'_d \\ u'_q \end{bmatrix}, A = \begin{bmatrix} -\frac{R_s}{L} & \omega \\ -\omega & -\frac{R_s}{L} \end{bmatrix}, B = \frac{1}{L}$.

As can be learnt from the above formula, the current equation contains the rotational speed information, so it can be taken as the adjustable model, and the PMSM itself as the reference model. The current estimated value model equation is:

$$\frac{d}{dt} \begin{bmatrix} \hat{i}'_d \\ \hat{i}'_q \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L} & \hat{\omega} \\ -\hat{\omega} & -\frac{R_s}{L} \end{bmatrix} \begin{bmatrix} \hat{i}'_d \\ \hat{i}'_q \end{bmatrix} + \frac{1}{L} \begin{bmatrix} u'_d \\ u'_q \end{bmatrix} \quad (15)$$

The current error equation can be obtained:

$$\begin{bmatrix} \frac{de_d}{dt} \\ \frac{de_q}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L} & \omega \\ -\omega & -\frac{R_s}{L} \end{bmatrix} \begin{bmatrix} e_d \\ e_q \end{bmatrix} - J(\omega - \hat{\omega}) \begin{bmatrix} \hat{i}'_d \\ \hat{i}'_q \end{bmatrix} \tag{16}$$

Where, $e_d = i'_d - \hat{i}'_d, e_q = i'_q - \hat{i}'_q, J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

By simplifying formula (16):

$$\begin{aligned} \frac{de}{dt} &= A_e e - w \\ v &= De \end{aligned} \tag{17}$$

Where, $A_e = A = \begin{bmatrix} -\frac{R_s}{L} & \omega \\ -\omega & -\frac{R_s}{L} \end{bmatrix}, e = \begin{bmatrix} e_d \\ e_q \end{bmatrix}, w = (\omega - \hat{\omega}) \begin{bmatrix} -\hat{i}'_q \\ -\hat{i}'_d \end{bmatrix}$. Taking $D = I$, then $v = De = Ie = e$.

4.2 Determination of the adaptive law

In MRAS algorithm, determination of the adaptive law is one of the core issues. In this paper, the POPOV ultra-stable theory is used for the design of the adaptive law. By combining the schematic diagram, the POPOV ultra-stable theory is briefly introduced. As shown in figure (2), the system is composed by thenon-linear time-changing frame and the linear constant-stable frame, the former of which is the feedback session. When the transfer function of the linear constant-stable frame is a positive real function, the adaptive algorithm obtained according to the POPOV ultra-stable theory will allow the system to remain stable.

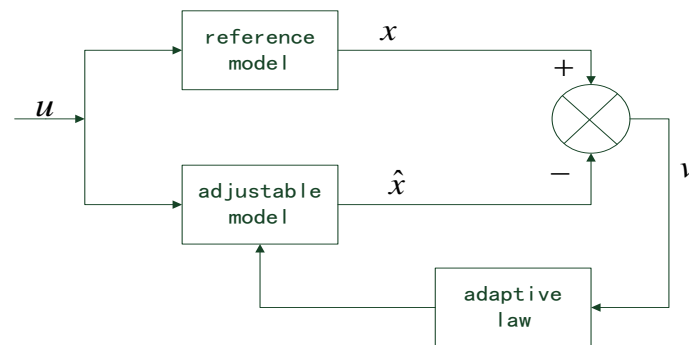


Figure 2 Schematic diagram of the POPOV ultra-stable theory

The core of POPOV ultra-stable theory is to find the relationship between the adaptive vector v and feedback vector w . The relationship between the two can be determined through the POPOV ultra-stable theorem. If the system meets:

- (1) The feedback session meet the POPOV inequality: $\eta(0, t_0) = \int_0^{t_0} v^T w dt \geq -\gamma_0^2, \forall t_0 \geq 0, \gamma_0^2$ is any positive number;
- (2) The transfer matrix is a strictly positive real matrix.

Then the system is stable.

As can be seen, $v = De = Ie = e = i' - \hat{i}', w = (\hat{\omega} - \omega) J \hat{i}'$. The adaptive law of the PMSM MRAS can be obtained by making use of the POPOV integral inequality reverse calculation:

$$\hat{\omega} = \int_0^t k_i \left[i_d \hat{i}_q - i_q \hat{i}_d - \frac{\psi_f}{L} (i_q - \hat{i}_q) \right] d\tau + k_p \left[i_d \hat{i}_q - i_q \hat{i}_d - \frac{\psi_f}{L} (i_q - \hat{i}_q) \right] + \hat{\omega}(0) \quad (18)$$

Where, $k_i > 0, k_p > 0$.

By inputting the motor parameters obtained through the forgetting factor recursive least squares to MRAS, the adverse effect on the motor parameter changes can be minimized to a large extent.

After getting the rotor speed, the rotor position $\theta = \int \omega dt$ can be obtained.

5. simulink simulation results and analysis

The simulink simulation model figure of the PMSM vector control system based on the improved MRAS is shown below.

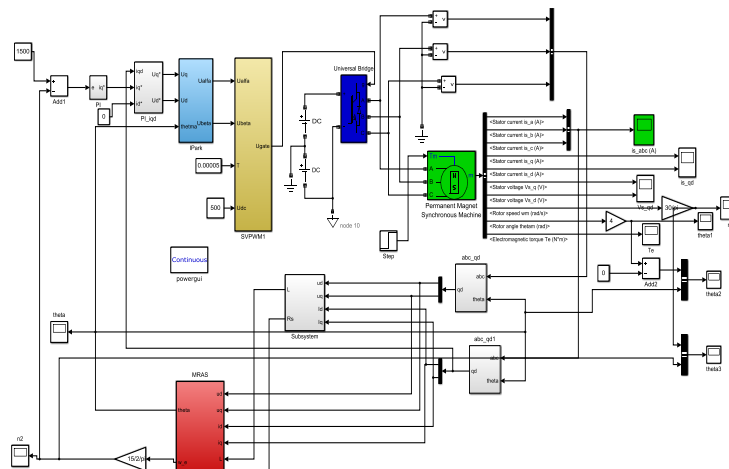


Figure 3 The PMSM vector control system simulation model based on the improved MRAS

To verify the improved MRAS performance designed by this article, the Simulink module in MATLAB2014a is used for simulation verification. The vector control system is applied, and the specific PMSM parameter setting is shown in the table below.

Table 1 The PMSM simulation parameter setting

Rated speed	1500 r / min
Rated power	550 W
Rated voltage	220 V
Initial inductance	8.5 mH
Initial resistance	2.875 Ω

The simulation time is set to be 0.4s. In the practical application of motors, the motor stator resistance and inductance almost do not change within 0.4s, so the motor parameter values are not changed in one simulation in order to make it closer to actual use, instead, the twice simulation way is used to verify the algorithms performance designed by the article. In the following analysis, the vector control system simulation contrast figure of the ordinary model reference adaptive algorithm and the improved model reference adaptive algorithm after the motor parameters change is given, in order to verify the performance of the improved MRAS.

5.1 Analysis of the ordinary MRAS vector control system simulation result after parameters change ($L = 7.5mH, R_s = 3.25\Omega$)

Figure (4) shows the simulation result contrast of the actual speed and estimated speed of ordinary MRAS. As can be seen, the actual speed and estimated speed of the vector control system fluctuate between 1300r / min and 1600r / min, while the actually set speed of the system is 1500r / min. The error is unacceptable in actual application. Figure (5) is the angle contrast figure obtained by the simulation

system, which reflects the distance between the rotor's estimated position and its actual position. The figure shows that there is huge difference between the estimated rotor angle and the actual rotor position, that is, the estimated value following the actual value is not ideal. Seen from the simulation result, the position estimation error is huge for the ordinary MRAS vector control system after the motor parameter changes because the parameters in MRAs do not change. The overall performance of the motor control system is significantly reduced, which cannot meet the high precision requirement of motors by modern industry.

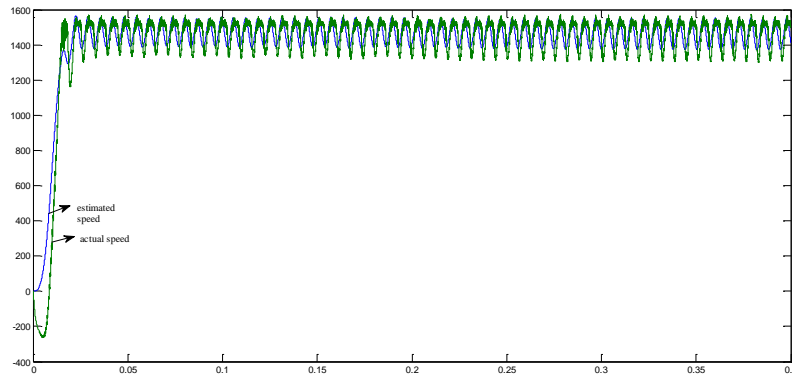


Figure 4 Contrast of the actual speed and estimated speed of ordinary MRAS

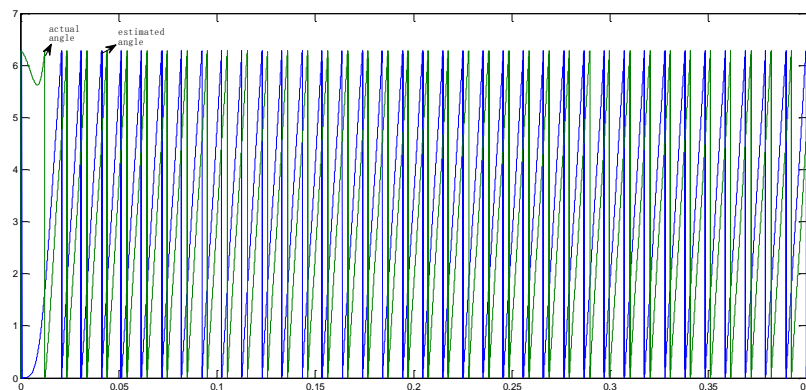


Figure 5 Contrast of the actual angle and estimated angle of ordinary MRAS

5.2 Analysis of the improved MRAS vector control system simulation result after parameters change ($L = 7.5mH$, $R_s = 3.25\Omega$)

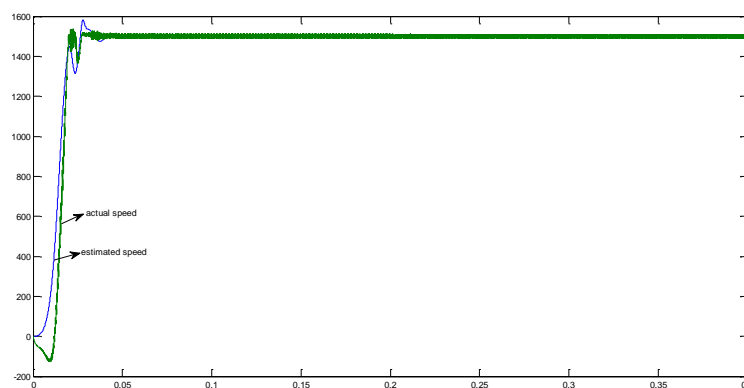


Figure 6 Contrast of the actual speed and estimated speed of ordinary MRAS

Figure (6) shows the contrast result of the actual speed and estimated speed of MRAS with parameter identification modules. As can be seen, the actual speed reaches the rated speed when $t = 0.04s$, the estimated speed is stable at $1500r/min$ when $t = 0.03s$. As can be seen, the system can make fast response. The curves of actual speed and estimated speed after $t = 0.04s$ can basically completely coincide, indicating the estimated speed can follow the estimated speed fast and the estimation effect

is positive. Figure (7) is the contrast of the rotor's estimated position and actual position. As can be seen, the estimated position basically coincides with the actual position when $t = 0.15s$, thus realizing the precise estimation. The above two figures have verified that improved MRAS has favorable position estimation performance. Adverse influence caused by PMSM parameter changes can basically be overcome, and the superior system performance is available.

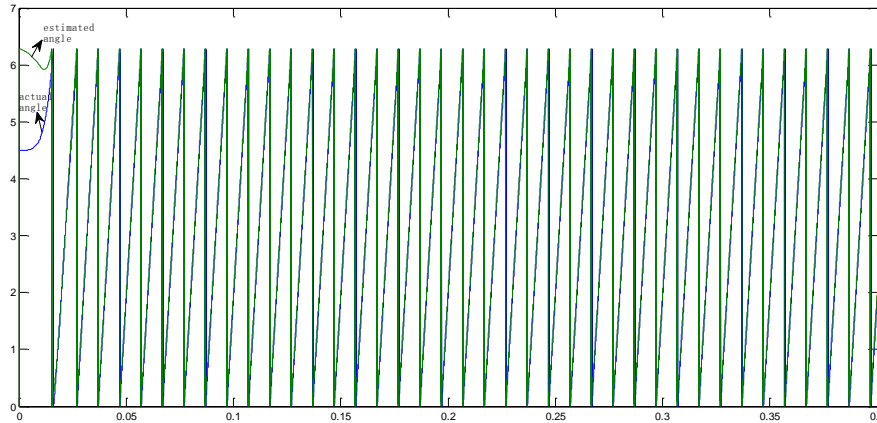


Figure 7 Contrast of the actual angle and estimated angle of improved MRAS

6. Conclusions

The innovation point of this paper is to integrate the forgetting factor recursive least square method with MRAS for online identification of motor parameters. The identified motor parameters are reported to MRAS timely, thus resolving the adverse influence of motor parameter changes in the PMSM running process effectively. The improved MRAS algorithm can help to improve the motor rotor position detection precision significantly, thus enhancing the PMSM vector control system performance to a large extent. So, it is with high research value.

References

- [1] Xu Junrui. Study on the Application of Permanent Magnet Synchronous Motors in Parameter Identification and Its Control [D]. Harbin: Harbin Institute of Technology: 2011
- [2] Liu Chencheng. Study and Realization of MRAS-Based Permanent Magnet Synchronous Motor Control System [D]. Chengdu: Electrical Machine Technology University, 2012
- [3] Yoon, S., Morimoto, and K. Ide, High bandwidth sensorless algorithm for AC machines based on square-wave-type voltage injection. IEEE Trans. Ind. Appl., vol.47, no.3, pp.1361-1370, May/June. 2011.
- [4] Xu J.F., Wang F.Y., Xie S.F., Xu J.P., Feng J.H. A new control method for permanent magnet synchronous machines with observer [C]. In: Proceedings of the IEEE 35th Annual Power Electronics Specialists Conference, 2001, 2: 1404-1408.
- [5] Gaur P., Aziz M., Mittal A.P., Singh B. Sensorless control schemes for permanent magnet synchronous motor [C]. In: Proceeding of the IEEE Region 10 Conference on Computers, Communications, Control and Power Engineering, 2005, 1-4.
- [6] Baik I.C., Kim K.H., Youn M.J. Robust nonlinear speed control of pm synchronous motor using adaptive and sliding mode control techniques [J]. IEEE Proceedings-Electric Power Applications. 1998, 145(4): 369-376.
- [7] Xu Zhenyu. Study on the Permanent Magnet Synchronous Motor Direct Torque Control Based on Model Reference Adaptation [D]. Changsha: Hunan University, 2012.
- [8] Li Cheng. Study on the Parameter Identification of Permanent Magnet Synchronous Motors [D]. Xi'an: Xi'an University, 2014.
- [9] Tang Renyuan. Modern Permanent Magnet Motor Theory and Design [M]. Beijing: China Machine Press, 1997.

- [10] Guo Xing. Study on the Rotor Speed Identification of Permanent Magnet Synchronous Motors Based on Improved MRAS [D]. Changsha: Hunan University, 2013.
- [11] Yanhui He, Yupeng Feng, and Yue Wang. Estimating the electrical parameters of induction motors at standstill using RLS method [J]. IEEE Power Electronics for Distributed Generation Systems. 2010:358-363.